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The Comparison of Standard Bootstrap and Robust Outlier Detections Procedure in Bilinear (1,0,1,1) Model

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ABSTRACT

Parameter estimation is the most important part in modelling and predicting time series. However, the existence of outliers in the data will affect the estimation, which consequently jeopardizes the validity of the model. Therefore, the existence of outliers in the data must be first detected before the next process can be performed. The best outlier detection procedure can ensure data are free of outliers and achieve the best value parameter estimation. One of the procedures is using the bootstrap method to obtain the variance of the estimated magnitude of outlier effects. The variance found directly from the bootstrap method is called the 'standard' variance. However, the bootstrap method is quite complex in obtaining the variance value. As alternatives, trimming methods involving robust estimators such as a median absolute deviation (MADn) and alternative median-based deviation called Tn in the 'robust' variance calculation are used to replace the 'standard' variance. This method involves direct calculation to obtain the value of the variance from the estimated magnitude of outlier effects. To see the effectiveness of this method, the bilinear (1,0,1,1) model and two robust detection procedures, namely, modified one-step M-estimator (MOM) with MADn and MOM with Tn were used. Later, these two procedures are evaluated and compared with the bootstrap method through simulation studies based on the probability of outlier detection. Through the findings obtained, in general, the standard bootstrap procedure performs better than the robust procedure performance in detecting the existence of outliers in the bilinear (1,0,1,1) model.

Keywords: Bilinear, Bootstrap, *MOM*, *MADn*, *Tn*, Variance.

1. INTRODUCTION

Time series models can be categorized into linear models and nonlinear models. The linear model is more popular among researchers due to its simplicity. However, not all linear models are sufficient or appropriate for time series data. Under certain circumstances, the nonlinear model may be more appropriate for the data. The bilinear model is the simplest model among nonlinear models because it is naturally a continuation of the linear model (Ramakrishnan and Morgenthaler, 2010). In this study, the bilinear model is chosen based on the advantages of its properties compared to other nonlinear models. Some examples of the application of bilinear models in various fields are the use of bilinear models to analyze the macroeconomic and financial series (Hristova, 2004), modelling revenue series (Usoro and Omekara, 2008) and estimating the rate of death of a particular disease (Shangodoyin, Ojo, Olaomi and Adebile, 2012). In bilinear time series model, parameter estimation is the most important part. However, with the existence of outliers in the time series data, it will affect the estimated value (Hordo, Kiviste, Sims and Lang, 2006). Therefore, the outlier detection procedure must be identified to obtain the best parameter estimation value. In general, there are four types of outliers (OT), namely, additional outlier (AO), innovational outlier (IO), level change (LC) and temporary change (TC). AO and IO are the common types of outlier found in bilinear time series data (Zaharim, Ahmad, Mohamed and

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Yahaya, 2007). AO is the type of outliers that affects a single observation at a time point (Abuzaid, Mohamed and Hussin, 2014). Meanwhile, IO is characterized by a single strange observation at a time point but also affects subsequent observations with the effect gradually dying out (Abuzaid et al., 2014).

In the meantime, the bootstrap method is used to obtain the magnitude of outlier effects in a bilinear model. This procedure is carried out through the process of drawing random samples with replacement. The bootstrap method is a computer-based method for estimating the standard error of $\hat{\theta}$ (Efron and Tibshirani, 1986). The classical bootstrap procedure yields classical bootstrap mean and variance. A study to detect AO and IO has been done in a bilinear model using the classical bootstrap procedure (Zaharim et al., 2007). In this study, the bootstrap method is used to calculate the estimate variance of the magnitude outlier effect. This variance is called the 'standard' variance. However, without the use of a computer, the calculation of the variance using bootstrap method seems too complicated. Therefore, in this paper, a substitution approach using trimming method was used which involves robust estimators of location i.e. modified one-step *M*-step (*MOM*), as well as robust scale estimators namely median absolute deviation (*MADn*) and *Tn*. Using this approach, the 'standard' variance is replaced by 'robust' variance. Trimming is a method of eliminating outliers from each distribution tail. This method is useful in achieving good efficiency and high power (Md Yusof, Othman and Syed Yahaya, 2008). Since the estimators chosen have the highest breaking point, which is least affected by extreme values, the method is deemed robust (Syed Yahaya, Othman and Keselman, 2004). Wilcox and Keselman (2003) introduced *MOM* as a measure of central tendency when testing the effects of treatment. This estimator is calculated based on data remaining from empirically determined trimming. The sample of this estimator can be classified as robust central tendency estimator which has the highest breakdown point as it is one of the sample median representatives. Meanwhile, *MADn* and *Tn* are among the best two robust estimators suggested by Rousseeuw and Croux (1993). *MADn* is a popular robust estimator scale with the highest breakdown point and least affected by extreme values. Meanwhile, *Tn* estimator also has the highest breakdown point of 50% and its efficiency is about 52%, making it more effective than *MADn* and a continuous influence function. This scale estimator is also simple and has a unique explicit formula.

