UNIVERSITI MALAYSIA PERLIS

Peperiksaan Akhir Semester Pertama
Sidang Akademik 2025/2026

Januari - Februari 2026

EMJ45303 – Advanced Power Electronics
[*Elektronik Kuasa Lanjutan*]

Masa : 3 jam

Please make sure that this question paper has **EIGHT (8)** printed pages including the front page and a separate set of **ONE (1) APPENDIX BOOK** before you start the examination.
*[Sila pastikan kertas soalan ini mengandungi **LAPAN (8)** muka surat yang bercetak termasuk muka hadapan dan satu set berasingan sebanyak **SATU (1) BUKU LAMPIRAN** sebelum anda memulakan peperiksaan ini.]*

This question paper has FOUR (4) questions. Answer **ALL** questions. Each question contributes 25 marks.
*[Kertas soalan ini mengandungi EMPAT (4) soalan. Jawab **SEMUA** soalan. Markah bagi tiap-tiap soalan adalah 25 markah.]*

Dicetak oleh Unit Peperiksaan & Pengijazahan, Bahagian Pengurusan Akademik, Pusat Pengurusan Akademik UniMAP

Question 1
[Soalan 1]

A new industrial actuator system requires a single-phase, fully controlled full-wave AC–DC converter to provide regulated DC power during operation. The converter is connected to a 240 V_{rms}, 50 Hz single-phase AC supply and drives a load consisting of a 10 Ω resistor connected in series with a 50 mH inductor. Your objective is to evaluate the steady-state behavior of the converter at two firing (delay) angles to verify load performance and grid power-quality compliance.
[Sistem penggerak perindustrian baharu memerlukan penukar AU–AT gelombang penuh fasa tunggal yang dikawal sepenuhnya untuk membekalkan kuasa AT yang dikawal selia semasa operasi. Penukar disambungkan kepada bekalan AU fasa tunggal 240 V_{pmkd}, 50 Hz sumber AT satu-fasa dan memacu satu beban yang terdiri daripada perintang 10 Ω disambung secara bersiri dengan induktor 50 mH. Objektif anda adalah untuk menilai kelakuan keadaan mantap penukar pada dua sudut picuan (lengah) untuk mengesahkan prestasi beban dan pematuhan kualiti kuasa grid.]

(a) Initially, the firing angle is set to α = 45° to provide a suitable DC voltage for normal operation of the actuator at a moderate load level. Evaluate the average output voltage, average load current, RMS load voltage, RMS load current, real power absorbed by the load and the power factor.
[Pada mulanya, sudut picuan ditetapkan kepada α = 45° untuk menyediakan voltan AT yang sesuai untuk operasi normal penggerak pada tahap beban sederhana. Nilaikan voltan keluaran purata, arus beban purata, voltan beban PMKD, arus beban PMKD, kuasa sebenar yang diserap oleh beban dan faktor kuasa.]

(20 Marks / *Markah*)

(b) Later, during a reduced-load operating period, the firing angle is increased to α = 75° in order to lower the DC output voltage and reduce power delivered to the actuator. Derive the time-domain expression for the load current and numerically determine the conduction interval and the extinction angle β.
[Kemudian, semasa tempoh operasi beban berkurangan, sudut picuan ditingkatkan kepada α = 75° untuk menurunkan voltan keluaran AT dan mengurangkan kuasa yang dihantar kepada penggerak. Terbitkan ungkapan domain masa untuk arus beban dan tentukan secara numerikal selang masa pengaliaran dan sudut kepupusan β.]

(5 Marks / *Markah*)

Question 2
[Soalan 2]

(a) Using appropriate circuit schematics, derive the steady-state voltage conversion ratio (transfer function) for a buck DC-DC converter operating in continuous conduction mode (CCM). Assume ideal components and apply the principle that the net change in inductor current over one switching cycle is zero under steady-state conditions. Your derivation should clearly justify each step and indicate the relationship between the input voltage, output voltage, and duty cycle.
[Menggunakan litar skema yang sesuai, terbitkan nisbah penukaran voltan keadaan mantap (fungsi pemindahan) untuk penukar AT-AT buck yang beroperasi dalam mod pengaliran berterusan (CCM). Andaikan komponen ideal dan gunakan prinsip bahawa perubahan bersih dalam arus induktor sepanjang satu kitaran pensuisan adalah sifar di bawah keadaan mantap. Terbitan anda haruslah mewajarkan setiap langkah dengan jelas dan menunjukkan hubungan antara voltan masukan, voltan keluaran dan kitar tugas.]

