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**MATHEMATICAL MODELLING OF MASS  
TRANSPORT IN LINEAR, SHUNT AND  
CONCENTRIC MICRODIALYSIS PROBES**

By

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## Pemodelan Matematik Pemindahan Massa dalam Prob Mikrodialisis

### ABSTRAK

Mikrodialisis ialah teknik bagi memperolehi dan menghantar bahan pada kawasan sasaran (yang mungkin merupakan tisu, organ dsb.), menggunakan satu alat yang kecil, dinamakan prob. Terdapat pelbagai jenis prob mikrodialisis seperti prob linear, prob tetolak dan prob konsentrik, dan pemilihan prob adalah bergantung kepada kawasan pengimplanan, sama ada untuk tisu (jenis tisu berbeza bagi rekabentuk prob berbeza) atau dalam medium tidak mengalir. Walaupun prob perlu dimasukkan secara fizikal ke dalam kawasan sasaran, prob mikrodialisis adalah secara relatifnya kecil dan ianya invasive pada tahap minimal (iaitu, mengakibatkan perubahan atau kecederaan minima kepada kawasan sasaran). Ini, bersama dengan pelbagai ciri mikrodialisis lain (contohnya, boleh digunakan pada hampir semua organ dan tisu, boleh digunakan pada pesakit yang hidup, sedar dan bergerak, dan sebagainya) membuatkan teknik ini amat popular. Naum begitu, Nonetheless, perolehan yang rendah, kesangsian berkaitan dengan resolusi masa dan penyediaan yang rumit bersama dengan keperluan untuk pra-percubaan bagi tujuan penentukuran, telah menghadkan aplikasi teknik ini. Kekangan ini adalah lazimnya dikaitkan dengan pembatasan pemindahan massa. Dalam tesis ini, sebuah rangka kerja matematik yang menggabungkan persamaan-persamaan aliran dan resapan telah dicadangkan bagi mewakili fenomena pemindahan massa dalam prob mikrodialisis. Rangka kerja matematik ini kemudiannya digunakan untuk menganalisa pengaruh yang mungkin dari parameter-parameter yang berkaitan terhadap perolehan glukos (iaitu, analit). Di dalam penyelidikan ini, telah didefinisikan bahawa di dalam membran bagi prob dan kawasan sekitar prob, proses pemindahan massa adalah bergantung hanya kepada resapan. Parameter model dan keadaan operasi telah diperolehi dari kajian persuratan. Dalam bahagian pertama, rangka kerja matematik telah dibina bagi mewakili pemindahan massa dalam dua prob mikrodialisis yang primitif, iaitu, prob linear dan prob tetolak. Menggunakan rangka kerja masing-masing, perolehan glukos dibawah keadaan operasi berbeza telah dibandingkan di antara prob-prob tersebut, yang telah didefinisikan untuk beroperasi dalam keadaan yang hampir sama. Oleh kerana tiada kerja-kerja matematikal yang telah dilakukan sebelum ini untuk menilai prestasi prob-prob ini, adalah menarik untuk dilihat bagaimana prestasi prob-prob ini jika kedua-duanya beroperasi dalam keadaan operasi yang hampir sama. Dapat dilihat dengan jelas bahawa rangka-rangka kerja matematik tersebut adalah cukup sensitif untuk menunjukkan perubahan kepekatan apabila parameter diubah. Keputusan-keputusan ini adalah lebih kurang sama dengan apa yang telah dibincangkan dalam kajian sebelumnya. Membandingkan prestasi kedua-dua prob yang beroperasi dalam keadaan operasi hampir serupa, prob tetolak telah menunjukkan perolehan glukos yang lebih tinggi, yang mencerminkan prestasi yang lebih baik. Pendekatan matematik dari rangka-rangka kerja matematik prob-prob linear dan tetolak yang lebih primitif ini telah dikembangkan untuk mewakili pemindahan massa dalam prob konsentrik yang lebih kompleks. Prob ini boleh dikatakan lebih popular dan yang paling lazim dirujuk dalam kajian-kajian sebelumnya berkaitan mikrodialisis. Rangka kerja ini kemudiannya digunakan untuk menilai pemindahan massa dalam prob dan kawasan sekitarnya. Keputusan dalam bentuk peratusan perolehan dan pekali pemindahan massa keseluruhan bagi keadaan operasi berbeza telah dibincangkan. Perbandingan telah dibuat dengan rangka kerja mikrodialisis Bungay berdasarkan pemindahan massa bagi