This study focuses on the detection of AO and IO in the bilinear (1,0,1,1) model. Two robust detection procedures i.e. *MOM* with *MADn* and *MOM* with *Tn* are introduced to evaluate the performance of outlier detection in bilinear (1,0,1,1) model. Through a simulation study, the performance of the two robust detection procedures and standard bootstrap detection procedure are compared and measured in terms of probability of outlier detection.

2. LITERATURE REVIEW

2.1 Robust estimators

In this study, two robust scale estimators, namely, *MADn* and *Tn* are used to calculate the deviation of observations, while the robust location estimator *MOM* is used to obtain the mean of observations.

(a) *MADn*

One of the scale estimators considered in this study is *MADn* which was suggested by Rousseeuw and Croux (1993). The formula of this estimator is given by:

$$MADn = 1.4826 \times med_i \left| x_i - med_j x_j \right| \quad (1)$$

where $X = (x_1, x_2, \ldots, x_n)$ represents a random sample of any distribution, $med_i\, x_i$ represents sample median, $i = 1,2,3,\ldots, n$ and $j = 1,2,3,\ldots, n$.

(b) *Tn*

Another scale estimator proposed for this study is *Tn* by Rousseeuw and Croux (1993). The formula for this robust scale estimator is given by:

$$Tn = 1.3800 \frac{1}{h} \sum_{k=1}^{h} \left\{ \underset{j \neq i}{med} \left| x_i - x_j \right| \right\}_{(k)} \tag{2}$$

where $h = \left[\frac{n}{2}\right] + 1$, $i = 1,2,3,\ldots, n$ and $j = 1,2,3,\ldots, n$.

(c) Modified one-step *M*-estimator (*MOM*)

Meanwhile, the robust location estimator *MOM* is given by:

$$\hat{\theta}_j = \sum_{i=i_1+1}^{n_j - i_2} \frac{Y_{(i)j}}{n_j - i_1 - i_2} \tag{3}$$

Where

$Y_{(i)j}$ = the i^{th} ordered observations in group *j*,
n_j = number (#) of observations for group *j*,
i_1 = number (#) of observations Y_{ij}, such that

$$\left(Y_{ij} - \hat{M}_j\right) < -2.24\left(MADn_j\right), \tag{4}$$

i_2 = number (#) of observations Y_{ij}, such that

$$\left(Y_{ij} - \hat{M}_j\right) > 2.24\left(MADn_j\right), \tag{5}$$

For *MOM* with *Tn*, just replace *MADn* in Equations (4) and (5) with *Tn*.

2.2 Bilinear Model

The bilinear (1,0,1,1) model is given by

$$Y_t = a\,Y_{t-1} + b\,Y_{t-1}\, e_{t-1} + e_t \tag{6}$$

where Y_t and e_t each represents the outlier-free observation and outlier-free residual at time *t*, where $t = 1,2,3,\ldots$. Both Y_t and e_t are also called the "original" observation and the "original" residual, respectively. e_t are assumed to follow a normal distribution with mean zero and variance σ^2. While, *a* and *b* are the coefficients of the model. Meanwhile, the bilinear (1,0,1,1) model with the existence of outlier is represented by

$$Y_t^* = aY_{t-1}^* + bY_{t-1}^* e_{t-1}^* + e_t^* \tag{7}$$

where Y_t^* is the contaminated observation and e_t^* represents the contaminated residual. Y_t^* and e_t^* exist when there is an outlier in the data at a certain time point *t*, where $t = 1,2,3,\ldots,n$.