(8 Marks / *Markah*)

(b) A 48 V DC bus is used in an industrial robot cell, and a buck DC–DC converter must be designed to supply an 18 V output that powers several motion-control boards represented by a 10 Ω load under rated conditions. The converter should meet the following design requirements: an output voltage of 18 V, a constant input voltage of 48 V, and a load resistance of 10 Ω. The peak-to-peak output voltage ripple must not exceed 0.5% of the output voltage, and the inductor current must remain continuous at the rated load. Ideal devices may be assumed for this analysis.
[Bas AT 48 V digunakan dalam sel robot perindustrian, dan penukar AT-AT buck mesti direka bentuk untuk membekalkan keluaran 18 V yang menguasakan beberapa papan kawalan gerakan yang diwakili oleh beban 10 Ω di bawah keadaan dinilai. Penukar hendaklah memenuhi keperluan reka bentuk berikut: voltan keluaran 18 V, voltan masukan malar 48 V, dan rintangan beban 10 Ω. Riak voltan keluaran puncak-ke-puncak tidak boleh melebihi 0.5% daripada voltan keluaran, dan arus induktor mesti kekal berterusan pada beban dinilai. Peranti ideal boleh diandaikan untuk analisis ini.]

(i) You are required to design the converter for a switching frequency of 40 kHz and determine the duty ratio, the minimum inductance required to maintain continuous conduction along with a practical selected value, the inductor ripple current, and the maximum, minimum, and rms inductor currents. Additionally, you must calculate the output capacitor value that satisfies the ripple requirement along with the capacitor RMS current, and determine the peak voltage stress across the switch, diode, inductor, and capacitor.
[Anda dikehendaki mereka bentuk penukar untuk frekuensi pensuisan 40 kHz dan menentukan nisbah tugas, kearuhan minimum yang diperlukan untuk mengekalkan pengaliran berterusan berserta nilai praktikal yang dipilih, arus riak induktor dan arus induktor maksimum, minimum dan pmkd. Selain itu, anda mesti mengira nilai kapasitor keluaran yang memenuhi keperluan riak berserta arus PMKD kapasitor dan menentukan tegasan voltan puncak merentasi suis, diod, induktor dan kapasitor.]

(12 Marks / *Markah*)

(ii) For cost considerations, the purchasing department intends to replace the originally selected inductor with a standard 68 μH component while keeping all other parameters unchanged. You must therefore recalculate the inductor current ripple and determine the new minimum inductor current. Based on these results, clearly indicate whether the converter will continue to operate in continuous-conduction mode (CCM) at the rated load, and discuss at least one practical consequence if the converter transitions into discontinuous-conduction mode (DCM) instead.
[Untuk pertimbangan kos, jabatan pembelian berhasrat untuk menggantikan induktor yang dipilih pada asalnya dengan komponen standard 68 μH sambil mengekalkan semua parameter lain tidak berubah. Anda mesti mengira semula riak arus induktor dan menentukan arus induktor minimum yang baharu. Berdasarkan keputusan ini, nyatakan dengan jelas sama ada penukar akan terus beroperasi dalam mod pengaliran berterusan (CCM) pada beban yang dinilai, dan bincangkan sekurang-kurangnya satu akibat praktikal jika penukar beralih ke mod pengaliran tidak berterusan (DCM).]

(5 Marks / *Markah*)

Question 3
[Soalan 3]

(a) In inverter design, understanding the relationship between switching patterns and the resulting output waveforms is essential for evaluation system performance. **Figure 3** show power stage output waveforms for a single-phase full-bridge inverter circuit using fixed switching scheme. Given the output frequency, f_o is 50 Hz and the series-inductive load of R is 25 Ω and L is 40 mH, analyse the total output power absorbed by the load, $P_{o(total)}$ and total harmonic distortion (%THD) of output current. Harmonic components up to the 9^{th} order may be considered.
*[Dalam rekabentuk penyongsang, pemahaman terhadap hubungan antara corak pensuisan dan gelombang keluaran yang terhasil adalah penting untuk menilai prestasi sistem. **Rajah 3** menunjukkan gelombang keluaran peringkat kuasa bagi sebuah litar penyongsang jambatan penuh fasa tunggal menggunakan skim pensuisan tetap. Diberi frekuensi keluaran, f_o adalah 50 Hz dan beban sesiri-induktif iaitu R adalah 25 Ω dan L adalah 40 mH, analisis jumlah kuasa keluaran yang diserap oleh beban, $P_{o(total)}$ serta jumlah herotan harmonik (%THD) arus keluaran. Pertimbangkan komponen harmonik sehingga tertib ke-9.]*

(15 Marks / *Markah*)

