Feature Extraction of Trademark Images Using Geometric Invariant Moment and Zernike Moment – A Comparison

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ABSTRACT

The qualities of features extracted determine the successfulness of any image related applications. This is particularly true especially when the natures of images are occluded and consist of various shapes and design styles, such as trademark images. In this study Geometric Invariant Moment and Zernike Moment techniques are utilized to extract sets of features from trademark images. The results obtained are analysed and compared in terms of intraclass invariance to various perturbations. It is found that a set of features produced by Zernike Moment technique is more robust when compared to features produced by Geometric Invariant Moment.

INTRODUCTION

Feature Extraction is crucially significant for the successfulness of the image-related applications [1][2][3][4][5][6]. It is still a dream for the computer to outperform human natural ability for visual interpretation, thus feature extraction still remains a challenging task in various realms of computer vision and image analysis.

Monochrome or one colour (black and white) trademark images belong to the category of silhouette image, where shape attribute is normally used to represent the image content. Moment-based techniques are chosen to perform the feature extraction due to the following reasons [7].

- a set of moments computed from a digital image represent global characteristics of the image shape,
- it also provides a lot of information regarding the different types of geometrical features inherent in the image and
- it produces a set of features that is invariant

The paper starts by describing the Geometric invariant moment technique in Section 2, Zernike moment technique in Section 3. Section 4 presents the implementation, Section 5 highlight Results and Discussion. The paper ends with a conclusion in Section 6.
METHODOLOGY

Geometric invariant moment techniques

Geometric invariant moment technique is chosen to extract image features since the features generated are Rotation Scale Translation (RST)-invariant. Geometric Moment Invariant was first introduced by Hu and it is also known in some literature as Hu’s Moment Invariants [8]. This moment was derived from the theory of algebraic invariant. Geometric Moment was successfully applied in aircraft identification, texture classification and radar images to optical images matching [9]. The properties of Geometrical Moment has the form of a projection of the \( f(x,y) \) function onto the monomial \( x^p y^q \). Properties of Geometric Moment as as follows;

For a 2-D continuous function \( f(x,y) \), the moment of order \((p+q)\) is defined as

\[
m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x,y) \, dx \, dy
\]

for \( p, q = 0, 1, 2, 3, \ldots \)

A unique theorem [8] states that if \( f(x,y) \) is piecewise continuous and has nonzero values only in a finite part of the \( xy \) plane, moment of all orders exist and the moment sequence \( m_{pq} \) is uniquely determined by \( f(x,y) \). Conversely, \( m_{pq} \) uniquely determines \( f(x,y) \). The central moments can be expressed as

\[
\mu_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-x_c)^p (y-y_c)^q f(x,y)
\]

where

\[
\bar{x} = \frac{m_{10}}{m_{00}} \quad \text{and} \quad \bar{y} = \frac{m_{01}}{m_{00}}
\]

\[
\mu_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y)
\]

For a grey scale image, Equation (1.2) becomes

For binary digital image, \( f(x,y) = 1 \) (indicating an object), thus Equation (1.3) becomes

\[
\mu_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q
\]

(1.4)
The centralised moments $\mu_{pq}$ can be expressed in terms of $m_{pq}$ as follows:

$$\begin{align*}
\mu_{00} &= m_{00} \\
\mu_{10} &= 0 \\
\mu_{01} &= 0 \\
\mu_{20} &= m_{20} - xm_{10} \\
\mu_{02} &= m_{02} - ym_{10} \\
\mu_{11} &= m_{11} - ym_{10} \\
\mu_{31} &= m_{30} - 3xm_{20} + 2xm_{10} \\
\mu_{21} &= m_{12} - 2ym_{11} - xm_{02} + 2ym_{10} \\
\mu_{21} &= m_{21} - 2xm_{11} - ym_{20} + 2xm_{01} \\
\mu_{03} &= m_{03} - 3ym_{02} + 2ym_{01}
\end{align*}$$

(1.5)

The normalized central moments are defined as:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}}$$

where,

$$\gamma = \frac{(p + q)}{2} + 1$$

for $(p+q) = 2, 3,...$

The resulting moment functions which are taken as representative features of the image as shown below are invariant with respect to translation rotation and scale change [8].

$$\phi_1 = \eta_{20} + \eta_{02}$$

(1.6)

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4 \eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})(\eta_{21} + \eta_{03})3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})]$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})]$$