2.2.1 AO Effects on Original Observations and Residuals

When there is no outlier in the data at a time point *t*, such that $t = 1,2,3,\ldots,n$, the observations (Y_t) is known as the "original" observations. If AO exists in the data, the symbol $Y_{t,AO}^*$ is used to signify the existence of the outlier, and is known as "AO effect on observation". The effect of this outlier exists only at a time point $t = d$ with ω a magnitude of outlier effect from bilinear (1,0,1,1) model. For a time point $t \neq d$, clearly $Y_{t,AO}^* = Y_t$ and the full formulation of AO effects on Y_t is given by:

$$Y_{t,AO}^* = \begin{cases} Y_t & for \quad t \neq d \\ Y_t + \omega & for \quad t = d \end{cases} \tag{8}$$

Equation (8) indicates that the effect of AO on Y_t occurs only at a one-time point $(t = d)$ while the rest of the time points are unaffected.

Meanwhile, the "original' residuals (e_t) are obtained when there is no outlier in the data at a time point *t*. The "AO effect on residual" is denoted by $e_{t,AO}^*$. $e_{t,AO}^* = e_t$ with a time point $t < d$, and the equation will be different with a time point $t \geq d$ and $k \geq 0$. Generally, for a time point $t = d + k$, the formulation for $e_{d+k,AO}^*$ is given by:

$$e_{d+k,AO}^* = e_{d+k} - \omega A_{k,AO} \tag{9}$$

where

$$A_{k,AO} = \begin{cases} -1 & for \quad k = 0 \\ (a_k + b_{k1} e_{d+k-1}) - \sum_{j=1}^{k} (b_{1j} Y_{d+k-1,AO}^*) A_{d+k-j,AO} & for \quad k \geq 1 \end{cases},$$

a and b are constant numbers. Based on Equation (9), several residuals for a time point $t \geq d$ should be affected.

2.2.2 IO Effects on Original Observations and Residuals

The IO effects on observations at a time point $t < d$ is given by $Y_{t,IO}^* = Y_t$ and the equation of IO effects on Y_t for $t \geq d$ is given by

$$Y_{d+k,IO}^* = Y_{d+k} + \omega A_{k,IO} \tag{10}$$

where

$$A_{k,IO} = \begin{cases} 1 & for \quad k = 0 \\ \sum_{m=1}^{k} (a_m + b_{m1} e_{d+k}) A_{k-m,IO} & for \quad k \geq 1 \end{cases}$$

Equation (10) shows that the existence of IO in bilinear (1,0,1,1) model not only affects Y_t at a one-time point but also some of the subsequent Y_t.

The symbol $e^*_{t,IO}$ is used when there is an IO effect on the original residual in bilinear (1,0,1,1) model. $e^*_{t,IO} = e_t$ with a time point $t < d$, and the equation will be different from a time point $t \geq d$ and $h \geq 0$. Generally, for a time point $t = d + h$, the equation for $e^*_{d+h,IO}$ is given by:

$$e^*_{d+h,IO} = e_{d+h} + \omega f_{d+h} \tag{11}$$

Where

$$f_{d+h} = \begin{cases} A_{0,IO} & for \quad h = 0 \\ A_{h,IO} - (\sum_{m=1}^{h} b_{m1} f_{d+h-m}) Y^*_{d+h-1,IO} - \sum_{k=1}^{h} (a_k + b_{k1} e_{d+h-k}) A_{h-k,IO} & for \quad h \geq 1 \end{cases}$$

The equation indicates that the existence of IO not only changes the residual at a time point ($t = d$) but also changes some of the subsequent residuals.

3. METHODOLOGY

3.1 Standard Bootstrap Detection Procedure

The full standard bootstrap procedure in detecting the existence of AO or IO in Y_t is described based on the following phases below.

Phase 1: Constructing null and alternative hypotheses

For a standard bootstrap detection procedure, the hypotheses are, H_0: $\omega = 0$ and H_1: $\omega \neq 0$ in bilinear (1,0,1,1) model with an outlier at a time point *t*. Then, the statistical test for the hypothesis is:

$$\hat{\tau}_{OT,standard,t} = \frac{\left(\hat{\omega}_{OT,t} - \overline{\tilde{\omega}}_{OT,standard,t}\right)}{\tilde{\sigma}_{OT,standard,t}} \tag{12}$$

Phase 2: Obtaining the magnitude of outlier effects

The statistics to measure the magnitude of outlier effects for AO and IO can be obtained using the least-squares method. Consider the following equation:

$$S = \sum_{t=1}^{n} e_t^2 = \sum_{t=1}^{d-1} e_t^2 + \sum_{k=0}^{n-d} \left(e^*_{d+k} - \{-1\}^k f_{d+k}(\omega)\right) \tag{13}$$

