Boundary Layer Flow and Heat Transfer of a Nanofluid over a Stretching/Shrinking Sheet with Suction and Slip Effect

by

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LIST OF ABBREVIATIONS

FEM	Finite Element Method
HAM	Homotopy Analysis Method
MHD	Magnetohydrodynamic

LIST OF SYMBOLS

a	Constant
C	Nanoparticle volume fraction
C_f	Skin friction coefficient
C_w	Nanoparticle volume fraction at the plate
C_∞	Nanoparticle volume fraction far from the plate
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion
f	Dimensionless stream function
k	Thermal conductivity
l	Slip coefficient
Le	Lewis number
n	Nonlinearity parameter
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Nu_x	Local Nusselt number
$\Pr$	Prandtl number
q_m	Mass flux
q_w	Heat flux
Re_x	Local Reynolds number
S	Suction parameter
Sh_x	Local Sherwood number

T	Temperature
T_w	Surface temperature
T_∞	Ambient temperature
u, v	Velocity components in the x and y directions
v_w	Suction or injection velocity
U_W	Shrinking/stretching velocity
x, y	Cartesian coordinates

Greek Symbols

α	Thermal diffusivity of the fluid
δ	Slip effect parameter
η	Similarity variable
θ	Dimensionless temperature
λ	Stretching/shrinking parameter
υ	Kinematic viscosity
$(\rho c)_f$	Heat capacity of the base fluid
$(\rho c)_p$	Effective heat capacity of the nanoparticles
τ	Parameter defined by $(\rho c)_p / (\rho c)_f$
τ_w	Skin friction
ϕ	Rescaled nanoparticle volume fraction

Subscripts

c	Critical
∞	Condition far away from the plate

Aliran Lapisan Sempadan dan Pemindahan Haba dalam Nanobendalir terhadap Permukaan Meregang/Mengecut dengan Sedutan dan Kesan Gelinciran

ABSTRAK

Dalam tesis ini, dipertimbangkan masalah aliran lapisan sempadan yang mantap dan pemindahan haba terhadap permukaan regangan dan pengecutan permukaan dengan syarat sempadan gelinciran halaju dan kesan sedutan. Bendalir yang dipertimbangkan dalam kajian ini, ialah nanobendalir. Kajian ini dimulakan dengan memformulasi model-model matematik yang mengawal aliran dan pemindahan haba. Seterusnya, persamaan-persamaan menakluk tak linear dalam bentuk persamaan pembezaan separa diturunkan kepada persamaan-persamaan pembezaan biasa menggunakan penjelmaan keserupaan yang bersesuaian. Sistem persamaan pembezaan biasa yang terhasil kemudiannya diselesaikan secara berangka menggunakan kaedah tembakan melalui aturcara yang telah siap dibina dalam perisian MAPLE 12. Nilai-nilai berangka bagi pekali geseran kulit, nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan dan nombor Sherwood setempat serta profil-profil halaju, suhu dan pecahan isipadu nanozarah diperoleh untuk nilai parameter menakluk iaitu parameter kesan gelinciran dengan nilai yang ditetapkan untuk parameter sedutan, parameter regangan/kecutan, parameter gerakan Brownan, parameter termoforesis, nombor Lewis dan nombor Prandtl. Keputusan berangka yang diperoleh dipersembahkan dalam bentuk jadual dan graf. Perbandingan keputusan dengan kajian terdahulu dibuat bagi kesahan keputusan kajian. Didapati bahawa ciri-ciri aliran dan pemindahan haba dipengaruhi oleh parameter kesan gelinciran. Peningkatan parameter kesan gelinciran menyebabkan pekali geseran kulit dan nombor Nusselt setempat menurun sementara nombor Sherwood setempat bertambah. Penyelesaian dual diperoleh bagi julat-julat tertentu parameter-parameter terlibat.

Boundary Layer Flow and Heat Transfer of a Nanofluid over a Stretching/Shrinking Sheet with Suction and Slip Effect

ABSTRACT

In this thesis, the problem of steady boundary layer flow and heat transfer over a stretching/shrinking surface with the velocity slip boundary condition and suction is considered. The fluid that considered in this study, namely the nanofluid. The study starts with the formulations of the mathematical models that governed the fluid flow and heat transfer. Next, the governing nonlinear equations in the form of partial differential equations are reduced into ordinary differential equations using an appropriate similarity transformation. The resulting system of ordinary differential equations is then solved numerically using a shooting method by means of the built-in programmme in the MAPLE 12 software. The numerical values of the skin friction coefficient, the local Nusselt number which represents the heat transfer rate at the surface and the local Sherwood as well as the velocity, temperature and nanoparticles volume fraction profiles, are obtained for the governing parameter namely slip effect parameter with the fixed values of suction parameter, stretching/shrinking parameter, the Brownian motion parameter, thermophoresis parameter, Lewis number and Prandtl number. The numerical results obtained are presented in the form of tables and graphs. The comparisons of results with previous studies are made to validate the results obtained. It is found the flow and heat transfer characteristics are influenced by slip effect parameter. The skin friction coefficient and the local Nusselt number decrease whereas the local Sherwood number increases with the increasing of slip effect parameter. The dual solutions are obtained for a certain range of the parameters involved.

