HYBRID METHOD OF FUZZY HOMOTOPY AND RUNGE KUTTA FEHLBERG METHOD FOR SOLVING FUZZY NONLINEAR EQUATIONS

by

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LIST OF ABBREVIATIONS

DE	Differential Equation
FBM	Fuzzy Bisection method
FBYM	Fuzzy Broyden's method
FFNM	Fuzzy Fixed Newton method
FFLPM	Fuzzy False Position method
FFPM	Fuzzy Fixed Point method
FHEM	Fuzzy Homotopy and Euler Method
FHM	Fuzzy Homotopy method
FHRK4M	Fuzzy Homotopy and Runge Kutta of Order four method
FHCM	Fuzzy Continuation Homotopy Method
FHCGM	Fuzzy Hybrid Conjugate Gradient Method
FHNM	Fuzzy Harmonic Newton method
FHRKFM	Fuzzy Homotopy and Runge Kutta Fehlberg Method
FHCGM	Fuzzy Hybrid Conjugate Gradient Method
FMNM	Fuzzy Midpoint Newton's method
FMSM	Fuzzy Modified Secant method
FNM	Fuzzy Newton's method
FNMFBYM	Hybrid method of Fuzzy Newton's method and Fuzzy Broyden's method
FSDM	Fuzzy Steepest Descent method
RK4M	Runge Kutta of Order four Method
RKFM	Runge Kutta Fehlberg Method

LIST OF SYMBOLS

$\mathbb{R}$	Real number
∂	Partial derivatives
$\in$	Element
λ	Family of problems

Kaedah Gabungan Homotopi Kabur dan Kaedah Runge Kutta Fehlberg untuk Menyelesaikan Persamaan Tak Linear Kabur

ABSTRAK

Sistem persamaan tak linear kabur memainkan peranan penting dalam menyelesaikan masalah dunia sebenar. Walau bagaimanapun, untuk menyelesaikan sistem persamaan tak linear kabur bukan satu tugas yang mudah. Hal ini jelas apabila berurusan dengan fungsi trigonometri, hiperbolik, dan eksponen. Oleh itu, kaedah berangka diperlukan untuk mengatasi masalah ini. Dalam disertasi ini, kaedah gabungan Homotopi Kabur dan Runge Kutta Fehlberg dicadangkan untuk menyelesaikan sistem persamaan tak linear kabur. Kaedah ini diperluaskan dari Kaedah Homotopi Kabur dan Penerusan yang dicadangkan dalam kajian lepas. Kaedah gabungan yang dicadangkan melibatkan secara numerik mencari penyelesaian daripada masalah yang diketahui dan diteruskan penyelesaian sehingga masalah tidak diketahui diperolehi. Algoritma kaedah gabungan yang dicadangkan dibangunkan dan diuji dengan contoh berangka. Hasilnya dianalisis dan dibandingkan dengan kaedah sedia ada dari segi ralat, ketepatan dan penumpuan. Berdasarkan kepada keputusannya, kaedah gabungan yang dicadangkan menunjukkan kurang ralat dan hasil yang baik dari segi ketepatan dan penumpuan berbanding kaedah yang sedia ada. Oleh itu, kaedah gabungan yang dicadangkan adalah lebih unggul dan boleh digunakan untuk menyelesaikan sistem persamaan tak linear kabur yang kompleks.

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Hybrid Method of Fuzzy Homotopy and Runge Kutta Fehlberg Method for Solving Fuzzy Nonlinear Equations.

ABSTRACT

The system of fuzzy nonlinear equations plays an important role in solving real-world problems. However, to solve the system of fuzzy nonlinear equations is not an easy task. This is patently obvious when dealing with trigonometric, hyperbolic, and exponential functions. Therefore, a numerical method is needed in order to overcome this issue. In this dissertation, a hybrid method of Fuzzy Homotopy and Runge Kutta Fehlberg is proposed to solve the system of fuzzy nonlinear equations. This method is extended from the Fuzzy Homotopy and Continuation Method proposed in the literature. The proposed hybrid method involves numerically finding the solution from known problems and continuing the solution until the unknown problem is found. The algorithm of the proposed hybrid method is developed and tested on a numerical example. The results are analyzed and compared with the existing methods in terms of errors, accuracy and convergence. Based on the results, the proposed hybrid method showed less error and good result in terms of accuracy and convergence compared to the existing methods. Therefore, the proposed hybrid method is superior and can be employed to solve complex system of fuzzy nonlinear equations.