Zernike moment technique

Zernike Moment (Z.M) is chosen since it is invariant to rotation and insensitive to noise. Another advantage of Z.M is the ease of image reconstruction because of its orthogonal
property [9][10][11][12][13]. Z.M is the projection of the image function onto orthogonal basis functions. Z.M also has a useful rotation invariance property where the magnitude of Z.M will not change for a rotated image. Another main property of Z.M is the ease of image reconstruction because of its orthogonal property [9]. The major drawback of Z.M is it’s computational complexity [7]. This is due to the recursive computation of Radial Polynomials. However in this study, we overcome the problem of computational complexity by adopting a non recursive computation of Radial Polynomials. The computation is based on the relationship between Geometric Moment Invariant and Z.M in order to derive Zernike Invariant Moment. Properties of Z.M are as follows:

The Z.M of order p with repetition q for a continuos image function $f(x,y)$ that vanishes outside the unit circle is as shown in Eq. 1.9.

$$Z_{pq} = \frac{p+1}{\pi} \int \int_{x^2+y^2 = 1} f(r, \theta) V_{pq} * r \, dr \, d\theta$$  \hspace{1cm} (1.9)

To compute a Z.M of a given image, the center of the image is taken as the origin and pixel coordinates are mapped to the range of unit circle, i.e. $x^2 + y^2 = 1$. The functions of $V_{pq}(r, \theta)$ denote Zernike Polynomials of order p with repetition q, and $*$ denotes a complex conjugate where the Zernike Polynomials are defined as functions of the polar coordinates $r, \theta$ [6]. Equations relating Zernike and Geometric Moment up to third order are given below

$$Z_{00} = (1/\pi) \, M_{00}$$
$$Z_{11} = (2/\pi) \, (M_{00} - i \, M_{10})$$
$$Z_{20} = (6/\pi) \, (M_{00} + M_{02}) - (3/\pi) \, M_{00}$$
$$Z_{22} = (3/\pi) \, (M_{20} - M_{02} - 2i \, M_{11})$$
$$Z_{31} = (12/\pi) \, (M_{00} + M_{12}) - (12/\pi) \, i \, (M_{03} + M_{21}) - (8/\pi) \, (M_{10} - i \, M_{01})$$
$$Z_{33} = (4/\pi) \, (M_{30} - 3M_{12}) + (4/\pi) \, i \, (M_{03} - 3M_{21})$$  \hspace{1cm} (1.10)

Rotation Invariants properties are illustrated as follows; from the definition of Z.M in Eq (1.9), when image undergoes a rotation by an angle $\alpha$, the transformed moment functions $Z'_{pq}$ are given by

$$Z'_{pq} = Z_{pq} \, e^{-iq\alpha}$$  \hspace{1cm} (1.11)

This simple property leads to the conclusion that the magnitudes of the Z.M of a rotated image function remain identical to those before rotation. Thus $|Z_{pq}|$, the magnitude of the Z.M, can be taken as a rotation invariant feature of the underlying image function. Rotation invariants and their corresponding expressions in Geometric Moment (G.M)are given below until the order of three:

$$|Z_{00}| = (1/\pi) \, M_{00}$$
$$|Z_{11}|^2 = (2/\pi)^2 \, (M_{00}^2 + M_{10}^2)$$
$$|Z_{20}|^2 = (3/\pi)^2 \, [2(M_{00} + M_{02}) - M_{00}]$$
$$|Z_{22}|^2 = (3/\pi)^2 \, [(M_{20} - M_{02})^2 + 4M_{11}^2]$$
$$|Z_{31}|^2 = (12/\pi)^2 \, [(M_{00} + M_{12})^2 + (M_{03} + M_{21})^2]$$
$$|Z_{33}|^2 = (4/\pi)^2 \, [(M_{30} - 3M_{12})^2 + (M_{03} - 3M_{21})^2]$$  \hspace{1cm} (1.12)
Implementation

Monochrome image samples are obtained from Trademark Ordinance are scanned using 200 dpi HP Laser and Lexmark Scanner [14][15]. The scanned image is then saved as .tiff format. The image is then converted into gray-level format; the noise is removed and it is thresholded using Otsu thresholding algorithm [16], and then it is saved using the raw format. Before the image file is closed, its width and height in pixels are noted, since the dimensions will become an input to the feature extraction phase. The trademark image samples utilized in this endeavor as shown in Appendix A, are divided into four categories based on image shape;

Category 1: Circle
Category 2: Rectangle
Category 3: Word-Embedded Mark
Category 4: Miscellaneous

Due to the analysis and observation of the trademark image samples, majority of them fall under the above shape categories. Furthermore, based on the literature study conducted, the moment-based feature extraction techniques performed well in extracting the global image shape [17]. We want to find out the invariance property displayed by G.M and Z.M features of different trademark image shapes.