CHAPTER 1

INTRODUCTION

1.1 Overview

Boundary layer flow and heat transfer of a nanofluid over a stretching/shrinking sheet with the suction and slip effect is investigated in this study. Some basic concept related to the boundary layer and heat transfer will be described briefly to get general idea on study that will be carried out.

Fluid mechanics is the branch of physics that studies the flow behavior of fluid such as liquid and gas. This movement of the fluid is due to the nature of fluid which cannot withstand the deformation forces. The phenomenon of fluid movement will continue to change as long as the force is applied to it. The force that causing the deformation or the movement of fluid is known as the shear force that acts tangentially to the surface (Kundu et al. 2011).

Ludwig Prandtl was a pioneer in introducing the boundary layer theory in year 1904 in 3rd International Mathematics Conference located in Heidelberg, German (Acheson 2001). Through his presentation on paper entitled "Fluid Flow in Very Little Friction", he described the boundary layer and its importance for drag. The main idea proposed by Prandtl was to divide the flow into two parts. The main part which is the largest is free fluid located far away than body surface that named nonviscous fluid. Meanwhile, the second part is thin layer located next to body surface (Acheson 2001). This layer specifically called as boundary layer in place occurrence of friction impact as a result the friction force need to be considered whereas for area beyond the boundary, friction force were small and can be neglected (Schlichting & Gersten 1979).

Heat transfer is a discipline that concerns the transfer of thermal energy from one medium to another medium due to temperature gradient. Some examples of temperature control are cooling of the nuclear reactors and cooling the exterior of the space vehicle when re-entry to Earth's atmosphere (Bejan 1993). Heat transfer has broad applications to the functioning of numerous device and also cooling or heating systems. Heat transfer principles may be used to preserve, increase or decrease temperature in a wide variety of circumstances. For example, heat transfer is widely used in engineering practices to provide desired temperature changes, as in heating of homes, industrial processes, cooling of equipment and others.

There are three main mechanisms of heat transfer namely conduction, convection and thermal radiation (Bejan 1993). In heat transfer by conduction, the transfer of thermal energy occurs from one medium of higher temperature to another medium of lower temperature due to a temperature gradient between both medium. The change of heat energy process will continue until both medium approaching thermal equilibrium that have same medium temperature. Heat transfer by conduction occurs in solid, liquid and gas. The second mechanism of heat transfer is convection. The heat transfer by convection refers to the movement of molecules within fluids such as liquids and gases. It happens due to temperature difference between fluid and body surface. Besides of conduction and convection, heat energy can also be transferred through the process of thermal radiation. Heat transfer of thermal radiation is electromagnetic wave (Bejan 1993). In engineering, thermal radiation is considered one of the fundamental methods of heat transfer, although it does not involve the transport of heat. In this thesis, the study only considers the process of heat transfer by convection.

This study involved the development of mathematical model and numerical solution for boundary layer flow and heat transfer of a nanofluid over a stretching/ shrinking sheet with the suction and slip effect. The effect on slip effect parameter and heat transfer characteristics will be studied and discussed.

1.2 Problem Statement

Most previous study on the flow and heat transfer which are studied in various surface geometry aspects consider no-slip boundary condition. No-slip boundary condition means the velocity of the fluid on a body surface is assumed to be equal to the velocity of the body. In fluid mechanics, no-slip boundary condition is identified as a key pillar in the Navier-Stoke theory. However, there is in certain circumstances which no-slip condition is not consistent with all physical characteristics. Therefore, it is essential to replace the no-slip boundary condition with the slip boundary condition. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances. With the slip on the boundary surface, the behavior of fluid flow and heat transfer charateristics is different with the flow of no-slip boundary condition (Nandy et al. 2014).

The slip effect on the boundary has important technological applications such as in polishing valves of artificial heart and internal cavities (Sahoo 2010). The effect of slip on the boundary also has many industrial applications and practical in flows at both macro and micro scale. At the macro level, the slip effect on the boundary occurs in the process of polymer extrusion due to the instability at the high pressure whereas at the micro level happens for microflows of liquid which is the boundary condition at the wall is largely influenced by surface properties (Anand 2014). Besides that, fluid flow in a variety of system of micro/nano applications such as hard disk drives, micropumps and microvalves are oriented by slip effect on the boundary surface (Ibrahim & Shankar 2013). On the other hand, the slip on the boundary mostly happens in the field of rheology and also polymer processing (Karapetsas & Mitsoulis 2013). Motivated by the above mentioned application, this present study extends the paper by Rana and Bhargava (2012) by consider velocity slip boundary condition.