CHAPTER 1 : INTRODUCTION

1.1 Introduction

This chapter introduces the research background about the research topic under consideration. It is followed by the problem statement, research objectives, scope and limitations of the study, research significant and organization of dissertation.

1.2 Research Background

A linear equation is a polynomial equation in which the unknown variables have a degree of one. All of the unknown variables in a linear equation are raised to the power of one. In the two dimensional case, they always form lines, planes, and points. Since their shapes are always perfectly straight, they are called linear equations. Linear equations have some useful properties where most of them are very easy to manipulate and solve. Although they are quite limited in what they can represent, it is often useful to try and approximate complicated systems using linear equations so that they will be easier to think about and deal with it.

A function $f : R^n \to R$ is defined as being nonlinear when it does not satisfy the superposition principle. That is:

$$f(x_1 + x_2 + ...) \neq f(x_1) + f(x_2) + ... ,$$

A system of nonlinear equations is a set of equations such as the following:

$$\begin{gathered} f_1(x_1, x_2, ..., x_n) = 0, \\ f_2(x_1, x_2, ..., x_n) = 0, \\ \vdots \\ f_3(x_1, x_2, ..., x_n) = 0, \end{gathered}$$

where $(x_1, x_2, ..., x_n) \in R^n$ and each f_i is a nonlinear real function $i = 1, 2, ..., n$.

A solution of a system of equations $f_1, f_2, ..., f_n$ in n variables is a point $(a_1, ..., a_n) \in R^n$ such that $f(a_1, ..., a_n) = \cdots = f_n(a_1, \ldots, a_n) = 0$. The systems of nonlinear equations cannot be solved as linear systems. Therefore, iterative methods are used. An iterative method is a procedure that is repeated to find the root or solution of a system of equations.

Solving a system of nonlinear equations becomes more complex, especially solving nonlinear equations analytically. This is because solving the system of nonlinear equations analytically usually involves trigonometric functions, hyperbolic functions, exponential functions or logarithmic functions. Therefore, a numerical method is needed to solve the system of nonlinear equations. The important properties of numerical methods that have been studied by researchers are convergence, accuracy and error. In mathematics, convergence means a sequence approach toward a definite value or point. A sequence converges when it keeps getting closer to a certain value. For example, $\frac{1}{x}$ and the term $\frac{1}{x}$ are $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and so on. The sequence converges to 0, because the terms get closer to 0. Accuracy means how close a measured value is to the actual (true) value. Accuracy increases when the measured value is close to true value.

Furthermore, error is defined as the difference between true value and an estimation, or approximation, of that value.

A crisp set is defined by crisp boundaries and it has a unique membership function. Fuzzy set is prescribed by vague or ambiguous properties; hence its boundaries are ambiguously specified. Furthermore, a fuzzy set can have an infinite number of membership functions to represent it. For example, the membership for a crisp set is only 0 and 1 while the membership for fuzzy set is defined as the interval $[0,1]$. Zadeh (1965) and Zimmermann (1996) were the earliest researchers to introduce and investigate the concept of a fuzzy number. Zadeh (1965) investigated classes of objects encountered in the real physical world that do not have a precisely defined criteria of membership. For example, when measuring the height of a person, the result obtained is a numerical value of a level of imprecision. Zadeh (1965) studied that it is easy to represent fuzzy numbers to indicate the same imprecise system of coefficients. Fuzzy numbers can also be used to solve the system of fuzzy nonlinear equations. In the literature, many researchers have proposed several methods for solving the system of fuzzy nonlinear equations. However, there are some weaknesses in the existing methods that can be improved for obtaining good results in terms of accuracy, convergence and error. In this dissertation, Fuzzy Homotopy and Runge Kutta Fehlberg method (FHRKFM) are proposed to solve the system of fuzzy nonlinear equations with concerns on accuracy, convergence and error. This method is extended from the Fuzzy Homotopy and Continuation method (FHCM) proposed by Ahmad et al., (2016).

