

KUKUM

**CLASSIFICATION OF BINARY INSECT
IMAGES USING FUZZY AND GAUSSIAN
ARTMAP NEURAL NETWORKS**

By

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ABSTRAK

Pengenalan dan pengelasan sesuatu objek merupakan rutin harian dalam kehidupan kita. Mata sebagai sebuah kamera mengambil imej objek yang tertentu kemudiannya menghantar imej tersebut kepada otak untuk dikenalpasti. Oleh yang demikian, sistem penglihatan manusia memberi ilham kepada para penyelidik untuk membina sistem penglihatan mesin. Sebagai satu bahagian penting dalam sistem penglihatan mesin, kajian ini tertumpu kepada dua (2) fasa penting iaitu; pengekstrakan fitur dan pengelasan. Dalam pengekstrakan fitur enam (6) jenis teknik momen tak varian yang berbeza telah dikaji untuk pengekstrakan fitur bentuk sejagat bagi imej serangga berbentuk binari. Ianya terdiri daripada Momen Geometrik tak varian (GMI), Momen Bersatu tak varian (UMI), Momen Zernike tak varian (ZMI), Momen Legendre tak varian (LMI), Momen Tchebichef tak varian (TMI) dan Krawtchouk tak varian (KMI). Fitur ini kemudiannya dihantarkan kepada rangkaian neural 'Fuzzy ARTMAP' (FAM) dan 'Gaussian ARTMAP' (GAM) untuk dikelaskan dan dikenalpasti. Dalam rangkaian neural GAM, rumus 'gamma threshold' diperkenalkan untuk mendapati nilai awal bagi taburan Gaussian semasa sesi latihan. Adalah didapati KMI merupakan teknik pengekstrakan fitur yang terbaik untuk mengekstrak maklumat bentuk sejagat bagi imej serangga jika dibandingkan dengan GMI, UMI, ZMI, LMI and TMI. Penemuan ini berdasarkan nilai terendah Jumlah Min Ralat Mutlak (TPMAE) (0.03%-1.01). Kaedah latihan dan pengujian untuk kedua-dua rangkaian neural adalah berdasarkan kepada teknik validasi 4-lipat bersilang. Adalah juga didapati pencapaian rangkaian neural FAM turut dipengaruhi oleh jenis teknik pernormalisasi yang digunakan. Teknik pernormalisasi Linear Pengskalaan Pembaikan (ILS) menghasilkan keputusan pengelasan yang tertinggi berbanding kaedah Jarak Unit (UR) serta Jarak Unit Pembaikan (IUR). Adalah juga didapati, rangkaian neural GAM merupakan teknik pengelasan serangga yang lebih baik jika dibandingkan dengan rangkaian neural FAM dengan menghasilkan ketepatan pengelasan sehingga 99.58% manakala ketepatan pengelasan bagi rangkaian neural FAM ialah 82%.

ABSTRACT

Object recognition and classification is an essential routine in our daily lives. Our eyes act as a camera capturing the image of particular object and sending it to the brain to be recognized. Thus, the eye vision system inspires researchers to create machine vision systems. As a significant part of the machine vision system, this research focused on two (2) important phases of the system; feature extraction and classification. As for the feature extraction six (6) different types of moment invariant techniques namely Geometric moment invariant (GMI), United moment invariant (UMI), Zernike moment invariant (ZMI), Legendre moment invariant (LMI), Tchebichef moment invariant (TMI) and Krawtchouk moment invariant (KMI) are used to extract the global shape features of the binary insect images. These features are then channeled to the Fuzzy ARTMAP (FAM) and Gaussian ARTMAP (GAM) neural network to be classified and recognized. In the GAM neural network, a gamma threshold is proposed to find the optimal value for gamma parameter acting as the initial value for a Gaussian distribution in the training phase. It is found that KMI is the best technique for features extraction of the global shape information of the insect images as compared to GMI, UMI, ZMI, LMI and TMI. The finding is based on the lowest value of Total Min Absolute Error (TPMAE) (0.03%-1.01). The training and testing method for both neural networks is based on 4-folds cross validation technique. It is also found that the performance of FAM neural network is influenced by the types of normalization technique utilized. The Improved Linear Scaling (ILS) normalization technique generated the highest classification rate by the FAM neural network when compared to Unit Range (UR) and Improved Unit Range (IUR). It is further found that GAM neural network is a better insect classification technique when compared to FAM neural network producing the classification accuracy up to 99.58% whereby the classification accuracy of FAM neural network is 82%.