rekabentuk dan parameter operasi yang berbeza. Keputusan mencadangkan bahawa rangka kerja konsentrik adalah sensitif terhadap perubahan parameter dan profil kepekatan yang diperolehi adalah hampir sama dengan rangka kerja Bungay yang diterima umum. Ini satu indikasi bahawa rangka kerja konsentrik yang dicadangkan boleh digunakan bagi mewakili fenomena pemindahan massa bagi prob tersebut. Keputusan dari kerja-kerja numerikal kemudiannya telah dibandingkan dengan kerja eksperimen. Keputusan tersebut menunjukkan bahawa data simulasi adalah sesuai dengan data eksperimen menggunakan prob konsentrik 5 mm dengan nilai faktor penghadangan membran  $\alpha$ , bernilai 10. Perbandingan telah dibuat sekali lagi dengan prob yang serupa, dengan dengan panjang yang berbeza (10 mm) dan kesesuaian juga didapati terbaik dengan nilai  $\alpha$  lebihkurang 10. Ini adalah justifikasi bahawa rangka kerja konsentrik boleh digunakan untuk mewakili fenomena pemindahan massa dalam prob yang tersebut, dan seterusnya, memberikan justifikasi berkenaan dengan kesahihan rangka kerja matematik bagi prob yang lebih primitif, sebelumnya.

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# Mathematical Modelling of Mass Transport in Microdialysis Probes

## ABSTRACT

Microdialysis is a technique for both recovering and administering substances at a target site (which may be tissues, organs, etc.), using a small equipment termed as probe. There are many types of microdialysis probes such as the linear probe, the shunt probe and the concentric probe, and the probe choices are depended on the site of implantation, whether it is for the tissue (different type of tissue for different probe design) or in the quiescent medium. Although the probe needs to be physically inserted into the site, the microdialysis probe is relatively small and is minimally-invasive (i.e., causing minimal changes or injuries to the target site). This along with many other features of microdialysis (e.g., can be performed on almost every organ and tissue, can be used on living, awake and even moving patients, etc.) make this technique very popular. Nonetheless, poor recovery, doubts related to temporal resolution, and tedious preparations plus the need of pre-runs for calibration, limits the application of this technique. These constraints are generally attributed to mass transfer limitations. In this thesis, a mathematical framework incorporating convection and diffusion equations have been proposed to represent transport phenomena in microdialysis probes. This mathematical framework is then used to analyse the possible influence of various relevant parameters on glucose (i.e., analyte) recovery. In this work, it is defined that within the probe's membrane and probe surrounding area (PSA), the transport process is solely driven by diffusion. The model parameters and operating conditions have been obtained from literature. In the first part, a mathematical framework was constructed to represent mass transport in two primitive microdialysis probes, namely the linear and shunt probes. Using the respective mathematical frameworks, glucose recoveries under various operating conditions were compared between the two probes, which were defined to operate under similar conditions. As there is no mathematical work that has been done to evaluate these both probes, it would be interesting to see how these two microdialysis probes may perform under similar operating conditions. It is clearly seen that the frameworks were sensitive enough to show concentration changes when parameters were varied. These results were comparable to what was discussed in literature. Comparing the two probe's performance under similar conditions, the shunt probe displayed higher glucose recoveries, which reflect better performance. The mathematical approach from the more primitive linear and shunt microdialysis probe frameworks was expanded to represent mass transport in the more complex concentric microdialysis probe. This probe is arguably the more popular and is the most commonly referred to in microdialysis literature. The framework is then used to evaluate mass transport within the probe and its surrounding area. Results on percentage recovery and overall mass transfer coefficient under various operating conditions have been discussed. Comparisons were made with the Bungay's microdialysis framework (BMF) on mass transport performance under different design and operating parameters. The results suggested that the concentric probe framework is sensitive to parameter changes, and the concentration profiles obtained are comparable to the widely accepted BMF. This is one indication that the proposed concentric probe framework can be used to represent mass transport phenomena in such probes. The results from the numerical efforts were then compared to experimental work. It was shown that the simulated data fits well with experimental data using a 5 mm concentric probe, for membrane

hindrance factor,  $\alpha$ , of 10. The comparison was done again using a similar probe, with a different length (10 mm) and fitting was also found best with  $\alpha$  of approximately 10. It is justified that the concentric mathematical framework can be used to represent mass transport phenomena in those probes, and in one way, justifies the validity of the mathematical frameworks for the more primitive probes, beforehand.

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