Equation (13) is then minimized with respect to ω, yielding the following measures of outlier effects:

$$\hat{\omega}_{OT} = \frac{\sum_{k=0}^{n-d}\left[\{-1\}^{l} e^{*}_{d+k} A_{k,OT}\right]}{\sum_{k=0}^{n-d} A^{2}_{k,OT}} \tag{14}$$

where

$$A_{k,AO} = \begin{cases} 1 & for \quad k=0 \\ -(a_k + b_{k1}e_{d+k-1}) - \sum_{j=1}^{k} b_{1j} Y^{*}_{d+k-j,AO} & for \quad k \geq 1 \end{cases}$$

and

$$A_{k,IO} = \begin{cases} 1 & for \quad k=0 \\ -\sum_{m=1}^{k} b_{1m} Y^{*}_{d+k-1,IO} A_{k-m,IO} & for \quad k \geq 1 \end{cases}$$

*OT = AO or IO

Phase 3: Obtaining a standard deviation of magnitude of outlier effects

Observing Equation (14), the complexity of the equation makes the determination of an algebraic expression for a standard deviation of ω_{OT} insurmountable. The bootstrap procedure is used to obtain the estimates of the standard deviation of ω_{OT}. The procedure, which is carried out through the process of drawing random samples with replacement from the residuals, is described as follows:

(a) Let $e_1, e_2, ..., e_n$ be the "original" residuals. Sampling with replacement is carried out from the "original" residuals giving a bootstrap sample of size *n*, say, $e^{*(1)} = e^{*}_1, e^{*}_2, \ldots, e^{*}_n$.
(b) Let B be the number of bootstrap samples. The process to obtain $e^{*(1)} = e^{*}_1, e^{*}_2, \ldots, e^{*}_n$ is repeated B times and the bootstrap samples of B sets is given by $e^{*(1)}, e^{*(2)}, ..., e^{*(B)}$.
(c) Calculate $\tilde{\omega}_M$ for each bootstrap sample $e^{*(M)}$, where $M = 1,2,...,B$.
(d) The sample standard deviation of $\tilde{\omega}_M$ is given by:

$$\tilde{\sigma}_{standard} = \left\{ \frac{\sum_{M=1}^{B}\left(\tilde{\omega}_M - \overline{\tilde{\omega}}_M\right)^2}{(B-1)} \right\}^{1/2} \tag{15}$$

where

$$\overline{\tilde{\omega}}_M = B^{-1}\sum_{M=1}^{B}\tilde{\omega}_M$$

Efron and Tibshirani (1986) has shown that as $B \to \infty$, $\tilde{\sigma}_{standard}$ approaches $\hat{\sigma}$, the bootstrap estimate of the standard deviation. Furthermore, it has been reported that a decent estimate can be obtained using $B = 25$ and $B = 200$ (Efron and Tibshirani, 1993).-In this paper, Equation (15) refers to the standard deviation with the bootstrap procedure.

Phase 4: Detecting the existence of AO or IO

The complete steps to detect the existence of AO or IO are described below.

(1) Compute statistical test value, $\hat{\tau}_{OT,S,t}$ based on the estimated ω in Phase 2 for each t, where $t = 1,2,3,\ldots,n$. S refers to the standard formula.
(2) The maximum value of $\hat{\tau}_{OT,S,t}$ is determined, which is represented by $\eta_{S,t} = \max_{t=1,2,\ldots,n} \left\{ \left| \hat{\tau}_{OT,S,t} \right| \right\}$.
(3) For any t if $\eta_{S,t} > CV$ (CV is a critical value), then H_0 is rejected.

Finally, the existence of AO or IO in Y_t is detectable.

3.2 Robust Estimators of the Magnitude of Outlier Effect

In the proposed robust detection procedures, instead of employing the bootstrap method to calculate the standard deviation of the magnitude of outlier effect, $\hat{\omega}$, this study proposes to separately use two robust scale estimators *MADn* and *Tn*, while the robust location estimator *MOM* is used to obtain a mean of the magnitude of outlier effect, $\bar{\tilde{\omega}}$.

The *MADn* for the standard deviation of $\hat{\omega}$ is given by:

$$\tilde{\sigma}_{MAD_n} = 1.4826 \times median \left| \hat{\omega} - \tilde{\hat{\omega}} \right| \tag{16}$$

where $\tilde{\hat{\omega}}$ is the median of $\hat{\omega}$.