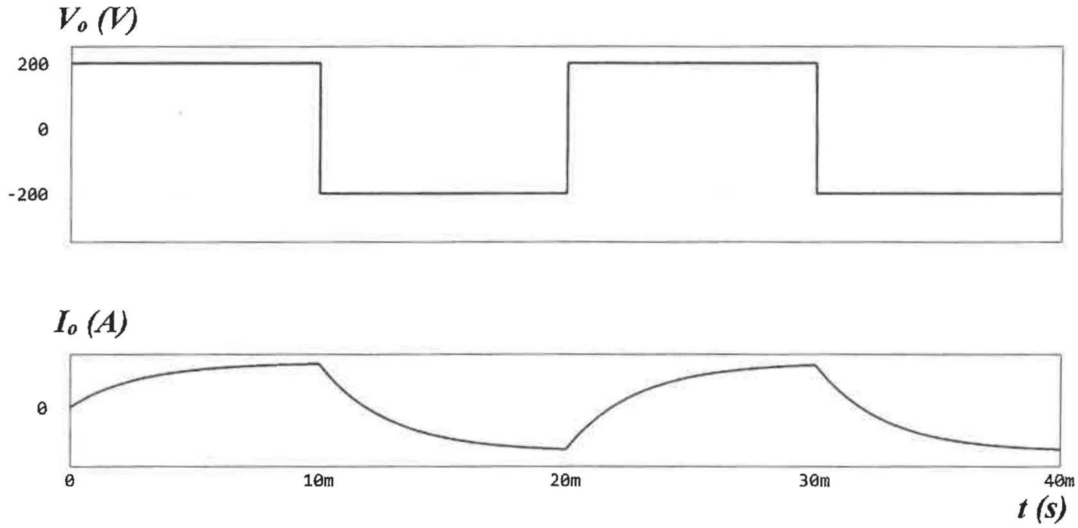

Figure 3: Power stage output waveforms
[Rajah 3: Gelombang-gelombang keluaran peringkat kuasa]

(b) The bipolar pulse width modulation (PWM) is then used as the switching scheme for the single-phase inverter. Considering a modulation index, m_a is 0.9, switching frequency, f_{sw} is 750 Hz and fundamental frequency, f_o is 50 Hz, investigate the total harmonic distortion (%THD) of output current. DC-link voltage and load are the same as stated in Question 3(a). **Table 3** may be used to assist your analysis. State your findings.

*[Modulasi lebar denyut bipolar (PWM) kemudiannya digunakan sebagai skim pensuisan penyongsang satu-fasa. Dengan mengambilkira indeks modulasi, m_a adalah 0.9, frekuensi pensuisan, f_{sw} adalah 750 Hz dan frekuensi asas, f_o adalah 50 Hz, nilaikan jumlah herotan harmonik (%THD) arus keluaran. Voltan-pautan AT dan beban adalah sama seperti yang dinyatakan dalam Soalan 3(a). **Jadual 3** boleh digunakan untuk membantu analisis anda. Nyatakan dapatan anda.]*

(10 Marks / *Markah*)

Table 3: Normalized Fourier coefficients V_n/V_{DC} for bipolar PWM

[Jadual 3: Pekali Fourier ternormal V_n/V_{DC} bagi PWM bipolar]

	m_a=1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
n =1	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
$n= m_f$	0.68	0.71	0.82	0.92	1.01	1.08	1.15	1.20	1.24	1.27
$n = m_f±2$	0.32	0.27	0.22	0.17	0.13	0.09	0.06	0.03	0.02	0.00

Question 4
[Soalan 4]

The standalone system in **Figure 4(a)** operates with photovoltaic (PV) modules as the primary energy source, generating a voltage, V_{PV} to the input of DC-DC converter. The regulated output of DC-DC converter, V_{out}, then establishes the DC-link voltage, V_{DC}. The stabilized DC-link voltage is then fed into a three-phase inverter which performs DC-AC power conversion to supply three-phase local loads.
*[Sistem kendiri dalam **Rajah 4(a)** beroperasi menggunakan modul fotovolta (PV) sebagai sumber tenaga utama yang menjana voltan, V_{PV} kepada masukan penyongsang AT-AT. Hasil keluaran terkawal daripada penukar AT-AT, V_{out}, kemudiannya membentuk voltan pautan-AT, V_{AT}. Voltan pautan-AT yang telah distabilkan ini seterusnya disalurkan kepada penukar tiga-fasa yang menjalankan penukaran kuasa AT-AU untuk dibekalkan kepada beban tiga-fasa setempat.]*

(a) Under peak operation conditions, the inverter exhibits unbalanced phase voltages (V_{an}, V_{bn} and V_{cn}) as shown in **Figure 4(b)**. Given the fundamental frequency, f is 50 Hz, DC-link voltage, V_{DC} is 600 V and load of one of the phases, R_b is 40 Ω. Predict the values of line-to-line voltages (V_{ab}, V_{bc} and V_{ca}) of the inverter. You need to provide analysis of equivalent circuits in each conduction sequence (from 0 to 2π).
*[Semasa operasi puncak, penyongsang mengalami keadaan voltan fasa tidak seimbang (V_{an}, V_{bn} and V_{cn}) seperti ditunjukkan dalam **Rajah 4(b)**. Diberi frekuensi asas, f adalah 50 Hz, voltan pautan-AT, V_{DC} adalah 600 V dan salah satu beban fasa mempunyai rintangan, R_b ialah 40 Ω. Ramalkan nilai voltan talian-ke-talian penyongsang (V_{ab}, V_{bc} and V_{ca}). Anda perlu memberikan analisis mengenai litar setara pensuisan bagi setiap jujukan pengaliran sepanjang satu kitar penuh (dari 0 hingga 2π).]*