1.3 Problem Statement

The studies of fuzzy nonlinear equations have been received much attention over a few years ago. Abbasbandy and Ezzati (2006a,2006b), Abbasbandy and Jafarian (2006), Shokri (2008a,2008b), Abbasbandy and Jafarian (2009), Mamat, Ramli, and Abdullah (2010), Zhou and Gan (2010), Waziri and Majid (2012), Umar (2015), Ali,et al. (2016) and Ahmad et al. (2016a) consider convergence for solving the solution of fuzzy nonlinear equations. While Ahmad et al. (2016b) consider their study with concerned onto accuracy of results. However, there are still medium of improvement that can be considered for obtaining better results. In tjis study, FHRKFM are proposed for improving the accuracy of result by referring numerical example from Ahmad et al. (2016b). In this research, there are several problems occur when solving fuzzy nonlinear equations numerically, which are divergence problems often occur in traditional numerical methods, most researcher focus on the convergence rather than accuracy of result and computer program that execute numerical methods might have error due to human error, truncation error and round off error. Therefore, in this study, accuracy, convergence and error are included and FHRKFM are proposed as alternative method compared to existing methods in order to find best solution for solving fuzzy nonlinear equations.

1.4 Objectives

The objectives of this study are:

(i) To solve fuzzy nonlinear equations using the FHRKFM method.

(ii) To estimate the accuracy and convergence of FHRKFM for solving fuzzy nonlinear equations.

(iii) To compare the performance of FHRKFM in term of error with the Fuzzy Homotopy and Euler method (FHEM) and Fuzzy Homotopy and Runge Kutta of Order Four method (FHRK4M).

1.5 Scope and Limitation of the Study

In this research, fuzzy nonlinear equations with the function of a single variable are considered. To solve this problem, the function should be continuous. Fuzzy equations for higher dimensions will not be considered due to the complexity of developing an algorithm to solve this equation.

1.6 Significant of the Study

Most researchers study convergence rather than accuracy for solving fuzzy nonlinear equations. Therefore, this hybrid method is developed to find the best method in accuracy for solving fuzzy nonlinear equations. This method is a new method that combines Homotopy method with Runge Kutta Fehlberg method (RKFM) which improves the accuracy of results.

1.7 Organisation of Dissertation

This dissertation is comprised of five chapters. The first chapter begins by presenting the problem statement, research objectives, scope and limitations of the study, research significant and dissertation organization to further clarify the idea and need of the study. Chapter 2 discusses existing methods that have been proposed by researchers for solving the system of fuzzy nonlinear equations. Next, Chapter 3 explains the methodology of this research study**.** Chapter 4 presents numerical results to show ability of FHRKFM to gives good result in terms of accuracy, convergence and error. Lastly, Chapter 5 will bring out the conclusion of the study and discuss whether the objectives of this study are achieved as well as recommendations and suggestions for future research.

1.8 Summary

This chapter presented the problem statement, research objectives, scopes and limitation of study, research significant and organisation of this dissertation to further clarify the idea and need of the study. The following chapter discusses the existing methods that have been proposed by researchers for solving fuzzy nonlinear equation and the basic concept about fuzzy set theory and fuzzy arithmetic.

CHAPTER 2 : LITERATURE REVIEW

2.1 Introduction

Solution of fuzzy nonlinear equation required to study the parametric form of fuzzy number when meet some imprecise coefficient. The existing methods proposed by researchers are presented to give an overview to solving fuzzy nonlinear equations. The rest of this chapter is devoted to the basic concept of fuzzy set theory and fuzzy arithmetic.

2.2 Preliminaries

In this section, some basic definitions of fuzzy set theory and fuzzy arithmetic are presented.