1.3 Objectives

The main objectives of the study are:

a) to apply the mathematical model on boundary layer flow over a stretching/shrinking surface in the presence of suction and slip effects;

b) to solve the mathematical model by using shooting method in MAPLE 12 software; and

c) to analyze the slip effect parameter on the fluid flow and heat transfer characteristics.

1.4 Scope of Study

In this study, a steady forced convection boundary layer flow over a nonlinearly stretching/shrinking sheet with the suction and velocity slip boundary condition is considered. This study focused on analysing the slip effect parameter on the fluid flow and heat transfer characteristics. The fluid that considered in this study is the nanofluid.

For the considered problem, the partial differential equations are reduced to ordinary differential equations using similarity transformation. The resulting of ordinary differential equations has been solved numerically using shooting method in MAPLE 12 software.

1.5 Significant of the Study

The significant of this study is the results of the fluid flow and heat transfer which is done by mathematical or numerical modeling that can be used as prediction results for a real flow phenomenon. This will help researchers to identify the constraints or problems that may arise before actual experiment is carried out. Therefore, the use of money, time and energy resources can be optimized as well as can avoid any form of waste. Besides that, the numerical results obtained can be made a comparison result to the results of the actual experiment. If the comparative results are good and satisfied, the mathematical model can be used as a solution for more complex flow problems. This is because not all physical aspects in the flow study can be examined experimentally due to the high cost and limited experimental equipment. Therefore, mathematical modeling is more appropriate.

Among the other significant of this study is the enrichment activity and knowledge enhancement. New findings from this study will help the other researchers to develop a higher level study. The results of the study in this thesis can be used as a reference and comparison to researchers in the future and also more new discoveries will be produced.

1.6 Project Outline

This report is separated into five chapters. Chapter 1 started by an introduction and the background of the study followed by the problem statement, objective, scope of study and project outline. The literature review of the study is stated in Chapter 2. It started by introduction, after that continue with the literature review on the forced convection flow towards stretching/shrinking sheet in a nanofluid, the slip effect and the methods that can be used to solve the considered problem. In Chapter 3, the formulation of the mathematical model, the governing partial differential equations and also the numerical method that used to solve the problem discussed. Then, result and discussion is well explained in Chapter 4. Lastly, in Chapter 5 will explain the conclusion and future work of this research report.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter will review the existing research on the related studies. In section 2.2 will discuss about the forced convection flow towards stretching/shrinking sheet in a nanofluid whereas in section 2.3 will discuss the study on flow with slip effect. Meanwhile, in section 2.4 will review the methods that can be used to solve the boundary layer equation include the advantages and disadvantages of existing methods.

2.2 Forced Convection Flow towards Stretching/Shrinking Sheet in a Nanofluid

The study of boundary layer flow and heat transfer on the stretching/shrinking sheet has received widespread attention among researches due to the large number of applications in industrial processes and engineering. Some of the examples are in the manufacturing industry and the production of polymer and rubber sheets. In this process, the quality of the final product depends on the rate of heat transfer on stretching surface (Khan & Pop 2010). The stretching surface occurs when the fluid velocity at the boundary moving away from a fixed point.

Crane (1970) is the first person who pioneered the study flow over a stretching by solving analytically the steady two-dimensional flow past a linearly stretching plate. An analytical form is presented by Crane (1970) for the steady boundary layer flow of an incompressible viscous liquid caused by the linear stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point.

Then, this study is extended by Gupta and Gupta (1977) by adding new dimensions with suction or blowing effects. Gupta and Gupta (1977) reported that stretching surface is not necessarily continuous. Most of the earlier investigations deal with the study of boundary layer flow past a stretching surface in which the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin.

Kumaran et al. (2009) studied the magnetohydrodynamic (MHD) flow towards a stretching permeable surface. An exact solution is obtained for a boundary layer flow over a stretching and linearly permeable surface. It can be concluded from this study that with the presence of a magnetic field, the streamlines are steeper and thus make the boundary layer thinner. Besides that, shear thinning reduces the wall shear stress.

In contrast to the stretching surface case, the case of shrinking surface causes the fluid velocity on the surface to move in the opposite direction towards a fixed point. Shrinking surface produces vorticity in the boundary layer. Therefore, an external force is required to confine the vorticity inside the boundary layer. The external force is the suction at the boundary. The vorticity is very important to enable the steady flow towards shrinking surface occurs (Bhattacharyya & Layek 2011).