(20 Marks / *Markah*)

(b) Evaluate how unbalanced loads alter the harmonic content in the phase and line voltages of a three-phase inverter. Explain how these harmonic distortions affects power quality and the overall system reliability.
[Nilaikan induktor beban tidak seimbang terhadap kandungan harmonik dalam voltan fasa dan voltan talian bagi sebuah penyongsang tiga fasa. Terangkan bagaimana gangguan-gangguan harmonik ini boleh menjejaskan kualiti kuasa serta kebolehharapan keseluruhan sistem.]

(5 Marks / *Markah*)

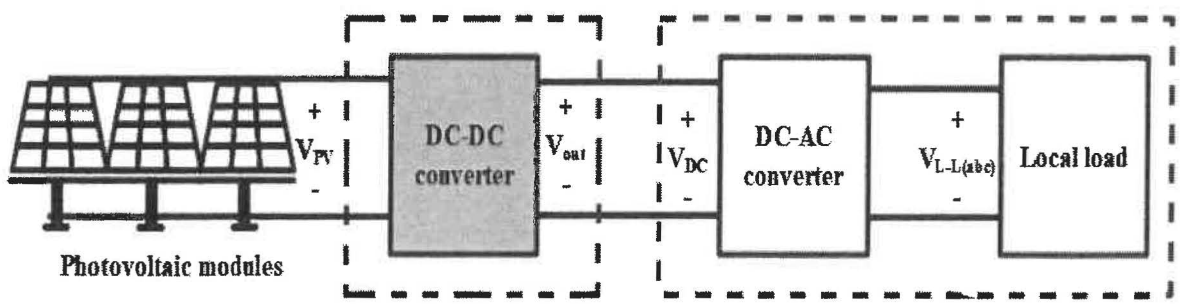

Figure 4(a): Standalone Photovoltaic System
[Rajah 4(a): Sistem Fotovolta Kendiri]