Definition 2.2.1: (Goetschel & Voxman (1983) and Dubois & Prade (1978)). A fuzzy number is a fuzzy set $U(x): \mathbb{R} \to I = [0,1]$ which satisfies

i. $U(x)$ is upper semi-continuous

ii. There are real number a, b, c, d such that $c \le a \le b \le d$

(a) $U(x)$ is monotonically increasing on $[c, a]$,

(b) $U(x)$ is monotonically decreasing on $[b, d]$,

(c) $U(x) = 1$,

iii. $U(x) = 0$ if x lies outside of interval $[c, d]$.

Definition 2.2.2: **(**Kajani, Asady & Vencheh (2005) and Dubois & Prade (1978)). A fuzzy number U in parametric form is a pair $(\underline{u}, \overline{u})$ of function $\underline{u}(r), \overline{u}(r)$ $0 \le r \le 1$, which satisfies the following conditions:

i. $\underline{u}(r)$ is bounded monotonic increasing left continuous function over $[0,1]$,

ii. $\overline{u}(r)$ is bounded monotonic decreasing left continuous function over $[0,1]$,

iii. $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

Definition 2.2.3: (Attari, Nasseri, Chitgar & Vahidi (2012)). A triangular fuzzy number is a fuzzy number represented by three points, $u = (a, b, c)$, and its membership function can be represented as

$$\mu(x) = \begin{cases} \dfrac{x-a}{c-a}, & a \le x < c, \\ \dfrac{x-b}{c-b}, & c \le x \le b, \end{cases}$$

where $c \ne a$, $c \ne b$ and hence its parametric form is

$$\left[\underline{\mu}^{r}, \overline{\mu}^{r}\right] = \left[a + (c-a)r, b + (c-b)r\right] \tag{2.1}$$

Definition 2.2.4: (Gao, Zhang & Cao (2009)). The operation of a fuzzy number can be generalized from that of a crisp interval. Let's have a look at the operations of interval $\forall a_1, a_3, b_1, b_3 \in \mathbb{R}$ and $A = [a_1, a_3], B = [b_1, b_3]$

Assume A and B are numbers that are expressed as intervals. The main operations of intervals are:

(i) Addition

$$[a_1, a_3](+)[b_1, b_3] = [a_1 + b_1, a_3 + b_3]$$

(ii) Subtraction

$$[a_1, a_3](-)[b_1, b_3] = [a_1 - b_3, a_3 - b_3]$$

(iii) Multiplication

$$[a_1, a_3](\bullet)[b_1, b_3]$$

$$= [a_1 b_1 \wedge a_1 b_3 \wedge a_3 b_1 \wedge a_3 b_3, a_1 b_1 \vee a_1 b_3 \vee a_3 b_1 \vee a_3 b_3]$$

(iv) Division

$$[a_1, a_3](/)[b_1, b_3]$$

$$= [a_1 / b_1 \wedge a_1 / b_3 \wedge a_3 / b_1 \wedge a_3 / b_3, a_1 / b_1 \vee a_1 / b_3 \vee a_3 / b_1 \vee a_3 / b_3]$$

(v) Inverse

$$[a_1, a_3]^{-1} = [1/a_1 \wedge 1/a_3, 1/a_1 \vee 1/a_3]$$

Definition 2.2.5: (Ahmad et al. (2016b)). Let $A = [a_1, a_2]$ be a closed interval in $\mathbb{R}^{+}$ and $k \in \mathbb{R}^{+}$. By identifying the scalar k as the closed interval $[k, k]$, the scalar multiplication $k.A$ is defined as follows

$$k.A = [k, k](\bullet)[a_1, a_2] = [ka_1, ka_2]$$

2.3 Existing fuzzy methods for solving fuzzy nonlinear equations

Many methods have been developed by researchers for solving fuzzy nonlinear equations. Several methods like Fuzy Homotopy method (FHM), Fuzzy Newton's method (FNM), Fuzzy Steepest Descent method (FSDM), Fuzzy Harmonic Newton's method (FHNM), Fuzzy Midpoint Newton's method (FMNM), Fuzzy False Position method (FFLPM), Fuzzy Broyden's method (FBYM), Fuzzy Hybrid Conjugate Gradient method (FHCGM), Hybrid of Fuzzy Newton's method and Fuzzy Broyden's method (FNMFBYM), Fuzzy Bisection method (FBM), Fuzzy Fixed Position method (FFPM) and Fuzzy Homotopy Continuation method (FHCM) are brieftly explained in next section.