CHAPTER 1

INTRODUCTION

1.1 Background

There are over a million types of insects in this world today with different types of colors and shape, thus making the task of insect classification and recognition a challenging ordeal. Furthermore, insect identification is difficult because it requires a detail understanding of insect taxonomy as well as the jargon and terms of morphological characteristics. Therefore, insect classification traditionally depends on taxonomists, but these professional individuals are not always available in all areas. Nevertheless, computer vision is a technology with the potential to make complete automation possible in insect classification, because it can fully utilize the huge potential offered by information technology, instead of relying on users to compare specimens to images or illustration.

Therefore, for the purpose of this research is to investigate the use of Moment Invariant technique as global shape descriptors with a combination of supervised ARTMAP neural network in order to classify insect images. Moment invariant had been proven as an effective technique especially for shape descriptor of binary or grey scaled images in many vision applications. Thus, six (6) difference types of moment invariant have been studied and compared for both inter and intraclass analysis that are Geometric moment invariant (GMI), United moment invariant (UMI), Zernike moment invariant (ZMI), Legendre moment invariant (LMI), Tchebichef moment invariant (TMI) and Krawtchouk moment invariant (KMI). Designing a suitable classifier is

significant in order to classify the extracted shape features. Hence, Fuzzy ARTMAP (FAM) and Gaussian ARTMAP (GAM) were used and the performance of both neural networks is compared. Nevertheless, the results gained from this work will hopefully be the basis of further research in developing an insect recognition system for more practical use.

1.2 Problem Statement

In practical environment, one might obtain several images which belong to the same object that contain different scaling and orientation factors compare to its original image. Thus, the feature vector produced by moment invariant techniques for the particular images will typically have some differences in their values. On the other hand, different objects definitely will cause the moment to produce dissimilar value of feature vectors. Therefore, in this research try to distinguish two terminologies which will help for further understand both the conditions.

a) Intraclass characteristics

Intraclass refers to the similarity between the values of feature vectors that are used for describing the same object. Hence, an effective moment techniques should have small error in generating different feature vectors for the same object.

b) Interclass characteristics

Interclass refers to the differences between the values of feature vectors that are used for describing dissimilar object.

Nevertheless, both characteristics are important factors that will influence the final classifications result of insect images. Therefore, this research will try to investigate both elements inside every feature vectors produced by moment invariant techniques applied. Both elements are also essential elements in determining the best technique for feature extraction and classification process. Figure 1.1 describes the differences between intra and interclass characteristics among feature vectors. Given

two set of data A and B whereas δ_s is the measurement for the similarity between data in the same group. While δ_d refer to the amount of differences between those two groups.

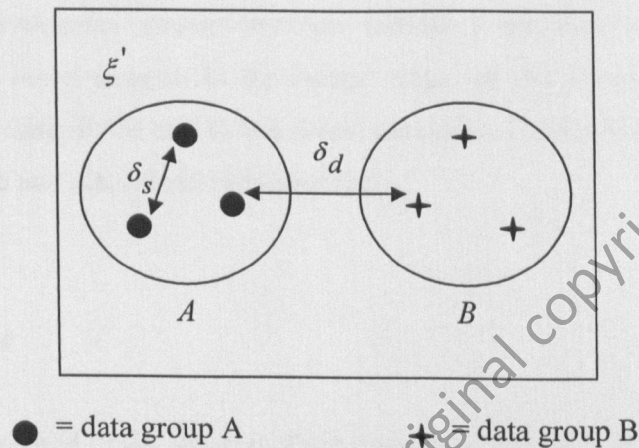


Figure 1.1

A comparison between intra and interclass characteristics

1.3 Research objective

There are five fundamental objectives of this work that are:

- a) To study the performance of six (6) different types of moment invariant technique that are used to extract the global shape features of binary insect images.
- b) To evaluate in terms of intraclass and interclass analysis between all six (6) types of moment invariant techniques based on the new error computation functions proposed.
- c) To study the performance of three (3) different types of normalization process required for FAM neural network.
- d) To perform the classification task of feature vectors belong to insect images using FAM and GAM neural networks.
- e) To evaluate the performance of FAM and GAM neural networks using 4-folds cross validation techniques.

1.4 Research scope

The scope of this research is limited to the use of only binary images. Basically, the overall work can be divided into two stages. The first stage focuses on the features extraction process whereas moment invariant technique was used to extract the global shape features of insect images. In the second stage, all the feature vectors produced will be classified using FAM and GAM neural networks. Finally, the best technique for features extraction and classification is obtained.

1.5 Thesis outline

This thesis is composed of six chapters. Each chapter is briefly described as follows:

- i) Chapter 1: Introduces the topics of this research and also presents an overview of the thesis which consists of the problem statement, research objective and scope.
- ii) Chapter 2: Presents a literature review of several previous works that related to moment invariant techniques and ARTMAP based neural networks. This chapter also discuss on the fundamental equations and algorithms for both methods.
- iii) Chapter 3: Discusses the details of methodology adopted in this work and also explained all the algorithms created in executing the moment invariant as features extraction technique.
- iv) Chapter 4: Explains in details all algorithms produced in this work that was used to implement both FAM and GAM neural networks as classifiers.
- v) Chapter 5: Discusses all the results for both intra and interclass analysis.
- vi) Chapter 6: Presents the conclusion of our research, contributions and the related of future works.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

There are over a million types of insects in this world today with different types of colors and shape, thus making the task of insect classification and recognition a challenging ordeal. Furthermore, insect identification is difficult because it requires a detail understanding of insect taxonomy as well as the jargon and terms of morphological characteristics. Therefore, insect classification traditionally depends on taxonomists, but these professional individuals are not always available in all areas. Nevertheless, computer vision is a technology with the potential to make complete automation possible in insect classification, because it can fully utilize the huge potential offered by information technology, instead of relying on users to compare specimens to images or illustration.

In the insect recognition domain, we found that there are few attempts in developing such application. Steinhage, V. (2001) classifies *Bombus Sylvarum* and *Bombus Veteranus* using initial substructure of the whole Bee venation structure of their forewing images. The substructure is represented using 15 parameters such as distance, angles and form parameter. The parameters are classified using Linear Discriminate Analysis (LDA) technique. The classification performance of 99.3% was achieved using Leave-One-Out Cross Validation (LOOCV) technique.

Nevertheless, this work is carried out in order to investigate the use of global shape information of insect images for recognition purposes. Xin, W. Y., et al. (2002) also used the same image properties in classifying an insect as illustrated in Figure 2.1. They also applied LDA to classify three (3) different butterfly species which are *H.Armigera*, *O.Fumacalis* and *E.Atrox* based on nine shape features. These features are referred to as the number of hole, region area, perimeter, eccentricity, form factor, roundness, circularity, sphericity and lobation. However the classification validation technique is doubtful. They claimed that the three species were correctly classified based on 75 binary images.

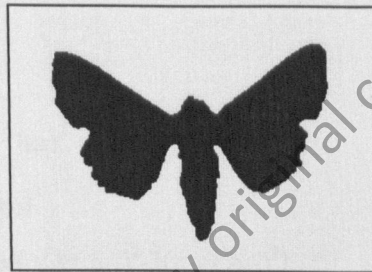


Figure 2.1: Example of butterfly binary images that was used by Xin, W. Y., et al. (2003)

However, this research utilized moment invariant as features extraction techniques while FAM and GAM are adopted as classifiers method. Therefore, in this chapter we will discuss the fundamental concept and some previous work conducted that are related to those methods.