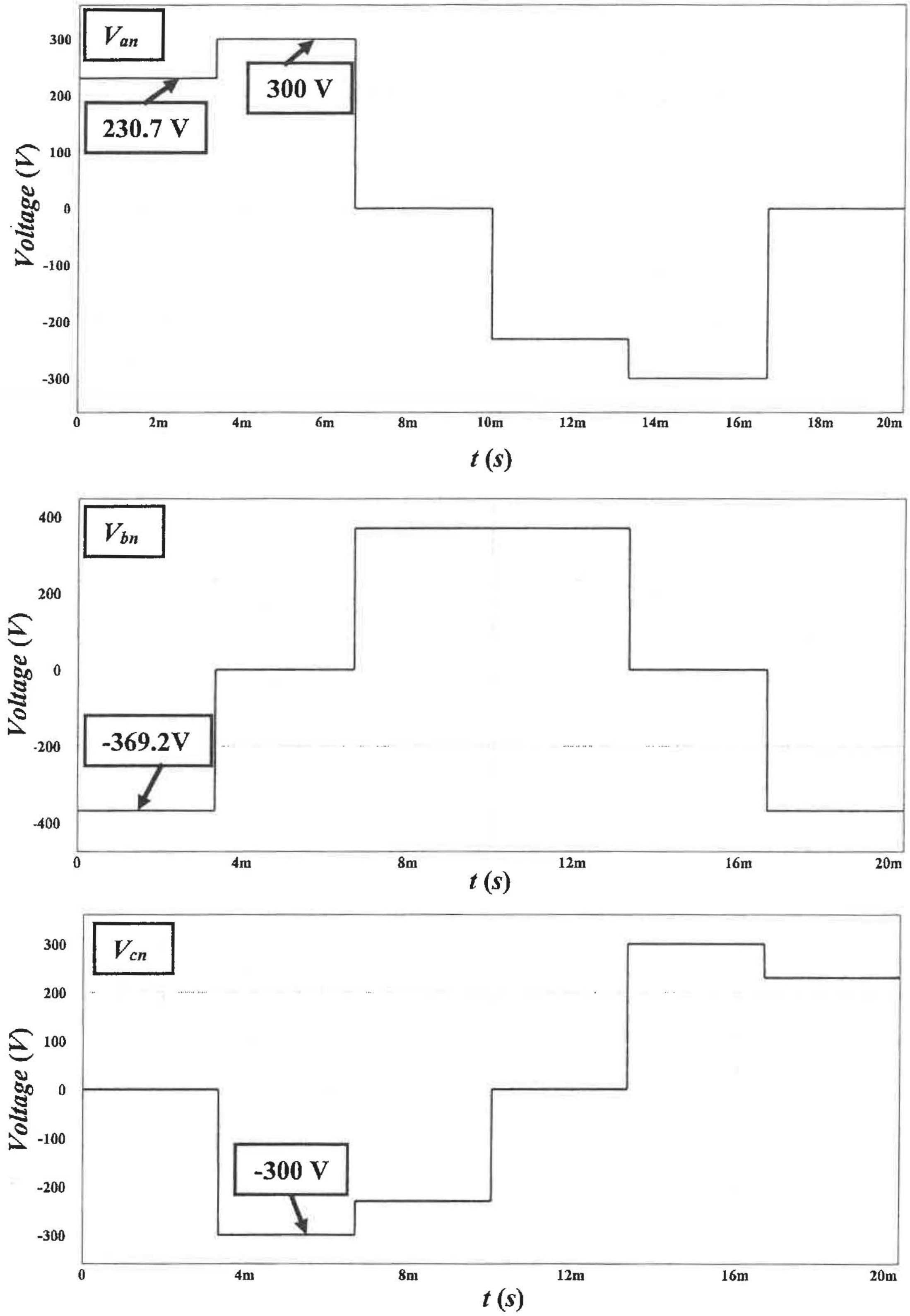

Figure 4(b): Distorted phase voltage waveforms

[Rajah 4(b): Gelombang-gelombang voltan fasa terherot]

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UNIVERSITI MALAYSIA PERLIS

Peperiksaan Semester Satu
Sidang Akademik 2025 / 2026

Januari - Februari 2026

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[*Elektronik Kuasa Lanjutan*]

Masa: 3 jam

APPENDIX
[LAMPIRAN]

Please make sure that this appendix book has **ELEVEN (11)** printed pages and this front page before you start the examination.
*[[Sila pastikan buku lampiran ini mengandungi **SEBELAS (11)** muka surat yang bercetak termasuk muka hadapan sebelum anda memulakan peperiksaan.]*

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List of Formula

1. Single Phase Rectifier

1.1 Uncontrolled Rectifier

1.1.1 Resistive Load

$$V_o = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{\pi}\Big|_{\text{halfwave}} \quad (1)$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 \, d(\omega t)} = \frac{V_m}{2}\Big|_{\text{halfwave}} \quad (2)$$

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}\Big|_{\text{halfwave}} \quad (3)$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R}\Big|_{\text{halfwave}} \quad (4)$$

$$P_{dc} = \frac{1}{2\pi}\int_0^{\pi} \frac{V_m^2}{R} \sin^2(\omega t) d(\omega t) = \frac{V_m^2}{4R}\Big|_{\text{halfwave}} = I_{rms}^2 R \quad (5)$$

$$pf = \frac{P}{S} = \frac{P_{dc}}{V_{s,rms} I_{s,rms}} \quad (6)$$

$$V_o = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{2V_m}{\pi}\Big|_{\text{fullwave}} \quad (7)$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 \, d(\omega t)} = \frac{V_m}{\sqrt{2}}\Big|_{\text{halfwave}} \quad (8)$$

$$I_o = \frac{V_o}{R} = \frac{2V_m}{\pi R}\Big|_{\text{fullwave}} \quad (9)$$

$$I_{rms} = \frac{V_m}{\sqrt{2}R} = \frac{I_m}{\sqrt{2}}\Big|_{\text{fullwave}} \quad (10)$$

$$P_{dc} = \frac{1}{\pi}\int_0^{\pi} \frac{V_m^2}{R}\sin^2(\omega t)d(\omega t) = \left.\frac{V_m^2}{2R}\right|_{\text{fullwave}} \qquad (11)$$
$$= I_{rms}^2 R$$

1.1.2 Resistive-Inductive Loads (Halfwave)

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\tau} & 0 \le \omega t \le \beta \\ 0 & \beta \le \omega t \le 2\pi \end{cases} \qquad (12)$$

$$A = \frac{V_m}{Z}\sin\theta \qquad (13)$$

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad (14)$$

$$\theta = \tan^{-1}\frac{\omega L}{R} \qquad (15)$$

$$\tau = \frac{\omega L}{R} \qquad (16)$$

$$\beta \approx \pi + \theta \quad \rightarrow L \Box R \qquad (17)$$

$$V_o = \frac{V_m}{2\pi}\int_0^{\beta}\sin(\omega t)d(\omega t) = \frac{V_m}{2\pi}(1 - \cos\beta) \qquad (18)$$