2.3.1 Fuzzy Homotopy method (FHM)

Recently, Abbasbandy and Ezzati (2006a) investigated FHM to solve fuzzy nonlinear equations. This method is a hybrid method between FHM and Fuzzy Newton's method (FNM) which is used to solve fuzzy nonlinear equations. The method starts by rewriting fuzzy nonlinear equations in parametric form. Then, FHM is used to convert it from the parametric form to a one parameter family of problem that is described in $\lambda \in [0,1]$. Then, FNM is used to solve the system of differential equation and the solution of fuzzy nonlinear equations is obtained. From the results, FNM can achieve super linear convergence if the initial guess is known. However, FNM fails if the solution is not exactly close to the initial guess. In other words, Fuzzy Newton's methods converge locally but do not provide guarantees to obtaining a solution.

2.3.2 Fuzzy Newton's method (FNM)

FNM is proposed by Abbasbandy and Ezzati (2006b) to solve fuzzy nonlinear equations. The solution of fuzzy nonlinear equations are expected to be a quadratic convergence when the initial value is known. In other words, this method fails if the initial guess is unknown or the solution is not exactly close to an exact solution. In addition, inverse matrix needs to be computed for each iteration for solving fuzzy nonlinear equations.

2.3.3 Fuzzy Steepest Descent's method (FSDM)

FSDM is proposed by Abbasbandy and Jafarian (2006) to solve fuzzy nonlinear equations. FSDM converges only linearly to the solution even if a poor initial guess is used. In other words, FSDM does not require an accurate initial guess to ensure convergence. FSDM different from FNM in that it does need an accurate initial guess to guarantee convergence. Hence, this method has improved the weakness of FNM for solving the system of fuzzy nonlinear equations.

2.3.4 Fuzzy Harmonic Newton's method (FHNM)

Shokri (2008a) proposed FHNM to solve fuzzy nonlinear equations. This method is known as implicit-type and Fuzzy Trapezoidal Newton's method. FHNM considers the function of two variables for solving system of fuzzy nonlinear equations.

Two examples are given to show the efficiency of FHNM for solving fuzzy nonlinear equations. Fuzzy Trapezoidal Newton's method used the same numerical example with Fuzzy Midpoint Newton's method. From the result, Fuzzy Midpoint Newton's method converges faster than Fuzzy Trapezoidal Newton's method.

2.3.5 Fuzzy Midpoint Newton's method (FMNM)

FMNM was proposed by Shokri (2008b) to increase the convergence of solving fuzzy nonlinear equations. FMNM also considers the function of two variables for solving the system of fuzzy nonlinear equations. FMNM uses the same numerical example as Fuzzy Trapezoidal Newton's method to solve fuzzy nonlinear equations. From the result, FMNM converges faster than Fuzzy Trapezoidal Newton's method.

2.3.6 Fuzzy Fixed Point method (FFPM)

Abbasbandy and Jafarian (2009) proposed FFPM to solve fuzzy nonlinear equations. FFPM focus on the convergence of results for solving fuzzy nonlinear equations. The solution of FFPM usually converges although a poor initial guess is used. FFPM is different from FNM in that it needs an accurate initial guess to guarantee convergence. FFPM converges very slowly if an inaccurate initial guess is used.

2.3.7 Fuzzy Broyden's method (FBYM)

Mamat, Ramli and Abdullah (2010) proposed FBYM to solve fuzzy nonlinear equations. Jacobian matrix is replaced by FBYM in the FNM to ease the update for each