The image samples are perturbed to produce seven variants as depicted in Figure 1.1, and to test the robustness of the feature extraction techniques adopted.

![Figure 1: An Image Example (Image im25 and its variations)](image-url)
The legend of the image variations is as follows:

- \( im25 \) an original image
- \( im25h \) the original image placed horizontally
- \( im25v \) the original image placed vertically
- \( im2590 \) the original image rotated to 90 degrees
- \( im25b \) the original image is enlarged by 50%
- \( im25s \) the original image is reduced by 50%
- \( im25rpl \) the original image is rippled to \(-160\) degrees
- \( im25twl \) the original image twirled to \(-72\) degrees

RESULTS AND DISCUSSION

A set of G.M and a set of Z.M features are extracted from image samples using Equation (1.6) and (1.12). Table 1, depicts an example of G.M Features Vectors of an original image and its variants belong to category 4. Table 2, depicts the Z.M features vectors of the same image.

**Table 1: The G.M Feature Vector of Image \( im25 \) and its Variants**

<table>
<thead>
<tr>
<th>Variants</th>
<th>( \varphi_1 )</th>
<th>( \varphi_2 )</th>
<th>( \varphi_3 )</th>
<th>( \varphi_4 )</th>
<th>( \varphi_5 )</th>
<th>( \varphi_6 )</th>
<th>( \varphi_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( im25 )</td>
<td>13.125976</td>
<td>24.450977</td>
<td>22.968588</td>
<td>23.027691</td>
<td>45.730389</td>
<td>0.000000</td>
<td>45.277840</td>
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<td>( im25h )</td>
<td>13.134906</td>
<td>24.493625</td>
<td>22.896047</td>
<td>23.004788</td>
<td>45.542001</td>
<td>0.000000</td>
<td>45.882536</td>
</tr>
<tr>
<td>( im25v )</td>
<td>13.135990</td>
<td>24.506779</td>
<td>22.954858</td>
<td>22.998057</td>
<td>45.992919</td>
<td>0.000000</td>
<td>43.095920</td>
</tr>
<tr>
<td>( im2590 )</td>
<td>13.874312</td>
<td>27.195126</td>
<td>27.565035</td>
<td>26.789870</td>
<td>53.638658</td>
<td>39.944855</td>
<td>0.000000</td>
</tr>
<tr>
<td>( im25b )</td>
<td>13.018402</td>
<td>23.274206</td>
<td>19.741464</td>
<td>16.195952</td>
<td>45.607242</td>
<td>0.000000</td>
<td>45.411974</td>
</tr>
<tr>
<td>( im25s )</td>
<td>13.133874</td>
<td>24.497487</td>
<td>22.764208</td>
<td>23.147620</td>
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<td>13.889930</td>
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<td>( im25twl )</td>
<td>12.828828</td>
<td>23.837837</td>
<td>22.429150</td>
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<td>48.860316</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

**Table 2: The Z.M Feature Vector of Image \( im25 \) and its Variants**

<table>
<thead>
<tr>
<th>Variants</th>
<th>( \varphi_1 )</th>
<th>( \varphi_2 )</th>
<th>( \varphi_3 )</th>
<th>( \varphi_4 )</th>
<th>( \varphi_5 )</th>
<th>( \varphi_6 )</th>
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<tr>
<td>( im25 )</td>
<td>0</td>
<td>0</td>
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<td>( im2590 )</td>
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<td>( im25b )</td>
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<td>0.799257</td>
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<td>0.000032</td>
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</table>

It is observed that Z.M orders 1 and 2 have null values and order 3 is significant. However with regard to G.M, higher order moment values of 4\(^{th}\) and sometimes 5\(^{th}\) and above tend to become insignificant.

Studies are performed to determine the sensitivity of Geometric Moment and Z.M techniques to various perturbations of each image selected from each image category. Based on Equation 1.13, mean \( \text{error}(m_e_{GM}) \) for G.M is computed for each image variants to determine the intraclass invariances. Mean error is computed by first obtaining the absolute difference
between the original and the perturbed image for each moment function value, represented by $e_i(G.M)$.

$$e_i(G.M) = |\varphi_i(\text{original}) - \varphi_i(\text{perturbed})| \quad i = 1, 2, 3, \ldots, 6$$

$$m_e_{G,M} = \frac{\sum_{i=1}^{6} e_i(G.M)}{6}$$

(1.13)

Table 3 depicts the values of the mean error of the G.M computed for an image belongs to Category 2.