$$V_{rms} = V_m\sqrt{\frac{1}{2\pi}\int_0^{\beta}\sin^2(\omega t)d(\omega t)}$$
$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\beta - \frac{\sin 2\beta}{2}\right]} \qquad (19)$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)d(\omega t)}$$
$$\approx \frac{V_m}{Z}\sqrt{\frac{\delta}{4\pi} - \frac{\sin(\delta)\times\cos(\delta+\theta)}{4\pi\times\cos(\theta)}} \qquad (20)$$

$$I_{o,avg} = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)d(\omega t)$$

$$= \frac{V_m}{Z}\left[\frac{1}{\pi}\right]\frac{\sin\left[\frac{\delta}{2}\right]}{\cos(\theta)} \approx \frac{V_m}{R}\left[\frac{1-\cos(\beta)}{2\pi}\right] \tag{21}$$

$$P_{dc} = I_{rms}^2 R \tag{22}$$

$$pf = \frac{P_{dc}}{V_s I_{rms}} \tag{23}$$

1.2 Controlled Rectifier

1.2.1 Resistive Load

$$V_o = \frac{V_m}{2\pi}\int_{\alpha}^{\pi}\sin(\omega t)d(\omega t) = \frac{V_m}{2\pi}(1+\cos\alpha)\Big|_{halfwave} \tag{24}$$

$$I_o = \frac{V_o}{R} = \frac{V_m}{2R\pi}(1+\cos\alpha)\Big|_{halfwave} \tag{25}$$

$$I_{o,rms} = \frac{V_m}{2R}\sqrt{1-\frac{\alpha}{\pi}+\frac{\sin(2\alpha)}{2\pi}}\Big|_{halfwave} \tag{26}$$

$$V_o = \frac{V_m}{\pi}\int_{\alpha}^{\pi}\sin(\omega t)d(\omega t) = \frac{V_m}{\pi}(1+\cos\alpha)\Big|_{fullwave} \tag{27}$$

$$I_o = \frac{V_o}{R} = \frac{V_m}{R\pi}(1+\cos\alpha)\Big|_{fullwave} \tag{28}$$

$$I_{o,rms} = \frac{V_m}{R}\sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\sin(2\alpha)}{4\pi}}\Big|_{fullwave} \tag{29}$$

1.2.2 Resistive-Inductive Loads

$$i(\omega t) = \frac{V_m}{Z}\sin(\omega t-\theta) + Ae^{-\omega t/\tau} \qquad ,\alpha \le \omega t \le \beta\Big|_{\text{halfwave}} \tag{30}$$

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t-\theta) - \sin(\alpha-\theta)e^{-(\omega t-\alpha)/\omega\tau}\right] \qquad ,\alpha \le \omega t \le \beta\Big|_{\text{halfwave}} \tag{31}$$

$$A = \left(-\frac{V_m}{Z} \sin(\alpha - \theta) \right) e^{\alpha/\omega\tau} \bigg|_{\text{halfwave}} \quad (32)$$

$$I_{o,avg} = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta) \bigg|_{\text{halfwave}} \quad (33)$$

$$I_{rms} = \frac{V_m}{Z}\sqrt{\frac{1}{2\pi}(\beta-\alpha) - \frac{\sin(\beta-\alpha)\cos(\alpha+\theta+\beta)}{\cos\theta}} \bigg|_{\text{halfwave}} \quad (34)$$

Full-wave, RL load Continuous

$$\alpha \le \tan^{-1}\left(\frac{\omega L}{R}\right) \quad (35)$$

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)d(\omega t) = \frac{2V_m}{\pi}\cos\alpha \quad (36)$$

$$I_0 = \frac{V_o}{R} \quad (37)$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$
$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right] \quad (38)$$
$$n = 2,4,6,\ldots$$

$$V_n = \sqrt{a_n^2 + b_n^2} \quad (39)$$

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega_o L|} \quad (40)$$

$$I_{rms} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2} \quad (41)$$

2. DC-DC Converter

2.1 Buck Converter

$$D = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T} \qquad (42)$$

$$V_o = DV_s \qquad (43)$$

$$I_L = I_o = \frac{V_o}{R} \qquad (44)$$

$$I_{L(\max)} = I_L + \frac{\Delta i_L}{2} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$
$$I_{L(\min)} = I_L - \frac{\Delta i_L}{2} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right) \qquad (45)$$

$$\Delta i_L = \left(\frac{V_s - V_o}{L}\right)DT = -\left(\frac{V_o}{L}\right)(1-D)T \qquad (46)$$

$$L = \left(\frac{V_s - V_o}{\Delta i_L}\right)DT = -\left(\frac{V_o}{\Delta i_L}\right)(1-D)T \qquad (47)$$

$$L_{\min} = \frac{(1-D)R}{2f} \qquad (48)$$

$$I_{L,rms} = \sqrt{I_L^2 + \left(\frac{\Delta i_L / 2}{\sqrt{3}}\right)^2} \qquad (49)$$