Physically, there are two conditions which the flow on a shrinking surface may exist. Firstly, by applying adequate suction effect to the boundary layer (Miklavčič & Wang 2006) and secondly by considering the stagnation flow (Wang 2008). The first analysis solution for viscous flow induced by a shrinking sheet with suction effect is given by Miklavčič & Wang (2006). They reported that suction is required to maintain the flow in the boundary layer due to the occurrence of vorticity generated by the shrinking surface. Then, the flow towards the shrinking surface is described by Goldstein (1965) as a backward flow. As a result, the flow towards the shrinking surface indicates different physical aspects compared to the stretching surface case.

The existence of variety usage of nanofluid in everyday life has attracted the interest among the researchers to explore this field. Choi (1995) is the first person who introduced the term "nanofluid" and observed that the addition of a small amount (less than 1% by volume) of nanoparticles to conventional liquids increased the thermal conductivity of the fluid up to approximately two times. "Nanofluid" is the term to describe a base fluid suspended with nanometer-sized particles for example water, ethylene glycol and oil. After this, nanofluid can be considered as the next generation of technology which can allow heat transfers through the performance of pure liquid.

As reported by Daungthongsuk and Wongwises (2007), the heat transfer characteristic of the base fluids is identified as a major obstacle to the high compactness and effectiveness of heat exchangers. One of the initiatives taken is by suspending the solid particles that have thermal conductivities several hundreds of times higher than base fluids. The suspended of solid particles and base fluids has good rheological properties and stability that can improve the thermal conductivity of the fluid.

There are some mathematical nanofluid models developed by researchers to study the problem of convective flow in a nanofluid. Among of them are mathematical nanofluid models proposed by Buongiorno (2006) and Tiwari and Das (2007). Buongiorno (2006) considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid, which are inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity. However, only Brownian diffusion and thermophoresis are found to be important slip mechanisms in a nanofluid (Nield & Kuznetsov 2009). Besides that, Buongiorno (2006) described Brownian motion as a random motion of nanoparticles within the base fluid and results from continuous collisions between nanoparticles and base fluid molecules. Meanwhile, the "thermophoresis" phenomenon refers to the process of immersed particle that occur due to temperature gradients in the fluids. For hot surfaces, thermophoresis tends to separate the boundary layer of the nanoparticles volume fraction from the surface due to nanoparticles diffusion (Malvandi 2013).

Tiwari and Das (2007) proposed mathematical nanofluid model where the nanoparticles when immersed in a fluid are capable of increasing the heat transfer capacity of base fluid. The presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics. As solid volume fraction increases, the effect is more pronounced. The variation of average Nusselt number is nonlinear with solid volume fraction. Besides that, nanoparticles are able to change the flow pattern of a fluid from natural convection to forced convection regime.

The first paper on the stretching sheet in a nanofluid is presented by Khan and Pop (2010). Khan and Pop (2010) are the first person who provide the numerical solution of the problem the laminar fluid flow which results from the stretching of a flat surface in a nanofluid. The effects of Brownian motion and thermophoresis are considered in this study. From this study, it can be concluded that the reduced Nusselt number is a decreasing function, whereas the reduced Sherwood number is an increasing function of the parameters of Prandtl number $\Pr$, Lewis number Le, Brownian motion parameter Nb and thermophoresis parameter Nt.

Hassani et al. (2011) studied the boundary layer flow over a stretching sheet in a nanofluid analytically by using homotopy analysis method (HAM). In this study, both the Brownian motion and thermophoresis effects are considered. It is found that the reduced Nusselt number is a decreasing function, whereas the reduced Sherwood number is an increasing function of each values of the parameters Prandtl number $\Pr$, Lewis number Le, Brownian motion parameter Nb and thermophoresis parameter Nt. The analytical solution that is investigated by Hassani et al. (2011) for Nusselt number and Sherwood number has good agreements with the numerical solution that is presented by Khan and Pop (2010).

The boundary layer flow in a nanofluid with a convective boundary condition is then studied numerically by Makinde and Aziz (2011). This study is used a convective heating boundary condition. The thermal boundary layer thickens with a rise in the local temperature as the Brownian motion, thermophoresis and convective heating. Then, it is found that as the convection Biot number increases, the local concentration of nanoparticles increases. For the convection Biot number decreases, the Lewis number will increase.

Kandasamy et al. (2011) explored the magnetohydrodynamic (MHD) boundary layer flow of a nanofluid past a vertical stretching permeable surface with the effects of suction and injection. Kandasamy et al. (2011) found that the thermophoresis parameter and