Estimation of Muscle and Joint Forces during Push-up

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Abstract-In these paper, methods for estimating the muscle and joint forces during push-up are discussed. Many simplifications/assumptions were required like rigid body bones, single line of action for muscles, planar assumption and known muscle insertion and origin. In general, the normalized biceps, brachialis, and brachioradialis forces calculated in this paper compare well with those available in the literature values. However, consistency of magnitude between both of these values suggest the simplified approach adopted in this analysis does provide a useful approximation in investigating musculoskeletal loads and muscle forces at the ankle joint during squatting exercises.

I. INTRODUCTION

The actual loads carried by joints have significant implications. Joint cartilage degeneration and capsuloligamentous laxity are often related to the magnitude and pattern of load transmission at the joint. In total joint arthroplasty, the wear and deformation of the articulating surfaces, the stress distribution in an implant, the mechanical behavior of the bone-implant interface, and the load-carrying characteristics of the remaining bone are intimately related to the joint load [1].

In fractures, the joint and muscle loads play an important role in the bone union. Knowledge of the magnitude and manner of joint loading encountered by the human body is important in determining the possible mechanism and prevention of injury during occupational and sports activities. Calculating internal muscle and joint forces provides additional useful information for the design of implants, surgery, and rehabilitation programs.

In the past two decades, numerous analytical and experimental techniques have been developed for estimating muscle and joint forces [1]. Analytically, the determination of muscle and joint forces involves two steps:

1. The determination of intersegmental forces and moments at the joint based on given or measured kinematic and kinetic data (inverse dynamic problem)
2. The partitioning of intersegmental forces and moments into muscle and joint constraint forces and moments (force distribution problem)

This study will discuss these two steps (Fig. 1).

II. METHODOLOGY

A. Determination of Intersegmental Forces and moments

In biomechanics, the unknown muscle and joint forces are commonly determined mathematically, because they cannot be easily measured directly. On the other hand, motion can be measured using experimental techniques. Determining the intersegmental forces and moments based on kinematic data requires solution of the inverse dynamic problem [2]. Derivation of the equations of motion can be based on either Newtonian or Lagrangian formulas.

A simplified solution assumes that kinematic effects are negligible, allowing a "quasi-static" analysis. Static equilibrium is a condition in which a body is at rest (i.e., with no motion in relation to surrounding objects) and the external and internal forces and moments are balanced [3].
Both translation and rotation equilibrium need to be maintained for each body segment. Thus, the equations for static equilibrium are

\[ \sum F = 0 \]  
\[ \sum M = 0 \]

for each segment of the body. The summation of forces and moments includes the intersegmental force, \( F_i \), and intersegmental moment, \( M_i \), and the externally applied force and moment on the segment distal to the joint. To illustrate the procedure, a simplified two-dimensional problem is considered.

### B. Distribution of Muscle and Joint Forces

After calculating intersegmental resultant force, \( F \) and moment, \( M \), we are able to determine the muscle force, \( F_m \), joint constraint force, \( F_j \), and joint constraint moment, \( M_j \), based on the concept of equilibrium,

\[ F_i = \sum F_m + F_j \]  
\[ M_i = \sum M_m + M_j \]

Partitioning of these intersegmental resultant forces and moments is generally called the force distribution problem. Unfortunately, the number of unknown variables of muscle forces and joint constraint forces and moments usually exceeds the number of available equations. This is primarily because of the redundant nature of anatomic structures. There are multiple muscles that can execute synergistic functions. Mathematically, this produces an indeterminate problem that has no unique solution. The difference between the number of unknown variables and the number of equations represents the degree of redundancy. In order to resolve this indeterminate problem, the degree of redundancy must be reduced by either introducing constraint equations or by decreasing the number of unknown variables [4].

Therefore, an important consideration in muscle and joint force determination is accounting for the number of unknown variables versus constraints in the equations of motion. In general, this decoupling procedure is achieved based on the concept of degrees of freedom (DOF) of the joint. Human joints, which have both capsuloligamentous and joint articular constraints, can move freely in several directions of translation or rotation. The possible modes of joint movement represent the rotation and translational DOF of the joint. For example, the shoulder joint and elbow joint have been considered to have three and one rotational DOF, respectively. Therefore, the associated moment equilibrium equations consist of only the unknown muscle force variables. With the muscle force determined, the joint constraint forces and moments can then be determined by using the remaining equations of motion [5].