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} \qquad (50)$$

$$C = \frac{1-D}{8L(\Delta V_o / V_o)f^2} \qquad (51)$$

$$i_{C,rms} = \frac{\Delta i_L / 2}{\sqrt{3}} \qquad (52)$$

$$V_{o(DCM)} = V_s\left(\frac{D}{D + D_1}\right)$$
$$D_{1(DCM)} = \frac{-D + \sqrt{D^2 + 8L/RT}}{2} \qquad (53)$$

$$I_{L(\max),DCM} = \Delta i_L = \left(\frac{V_s - V_o}{L}\right) DT = \frac{V_o D_1 T}{L} \quad (54)$$

2.2 Boost Converter

$$D = 1 - \frac{V_s}{V_o} \quad (55)$$

$$I_L = \frac{V_s}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s} \quad (56)$$

$$I_{L(\max)} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L}$$
$$I_{L(\min)} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \quad (57)$$

$$L_{\min} = \frac{RT}{2}(1-D)^2 D \quad (58)$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \quad (59)$$

$$V_{o(DCM)} = V_s \left(\frac{D + D_1}{D_1}\right) \quad (60)$$

$$D_{1(DCM)} = \left(\frac{V_o}{V_s}\right)\left(\frac{2L}{RDT}\right) \quad (61)$$

$$\frac{V_{o(DCM)}}{V_s} = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2D^2 RT}{L}}\right) \quad (62)$$

2.3 Buck-boost Converter

$$D = \frac{|V_o|}{V_s + |V_o|} \quad (63)$$

$$I_L = \frac{V_o^2}{V_s RD} = \frac{P_o}{V_s D} = \frac{V_s D}{R(1-D)^2} \quad (64)$$

$$L_{\min} = \frac{RT(1-D)^2}{2} \quad (65)$$

$$I_{L(\max)} = I_L + \frac{\Delta i_L}{2} = \frac{V_s D}{R(1-D)^2} + \frac{V_s DT}{2L}$$
$$I_{L(\min)} = I_L - \frac{\Delta i_L}{2} = \frac{V_s D}{R(1-D)^2} - \frac{V_s DT}{2L} \quad (66)$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \quad (67)$$

3. DC-AC Converter

$$i_o(t) = \begin{cases} \frac{V_{dc}}{R} + \left(I_{\min} - \frac{V_{dc}}{R}\right)e^{-t/\tau} & \text{for } 0 < t < \frac{T}{2} \\ \frac{-V_{dc}}{R} + \left(I_{\max} + \frac{V_{dc}}{R}\right)e^{(t-T/2)/\tau} & \text{for } \frac{T}{2} < t < T. \end{cases} \quad (68)$$

3.1 Single-phase full-bridge inverter

$$v_o(t) = \sum_{n=1,3,5,..}^{\infty} V_n \sin n\omega t \quad (69)$$

$$V_n = \frac{4V_{dc}}{n\pi} \quad (70)$$

$$i_o(t) = \sum_{n=1,3,5,..}^{\infty} I_n \sin(n\omega t - \theta_n) \quad (71)$$

$$I_n = \frac{V_n}{|Z_n|} = \frac{4V_{dc}}{n\pi|Z_n|} = \frac{4V_{dc}}{n\pi\sqrt{R^2 + (n\omega_0 L)^2}} \quad (72)$$

$$P_n = I_{n,rms}^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R \quad (73)$$

$$V_{o1,rms} = \frac{V_n}{\sqrt{2}} = \frac{4V_{dc}}{\sqrt{2}n\pi} \quad (74)$$

$$THD_v = \frac{\sqrt{\sum_{n=2}^{\infty}(V_{n,rms})^2}}{V_{o1,rms}} = \frac{\sqrt{V_{o,rms}^2 - V_{o1,rms}^2}}{V_{o1,rms}} \quad (75)$$

$$THD_i = \frac{\sqrt{\sum_{n=2}^{\infty}(I_{n,rms})^2}}{I_{o1,rms}} = \frac{\sqrt{I_{o,rms}^2 - I_{o1,rms}^2}}{I_{o1,rms}} \quad (76)$$

$$m_f = \frac{f_{sw}}{f} \quad (77)$$

$$m_a = \frac{V_{m,reference}}{V_{m,carrier}} \quad (78)$$

$$V_{n,peak} = m_a \times V_{dc} \quad (79)$$

3.2 Three-phase inverter

$$v_{ab} = \sum_{n=1,5,7,..}^{\infty} \frac{4V_{dc}}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t + \frac{\pi}{6}\right) \quad (80)$$

$$v_{bc} = \sum_{n=1,5,7,..}^{\infty} \frac{4V_{dc}}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{\pi}{2}\right) \quad (81)$$

$$v_{ca} = \sum_{n=1,5,7,..}^{\infty} \frac{4V_{dc}}{n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{7\pi}{6}\right) \quad (82)$$