2.2 Moment invariant

In 1961, Hu introduced the moment invariant based on the theory of algebraic function. He derived a set of moment invariants, which are translation, scaling and rotation independent. This method is also known as Geometric Moment Invariant (GMI). The $(p+q)^{th}$ geometric moment for $p, q = 0, 1, 2, 3, \dots$ are define in (2.1). Where $h(x,y)$ is an image of the size $N \times M$. To make these moments invariant to translation, central moments (2.2) is derived based on (2.1). Where $\bar{x} = m_{10}/m_{00}$ and $\bar{y} = m_{01}/m_{00}$. In order to produce the invariant properties with scaling factor, the central

moment than be normalized using (2.3). The seven (7) functions of central moments that are invariant to rotational and scale differences are shown in (2.4).

$$m_{pq} = \sum_{x=1}^N \sum_{y=1}^M x^p y^q h(x, y) \quad (2.1)$$

$$\mu_{pq} = \sum_{x=1}^N \sum_{y=1}^M (x - \bar{x})^p (y - \bar{y})^q h(x, y) \quad (2.2)$$

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{p+q+2}} \quad (2.3)$$

$$\begin{aligned} \phi_1 &= (\eta_{20} + \eta_{02}) \\ \phi_2 &= (\eta_{20} - \eta_{02})^2 + 4 \eta_{11}^2 \\ \phi_3 &= (\eta_{30} - 3 \eta_{12})^2 + (3 \eta_{21} - \eta_{03})^2 \\ \phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\ \phi_5 &= (\eta_{30} - 3 \eta_{12})(\eta_{30} + \eta_{12})[3(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\ &\quad + (3 \eta_{21} + \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ \phi_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4 \eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\ \phi_7 &= (3 \eta_{12} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12}) - 3(\eta_{21} + \eta_{03})] \\ &\quad - (\eta_{30} - 3 \eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \end{aligned} \quad (2.4)$$

2.3 Other types of moment

Since the introduction of GMI, many other works were conducted to improve the invariant properties of GMI. These contributed to the development of other types of moment techniques. Therefore, the next paragraph will explain the associated techniques applied in this work that is based on moment invariant.

2.3.1 United Moment Invariant (UMI)

Sun, Y. et al. (2003) in his paper analyzed three conditions that relate the GMI with the effect of scaling factors. These can be revealed in equations (2.3), (2.5) and

(2.6). In order to eliminate the influence of μ_{00} and the scaling factor in discrete images ρ , he derived eight (8) sets of UMI base on GMI as shown in equation (2.7).

$$\eta' = \rho^{p+q} \eta_{pq} \quad \dots (2.5)$$

$$\eta'' = \frac{\mu_{pq}}{\mu_{00}^{p+q+1}} \quad \dots (2.6)$$

$$\begin{aligned} \theta_1 &= \sqrt{\phi_2} \div \phi_1 & \theta_5 &= (\phi_1 \times \phi_6) \div (\phi_2 \phi_3) \\ \theta_2 &= \phi_6 \div (\phi_1 \times \phi_4) & \theta_6 &= ((\phi_1 + \sqrt{\phi_2}) \phi_3) \div \phi_6 \\ \theta_3 &= \sqrt{\phi_5} \div \phi_4 & \theta_7 &= (\phi_1 \times \phi_5) \div (\phi_3 \times \phi_6) \\ \theta_4 &= \phi_5 \div (\phi_3 \times \phi_4) & \theta_8 &= (\phi_3 + \phi_4) \div \sqrt{\phi_5} \end{aligned} \quad (2.7)$$