Decoupling of the constraint equations makes the procedure for solution easier and more comprehensible. However, under some conditions, it is inappropriate to use the decoupling procedure to solve muscle and joint forces independently.
**Reduction Method**

The goal of the reduction method is to reduce the degree of redundancy by reducing the number of unknown forces until the number of unknown forces is equal to the number of equations. Muscles with similar functions or common anatomic insertions and orientations can be grouped together, whereas qualitative electromyographic data can be used to eliminate inactive muscles. This would make the calculation become more easier. Although this method gives the joint force, the detailed behavior of individual muscles is lost from the solution.

**Optimization Method**

The distribution problem at a joint is, typically, an indeterminate problem, because the number of muscles, ligaments, and articular contact regions available to transmit force across a joint, in many cases, exceeds the minimum number of equations required to generate a determinate solution. An infinite number of possible solutions exists for the indeterminate equations. Determinate solutions are obtainable only with significant simplification of the functional anatomy. One method of solution without such simplification is that of seeking an optimum solution, that is, a solution that maximizes or minimizes some process or action [1].

In order to solve an optimization problem, its format must be specified. This is done by:
1. defining the cost function,
2. identifying the constraint functions,
3. specifying the design variables, and
4. setting the appropriate bounds for the design variables.

However selection and justification of optimal criteria have been major problems. A wide variety of cost functions have been used with different degrees of success, and these are discussed briefly in the next two sections. The criteria are grouped according to the nature of the optimization method—linear or nonlinear programming. Further, these criteria may include single or multiple objective functions. That why we do not use this method for this study.

**III. RESULTS AND DISCUSSION**

**A. Determination of Intersegmental Forces and Moments**

In our experiment the subject’s body weight (W) is 686 N, a 343-N weight (0.5 x mass(m) x gravity(g)) is held in the right hand a distance of 22 cm from the elbow joint center, and the 11-N forearm weight(0.016 x mg) having its center of mass 11 cm from the elbow. The forearm weight is taken from anthropometric data with segment weight/total body weight is 0.016mg [6]. The hand is externally rotated with θ = 30° (Fig. 2).

![Fig 2. Free body diagram of forearm during push-up with hand externally rotated of 30°](image)

The intersegmental force can be calculated from the force equilibrium equations:

\[ F_{lx} + L_x + W_x = 0 \]  \hspace{1cm} \text{(5)}

\[ F_{ly} + L_y + W_y = 0 \]  \hspace{1cm} \text{(6)}

where,

\[ W_x = W \sin 30°, \]

\[ W_y = -W \cos 30°, \]

\[ L_x = L \sin 30°, \]

\[ L_y = -L \cos 30°, \]

W = 11 N, and L = 343 N, which gives \( F_{lx} \) and \( F_{ly} \) to be –177 N and 306.5 N, respectively.

The negative sign indicates that the calculated force is in the negative direction of the coordinate system.

Similarly, the intersegmental moment \( M_{iz} \) can be calculated from the moment equilibrium equation,
\[ M_{lz} + L_y \times 22 + W_y \times 11 = 0 \]  \hspace{1cm} (7)

giving

\[ M_{lz} = 6638.5 \text{ N.cm} \]

**B. Reduction Method**

In push up, we assume that all of the elbow flexors are grouped as one muscle (Fig. 3). The moment equilibrium equation for flexion-extension consists of only two variables, the intersegmental moment, which has been calculated from the inverse dynamic problem, and the unknown muscle force. 45° and 4.4cm moment arm is simply taken from Murray [7]

\[ g \text{giving} \]

\[ M_{lz} = 6638.5 \text{ N.cm} \]

In this simplified consideration, the flexor force, \( F_F \) can be uniquely determined from

\[ M_{lz} = 6638.5 \text{ N.cm} = F_F \times \sin 45^\circ \times 4.4 \text{cm} \]

hence

\[ F_F = 2133 \text{ N} \]

With the muscle force determined, the joint constraint forces, \( F_{jx} \) and \( F_{jy} \) can then be determined based on the two force equilibrium equations,

\[ F_i = \sum F_m + F_j \]

\[ F_{jx} = -177 \text{N} = F_{jx} - F_F \times \cos 45^\circ \]

hence

\[ F_{jx} = 1331 \text{ N} \]

To the right: and

\[ F_{jy} = 306.5 \text{N} = F_{jy} + F_F \times \sin 45^\circ \]

hence

\[ F_{jy} = -1202 \text{ N} \]

That is, 1202 N downward

An alternative method of reducing the degree of redundancy is to increase the number of constraint equations. This is usually accomplished by assuming a force distribution between muscles based on anatomic consideration of the muscles' physiological cross-sectional areas.