$$v_{an} = \sum_{n=1,5,7,..}^{\infty} \frac{4V_{dc}}{\sqrt{3}n\pi} \sin\frac{n\pi}{3} \sin n(\omega t) \quad (83)$$

$$v_{bn} = \sum_{n=1,5,7,..}^{\infty} \frac{4V_{dc}}{\sqrt{3}n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{2\pi}{3}\right) \quad (84)$$

$$v_{cn} = v_{an} = \sum_{n=1,5,7,..}^{\infty} \frac{4V_{dc}}{\sqrt{3}n\pi} \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{4\pi}{3}\right) \quad (85)$$

$$V_L = \sqrt{\frac{2}{3}} V_{dc} \quad (86)$$

$$V_{L1} = \frac{4V_{dc} \sin 60^o}{\sqrt{2}\pi} \quad (87)$$

$$V_P = \frac{V_L}{\sqrt{3}} \quad (88)$$

4. AC Voltage Controller

$$V_{o,rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} (V_m \sin \omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \quad (89)$$

$$I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{V_m}{R\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \quad (90)$$

$$pf = \frac{P}{S} = \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} \tag{91}$$

$$I_{SCR,Avg} = \frac{1}{2\pi}\int_{\alpha}^{\pi} \frac{V_m \sin \omega t}{R} d\omega t = \frac{V_m}{2\pi R}\left(1 + \cos \alpha\right) \tag{92}$$

$$I_{SCR,rms} = \frac{V_m}{\sqrt{2}\sqrt{2}R}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} = \frac{I_{o,rms}}{\sqrt{2}} \tag{93}$$

Appendix 1

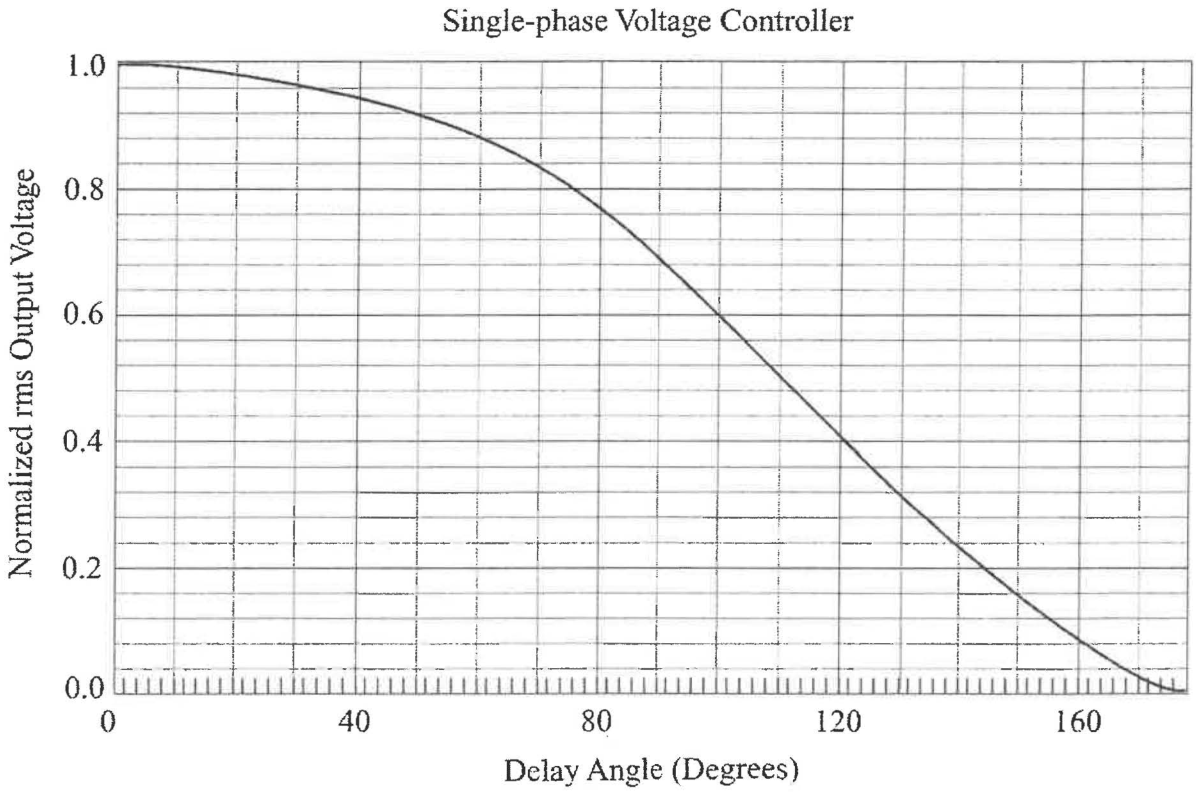

Figure 1: Normalized rms output voltage versus delay (firing) angle for a single-phase ac voltage regulator with resistive load