2.3.2 Zernike Moment Invariant (ZMI)

Zernike Moments were first introduced by Teague (1980), based on continuous orthogonal functions called Zernike polynomials. Equation (2.8) provides a convenient way to express Zernike moments in terms of geometric moments in Cartesian form. Then Zernike Moment invariant (ZMI) functions are derived from equation (2.8) which is invariant against rotation and scaling factors. The value of $f(x, y)$ refer to the pixel density of $N \times N$ image size.

$$Z_{mn} = \frac{n+1}{\pi} \sum_{k=m}^n B_{nmk} \sum_{x=1}^N \sum_{y=1}^M (x-iy)^m (x^2+y^2)^{(k-m)/2} f(x, y) \quad (2.8)$$

$$B_{nmk} = \frac{(-1)^{(n-k)/2} \left(\frac{n+k}{2}\right)!}{\left(\frac{n-k}{2}\right)! \left(\frac{k+m}{2}\right)! \left(\frac{k-m}{2}\right)!} \quad (2.9)$$

$$\varphi_1 = Z_{p0} \quad ; \quad \varphi_2 = |Z_{pq}|^2 \quad (2.10)$$

2.3.3 Legendre Moment Invariant (LMI)

The Legendre moment was also introduced by Teague (1980) which is produced based on Legendre polynomials. The Legendre moments of order $(p+q)$ can be expressed in terms of geometric moments as shown in eq. (2.11) whereas in eq. (2.12) $|x| \leq 1$ and $(n-k)$ is even. The purpose of v_{pq} is to give TMI equation invariant against translation, scaling and rotation factors.

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{i=0}^p \sum_{j=0}^q a_{pi} a_{qj} m_{ij} \quad (2.11)$$

$$a_{pi} = P_n(x) = \sum_{k=0}^n (-1)^{(n-k)/2} \frac{1}{2^n} \frac{(n+k)! x^k}{\left(\frac{n-k}{2}\right)! \left(\frac{n+k}{2}\right)! k!} \quad (2.12)$$

$$v_{pq} = M_{00}^{-\gamma} \sum_{x=1}^N \sum_{y=1}^N [(x-\bar{x}) \cos \phi + (y-\bar{y}) \sin \phi]^p \times [(y-\bar{y}) \cos \phi - (x-\bar{x}) \sin \phi]^q f(x, y) \quad (2.13)$$

Where:

$$\gamma = \frac{n+m}{2} + 1 \quad (2.14)$$

$$\phi = 0.5 \tan^{-1} \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \quad (2.15)$$

2.3.4 Tchebichef Moment Invariant (TMI)

TMI was introduced by R. Mukundan (2001) which is produced based on discrete Tchebichef polynomials. The $(p+q)$ order of TMI can be calculated using equation (2.16) whereas the computations of \tilde{v}_{ij} is shown in (2.20).

$$T_{pq} = A_p A_q \sum_{k=0}^p C_k(p, N) \sum_{l=0}^q C_l(q, N) \times \sum_{i=0}^k \sum_{j=0}^l s_k^{(i)} s_l^{(j)} v_{ij} \quad (2.16)$$

$$A_p = \frac{N^p (N-p-1)! (2p+1)!}{(2p)! (N+p)!} \quad (2.17)$$

$$C_k(p, N) = (-1)^{p-k} \frac{p!}{k!} \binom{N-1-k}{p-k} \binom{p+k}{p} \quad (2.18)$$

$$\sum_{i=0}^k s_k^{(i)} x^i = \frac{x!}{(x-k)!} \quad (2.19)$$

$$\tilde{v}_{ij} = \sum_{p=0}^i \sum_{q=0}^j \binom{i}{p} \binom{j}{q} \left(\frac{N^2}{2}\right)^{\frac{p+q}{2}+1} \left(\frac{N}{2}\right)^{i+j-p-q} v_{pq} \quad (2.20)$$

$$\binom{x}{y} = \frac{x!}{(x-y)! y!} \quad (2.21)$$

2.3.5 Krawtchouk Moment Invariant (KMI)

Krawtchouk moment invariants were derived by Yap, P.T et al. (2003) using the concept of Krawtchouk polynomial function with the implementations of linear combinations of Geometric Moment. The $(p+q)$ order of Krawtchouk moment is given by (2.22).