Meanwhile, the *Tn* for the standard deviation of $\hat{\omega}$ is given by:

$$\tilde{\sigma}_{T_n} = 1.3800 \times \frac{1}{h} \sum_{k=1}^{h} \left\{ \underset{M \neq M'}{median} \left| \hat{\omega}_{(M)} - \hat{\omega}_{(M')} \right| \right\}_{(k)} \tag{17}$$

where $h = \left[\frac{n}{2} \right] + 1$.

While the Modified one-step *M*-estimator (*MOM*) for the mean of the magnitude of the outlier effect is given by:

$$\bar{\hat{\omega}}_{MOM,j} = \sum_{i=i_1+1}^{n_j - i_2} \frac{\hat{\omega}_{(i)j}}{n_j - i_1 - i_2} \tag{18}$$

where

$\hat{\omega}_{(i)j}$ = the i^{th} ordered $\hat{\omega}$ in group *j*.
n_j = number (#) of $\hat{\omega}$ for group *j*.
i_1 = number (#) of $\hat{\omega}_{ij}$, such that

$$\left(\hat{\omega}_{ij} - \tilde{\hat{\omega}}_j \right) < -2.24 \left(MADn_j \right). \tag{19}$$

i_2 = number (#) of $\tilde{\omega}_{ij}$, such that

$$\left(\hat{\omega}_{ij} - \tilde{\tilde{\omega}}_j\right) > 2.24\left(MADn_j\right). \tag{20}$$

This yields the formula for *MOM* with *MADn.* To obtain the formula for MOM with Tn, the $MADn$ in Equations (19) and (20) is replaced with Tn.

3.3 Robust Bootstrap Detection Procedure

Meanwhile, to detect AO and IO in Y_t, the robust bootstrap detection procedure is used and described in the following phases below.

Phase 1: Constructing the null and alternative hypotheses

For a robust detection procedure, the hypotheses are, H_0: $\omega = 0$ and H_1: $\omega \neq 0$ in bilinear (1,0,1,1) model with an outlier at a time point *t*. Then, the statistical test for the hypothesis is:

$$\hat{\tau}_{OT,MADn,t} = \frac{\left(\hat{\omega}_{OT,t} - \bar{\hat{\omega}}_{OT,MOM,t}\right)}{\tilde{\sigma}_{OT,MADn,t}} \tag{21}$$

$$\hat{\tau}_{OT,Tn,t} = \frac{\left(\hat{\omega}_{OT,t} - \bar{\hat{\omega}}_{OT,MOM,t}\right)}{\tilde{\sigma}_{OT,Tn,t}} \tag{22}$$

where OT represents the type of outlier, AO or IO and $t = 1,\ldots,n$.

Phase 2: Obtaining the magnitude of outlier effects

This step is the same as Phase 2 of the standard bootstrap detection procedure.

Phase 3: Obtaining robust variance of the magnitude of outlier effects

Calculate $\tilde{\sigma}_{OT,MADn,t}$ using a robust formula of *MADn* in Equation (16) and calculate $\tilde{\sigma}_{OT,Tn,t}$ using a robust formula of *Tn* in Equation (17), while for $\bar{\tilde{\omega}}_{OT,MOM,t}$ is obtained from *MOM* in Equation (18).

Phase 4: Detecting the existence of AO or IO

The complete steps to detect the existence of AO or IO are described as follows:

(1) Compute statistical test value, $\hat{\tau}_{OT,F,t}$ based on Equation (21) and Equation (22) for each t, where $t = 1,2,\ldots,n$. *F* refers to *MADn* and *Tn* formula in Equation (16) and Equation (17), respectively.
(2) The maximum value of $\hat{\tau}_{OT,F,t}$ is determined, which is represented by $\eta_{F,t} = \max_{t=1,2,\ldots,n}\left\{\left|\hat{\tau}_{OT,F,t}\right|\right\}$.
(3) For any *t*, where $t = 1,2,\ldots,n$, if $\eta_{F,t} > CV$ (CV is a critical value), then H_0 is rejected.