**TABLE I**

<table>
<thead>
<tr>
<th>Muscle</th>
<th>PCSA(cm²)</th>
<th>Moment Arm (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brachialis</td>
<td>7</td>
<td>3.4</td>
</tr>
<tr>
<td>Biceps</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>Brachioradialis</td>
<td>1.5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

In the model, three muscles (biceps, brachialis, and brachioradialis) are considered for this calculation (Fig. 4). From the figure shown, the lever arms of the biceps (BIC), brachialis (BRA), and brachioradialis (BRD) are 4.6 cm, 3.4 cm, and 7.5 cm, respectively [6].

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**Fig. 4 Model of three muscles (the biceps, brachialis, and brachioradialis)**

Muscle forces are needed to maintain the forearm in the position shown can be determine By equilibrium of moments about the elbow,

\[ M_{lz} = 6638.5 \text{ N.cm} \]

\[ = 4.6 \times F_{BIC} + 3.4 \times F_{BRA} + 7.5 \times F_{BRD} \]
Note that there is only one equation, but there are three unknowns. It is not possible to solve this problem, which is termed an *indeterminate* problem. That is to say, the equations contain more unknown values of forces in the anatomic structures than there are equations to describe the joint behavior.

Physiologic constraints allow us to eliminate any solutions in which muscle forces are negative or unrealistically high. Still, there is an infinite number of solutions that can satisfy the equation, but it may be difficult or impossible to know which solution is the correct one.

In order to distribute the forces in the muscles and joint to obtain a unique solution, we may have to introduce additional constraint equations. For example, based on physiological considerations, the muscle force might be assumed to be proportional to the physiological cross-sectional areas (PCSA) of the muscles [6]:

\[
\frac{F_{BIC}}{F_{BRD}} = \frac{4.6}{1.5}
\]
\[
F_{BRD} = \frac{1.5}{4.6} F_{BIC}
\]

and

\[
\frac{F_{BIC}}{F_{BRA}} = \frac{4.6}{7.0}
\]
\[
F_{BRA} = \frac{7.0}{4.6} F_{BIC}
\]

Now there are three equations, which will make the problem uniquely solvable.

\[
6638.5 = 4.6F_{BIC} + 3.4 \left(\frac{7.0}{4.6}\right)F_{BIC} + 7.5 \left(\frac{1.5}{4.6}\right)F_{BIC}
\]
\[
F_{BIC} = 543 \text{ N}
\]

From Lou et al.[2] the normalized muscle force during anterior position is 20.601 (N/kg). The normalized muscle force fraction of forces for each muscle can be obtained by assuming a force distribution between muscles based on anatomic consideration of the muscles’ physiological cross-sectional areas. This result are stated in Table II.

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Force Force (N)</th>
<th>Normalized Muscle Force (N/kg)</th>
<th>Calculated</th>
<th>Lou et al. [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biceps</td>
<td>543.0</td>
<td>7.78</td>
<td>20.601</td>
<td></td>
</tr>
<tr>
<td>Brachialis</td>
<td>826.3</td>
<td>12.04</td>
<td>11.00</td>
<td></td>
</tr>
<tr>
<td>Brachioradialis</td>
<td>177.1</td>
<td>2.53</td>
<td>2.35</td>
<td></td>
</tr>
</tbody>
</table>

In general, the normalized biceps, brachialis, and brachioradialis forces derived in this analysis are at slightly difference with literature. The discrepancies between calculated and literature shown above are expected as the one from literature results (biceps, brachialis, and brachioradialis muscles force) are obtained from mean value of push-up during neutral position analysis while the analysis we did is during external rotation of push-up.

These values suggest the simplified approach adopted in this analysis does provide a useful approximation in investigating musculoskeletal loads and muscle forces at the ankle joint during squatting exercises.

**CONCLUSION**

Solving for muscle forces and joint reaction forces for even quite simple tasks is relatively complex. Many simplifications/assumptions were required like rigid body bones, single line of action for muscles, planar assumption and known muscle insertion and origin.

The slightly difference result achieved compare to literature suggest the simplified approach adopted in this analysis does provide a useful approximation in investigating musculoskeletal loads and muscle forces at the ankle joint during squatting exercise.

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