$$\tilde{Q}_{nm} = \Omega_{nm} \sum_{i=0}^n \sum_{j=0}^m a_{i,n,p_1} a_{j,m,p_2} \tilde{v}_{ij} \quad (2.22)$$

$$\Omega_{nm} = [\rho(n; p_1, N-1) \rho(m; p_2, N-1)]^{-0.5} \quad (2.23)$$

$$\rho(n; p, N) = \binom{N-1}{n} \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n} \quad (2.24)$$

$$\sum_{k=0}^n a_{k,n,p} x^k = \sum_{k=0}^n \frac{(-n)_k (-x)_k}{(-N)_k} \times \frac{p^{-k}}{k!} \quad (2.25)$$

2.4 Application of moment invariant

Moment invariant has been widely used over the years as features extraction technique for recognition and classification in many areas of image analysis [Mukundan, R. et al. 2001, Paschalakis, S. et al. 1999]. This method is successfully adopted along with other techniques in order to produce an efficient system which is used as image recognition and classification. Thus, the next paragraph will explain some of the work done that applied moment as the features extraction techniques.

2.4.1 Optical character recognition (OCR)

It is found that moment invariant was one of the common used methods to extract the shape of character images. Since character can be created in various forms, thus applying the moment techniques seem become an interesting subject to be used. This is because moment invariant preserves the invariant properties against translation, position and rotation. Table 2.1 illustrates some of the works done that applied moment invariant techniques in OCR application.

Table 2.1: Example works that applied moment invariant in OCR

	Authors	Types of Images	Types of moment applied
1	Belkasim S.O. et al. (1989)	Handwritten numbers	GMI,ZMI
2	M.Majid et al. (1994)	Arabic text	GMI
3	W.Y.Kim et al. (1994)	Alphanumeric Machine-Printed Character	GMI,ZMI
4	F.Pang et al. (1994)	Handwritten numbers	GMI
5	H.Lim et al. (1996)	Chinese character	ZMI
6	M.Deghan et al. (1997)	Farsi handwritten	GMI,ZMI,LMI
7	M.Sabaei et al. (1997)	Farsi handwritten	GMI,ZMI,LMI
8	Q.Chen et al. (2003)	Alphabet	GMI
9	M. Sarfraz et al. (2003)	Arabic text	GMI

2.4.2 Military research

Moment invariant was also used in the military field research. Basically, this technique is applied to extract the shape information of interested object such as tanks, air-craft and ships to be recognized. Table 2.2 describes some of the work done using moment invariant in military research.

Table 2.2: Example works that applied moment in military research

	Authors	Types of Images	Types of moment applied
1	A.D Kulkani et al. (1990)	Air-Craft	GMI
2	A.McAulay et al. (1991)	Air-Craft	GMI
3	P.Pejnovic et al. (1992)	Tank, helicopter Air-Craft, ship	ZMI
4	X.Yan et al (1995)	Tank	GMI

2.4.3 Face and Facial recognition

Moment invariant is also applied in face and facial recognition as shown in Table 2.3. Most of the works applied moment to extract the global shape information of the grey scale face images. However, Phiasai, T. et al. (2001) and Zhu, Y. et al. (2000) adopted moment to extract the local shape information such as nose and eyes. Phiasai, T. also demonstrated that the combination of Principal Component Analysis (PCA) and moment invariant will increase the face recognition rate.

Table 2.3: Example works that applied moment in face and facial recognition

	Authors	Objective	Types of moment applied
1	Y.Zhu et al. (2000)	Facial recognition	GMI
2	T.Phiasai et al. (2001)	Face recognition	GMI
3	J.Haddadnia et al. (2001)	Face recognition	ZMI, LMI
4	A.Saradha et al. (2001)	Face recognition	GMI, ZMI, LMI