Finally, the existence of AO or IO in Y_t is detectable using the two robust detection procedures of *MOM* with *MADn* and *MOM* with *Tn.*

4. SIMULATION AND RESULTS

A simulation study has been conducted to observe the performance of the investigated procedures in detecting AO and IO. The robust outlier detection procedures performance using both *MOM* with *MADn* and *MOM* with *Tn* are compared to the standard bootstrap detection procedure. The effectiveness of the proposed procedure is measured by the probability of outlier detection. The data were simulated using S-Plus package. To investigate the performance of the proposed detection procedures, the combinations of the following factors are considered:

(a) Two types of outliers: AO and IO.
(b) Five underlying bilinear (1,0,1,1) model with different combinations of coefficients (a,b) for both types of outliers.
(c) A single outlier will be introduced at a time point $t = 40$ in sample size (n) of 100.
(d) $B = 100$ for the number of sets of bootstrap samples.
(e) Two different values of the magnitude of the outlier effect: $\omega = 3$, 5.
(f) Five different levels of critical values (CV): $CV = 2.0$, 2.5, 3.0, 3.5, 4.0.

For each given bilinear (1,0,1,1) model, 100 series of length 100 is generated using *rnorm* procedure in S-Plus. The series is generated to contain only one of the outlier types. For the magnitude of the outlier effect (ω), this study uses $\omega = 3$ and $\omega = 5$. These values are selected to see the change of the probability of outlier detection when the value of the outlier effect is increased from 3 to 5. These values are selected based on earlier studies by Mohamed et al. (2011). The performance of outlier detection procedure in bilinear (1,0,1,1) model can be observed in Tables 1-5 for AO while Tables 6-10 for IO. In the tables, the values in columns 3-5 represent the probability of outlier detection of the respective type of outlier with the correct location at a time point $t = 40$.

The results across the tables show that the performance of the standard bootstrap outlier detection procedure is better than the proposed robust outlier detection procedure for almost all the models used. For example, in Tables 1, 2 and 6 it can be observed that the results for all the critical values used are in favour of the standard bootstrap method. The same situation happens when the magnitude of outlier effect is $\omega = 3$ combined with a critical value greater than 2.5, the performance of standard bootstrap detection procedure is the best compared to the robust detection procedure for all models used. A very low proportion of correct detection is observed especially in the case of robust detection procedure when the critical values are increased to 3.5 and 4.0. This happens because the values of the outlier for those cases are not as large as that critical value. Critical value values between 2.0 to 3.0 are considered to be most appropriate to detect the existence of outliers for both detection procedures.

In this study, the coefficient values between 0.1 and 0.5 are selected as a sufficient condition for the existence of a stationary process for the model, (Ismail, 2009). The selection of a combination of coefficients (a, b) may affect the probability of detection. When the coefficient value is increased, the probability of detection decreases for both AO and IO cases due to the larger value of coefficients used as shown in tables. Meanwhile, the performance of the proposed outlier detection procedures is better when larger ω is used. In general, the comparison reveals that the standard bootstraps detection procedure performs better than the proposed robust detection procedure in detecting the AO and IO.

Table 1 The performance in detecting AO in bilinear (1,0,1,1) model with coefficient (a=0.1, b =0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.62	0.39	0.47
	2.5	0.62	0.16	0.23
	3.0	0.51	0.05	0.10
	3.5	0.26	0.00	0.05
	4.0	0.15	0.00	0.02
5	2.0	0.96	0.95	0.94
	2.5	0.96	0.81	0.81
	3.0	0.93	0.55	0.64
	3.5	0.85	0.27	0.42
	4.0	0.71	0.12	0.21

Table 2 The performance in detecting AO in bilinear (1,0,1,1) model with coefficient (a=0.3, b =0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.66	0.35	0.39
	2.5	0.59	0.15	0.23
	3.0	0.38	0.04	0.12
	3.5	0.20	0.01	0.03
	4.0	0.07	0.01	0.02
5	2.0	0.97	0.94	0.93
	2.5	0.97	0.80	0.83
	3.0	0.95	0.46	0.60
	3.5	0.85	0.21	0.38
	4.0	0.64	0.08	0.18

Table 3 The performance in detecting AO in bilinear (1,0,1,1) model with coefficient (a=0.1, b =0.5)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.41	0.32	0.39
	2.5	0.38	0.15	0.26
	3.0	0.34	0.07	0.13
	3.5	0.21	0.03	0.04
	4.0	0.12	0.01	0.01
5	2.0	0.60	0.71	0.71
	2.5	0.60	0.60	0.65
	3.0	0.60	0.29	0.56
	3.5	0.59	0.13	0.34
	4.0	0.51	0.03	0.17

Table 4 The performance in detecting AO in bilinear (1,0,1,1) model with coefficient (a=-0.1, b =-0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.53	0.43	0.47
	2.5	0.51	0.20	0.27
	3.0	0.38	0.05	0.09
	3.5	0.19	0.01	0.02
	4.0	0.09	0.00	0.01
5	2.0	0.95	0.95	0.95
	2.5	0.95	0.76	0.77
	3.0	0.95	0.55	0.58
	3.5	0.90	0.25	0.36
	4.0	0.75	0.19	0.17

Table 5 The performance in detecting AO in bilinear (1,0,1,1) model with coefficient (a=-0.3, b =0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.70	0.52	0.52
	2.5	0.67	0.17	0.33
	3.0	0.48	0.06	0.19
	3.5	0.24	0.02	0.08
	4.0	0.12	0.01	0.03
5	2.0	0.98	0.98	0.94
	2.5	0.98	0.80	0.82
	3.0	0.98	0.54	0.66
	3.5	0.89	0.25	0.47
	4.0	0.73	0.10	0.23

Table 6 The performance in detecting IO in bilinear (1,0,1,1) model with coefficient (a=0.1, b =0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.60	0.39	0.47
	2.5	0.58	0.16	0.23
	3.0	0.36	0.05	0.10
	3.5	0.18	0.00	0.05
	4.0	0.01	0.00	0.02
5	2.0	0.98	0.95	0.94
	2.5	0.98	0.81	0.81
	3.0	0.96	0.55	0.64
	3.5	0.89	0.27	0.42
	4.0	0.69	0.12	0.21

Table 7 The performance in detecting IO in bilinear (1,0,1,1) model with coefficient (a=0.3, b =0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.55	0.35	0.39
	2.5	0.55	0.15	0.23
	3.0	0.41	0.04	0.12
	3.5	0.23	0.01	0.03
	4.0	0.09	0.01	0.02
5	2.0	0.92	0.94	0.93
	2.5	0.92	0.80	0.83
	3.0	0.90	0.46	0.60
	3.5	0.74	0.21	0.38
	4.0	0.60	0.08	0.18

Table 8 The performance in detecting IO in bilinear (1,0,1,1) model with coefficient (a=0.1, b =0.5)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.29	0.32	0.39
	2.5	0.21	0.15	0.26
	3.0	0.18	0.07	0.13
	3.5	0.12	0.03	0.04
	4.0	0.07	0.01	0.01
5	2.0	0.65	0.71	0.71
	2.5	0.65	0.60	0.65
	3.0	0.63	0.29	0.56
	3.5	0.62	0.13	0.34
	4.0	0.50	0.03	0.17

Table 9 The performance in detecting IO in bilinear (1,0,1,1) model with coefficient (a=-0.1, b =-0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.49	0.43	0.47
	2.5	0.49	0.20	0.27
	3.0	0.37	0.05	0.09
	3.5	0.15	0.01	0.02
	4.0	0.07	0.00	0.01
5	2.0	0.94	0.95	0.95
	2.5	0.94	0.76	0.77
	3.0	0.91	0.55	0.58
	3.5	0.85	0.25	0.36
	4.0	0.67	0.19	0.17

Table 10 The performance in detecting IO in bilinear (1,0,1,1) model with coefficient (a=-0.3, b =0.1)

ω	C.V	Standard	*MOM* with *Tn*	*MOM* with *MADn*
3	2.0	0.52	0.52	0.52
	2.5	0.50	0.17	0.33
	3.0	0.36	0.06	0.19
	3.5	0.23	0.02	0.08
	4.0	0.08	0.01	0.03
5	2.0	0.95	0.98	0.94
	2.5	0.95	0.80	0.82
	3.0	0.92	0.54	0.66
	3.5	0.86	0.25	0.47
	4.0	0.63	0.10	0.23

5. CONCLUSION

This paper proposed two outlier detection procedures for bilinear (1,0,1,1) model to detect AO and IO. The procedures are the standard bootstrap and the robust *MOM* with *Tn* and *MOM* with *MADn*. The robust procedures are compared with the standard bootstrap procedures under various factors. Based on the simulation results, the probability of outlier detection using *MOM* with *Tn* and *MOM* with *MADn* is almost identical, otherwise, the best result is the standard bootstrap procedure. In general, the performance of the standard bootstrap detection procedure is better than both robust detection procedures based on the results obtained.

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