**Table 3: Mean Error of the G.M for an Image belongs to Category 2**

<table>
<thead>
<tr>
<th></th>
<th>$\varphi_1$</th>
<th>$\varphi_2$</th>
<th>$\varphi_3$</th>
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<td>15.68733</td>
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<td>0.032751</td>
<td>0.015921</td>
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<td>Mean error</td>
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<td></td>
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<tr>
<td>Error</td>
<td>0.102568</td>
<td>1.927353</td>
<td>0.394337</td>
<td>0.981435</td>
<td>1.694863</td>
<td>0.147332</td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>0.87465</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Similarly as for Z.M, absolute difference between original and perturbed image is first computed, then followed by getting the mean error \( m_{e_{Z,M}} \) using Equation 1.14.

\[
e_i(Z,M) = |\phi_i(\text{original}) - \phi_i(\text{perturbed})| \quad i = 3, 4, \ldots, 6
\]

\[
m_{-e_{Z,M}} = \frac{\sum_{i=3}^{i=6} e_i(Z,M)}{4}
\]  

(1.14)

**Table 4**: depicts the values of the mean error of the Z.M computed for the same image.

**Table 4: Mean Error of the Z.M of an image belongs to Category 2**

<table>
<thead>
<tr>
<th></th>
<th>( \phi_1 )</th>
<th>( \phi_2 )</th>
<th>( \phi_3 )</th>
<th>( \phi_4 )</th>
<th>( \phi_5 )</th>
<th>( \phi_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>im2</td>
<td>0</td>
<td>0</td>
<td>0.613385</td>
<td>0.009635</td>
<td>0.01039</td>
<td>0.001653</td>
</tr>
<tr>
<td>im2h</td>
<td>0</td>
<td>0</td>
<td>0.616112</td>
<td>0.009891</td>
<td>0.010391</td>
<td>0.00168</td>
</tr>
<tr>
<td>error</td>
<td>0</td>
<td>0</td>
<td>0.002727</td>
<td>0.000256</td>
<td>0.000001</td>
<td>0.000026</td>
</tr>
<tr>
<td>mean error</td>
<td>0.000747</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| im2    | 0            | 0            | 0.613385     | 0.009635     | 0.01039      | 0.001653     |
| im2v   | 0            | 0            | 0.695523     | 0.012745     | 0.021135     | 0.002812     |
| error  | 0            | 0            | 0.082138     | 0.00311      | 0.00311      | 0.001159     |
| mean error | 0.022379   |              |              |              |              |              |

| im2    | 0            | 0            | 0.613385     | 0.009635     | 0.01039      | 0.001653     |
| im290  | 0            | 0            | 0.664766     | 0.005132     | 0.001403     | 0.000099     |
| error  | 0            | 0            | 0.051381     | 0.004503     | 0.008987     | 0.001554     |
| mean error | 0.016606   |              |              |              |              |              |

| im2    | 0            | 0            | 0.613385     | 0.009635     | 0.01039      | 0.001653     |
| im2b   | 0            | 0            | 0.571565     | 0.007125     | 0.005132     | 0.000644     |
| error  | 0            | 0            | 0.04182      | 0.00251      | 0.005258     | 0.001009     |
| mean error | 0.012649   |              |              |              |              |              |

| im2    | 0            | 0            | 0.613385     | 0.009635     | 0.01039      | 0.001653     |
| im2s   | 0            | 0            | 0.566212     | 0.006901     | 0.004786     | 0.000614     |
| error  | 0            | 0            | 0.047173     | 0.002734     | 0.005604     | 0.001039     |
| mean error | 0.014138   |              |              |              |              |              |

| im2    | 0            | 0            | 0.613385     | 0.009635     | 0.01039      | 0.001653     |
| im2ripl| 0            | 0            | 0.58891      | 0.00706      | 0.005056     | 0.000645     |
| error  | 0            | 0            | 0.044775     | 0.002575     | 0.005334     | 0.001008     |
| mean error | 0.013423   |              |              |              |              |              |

| im2    | 0            | 0            | 0.613385     | 0.009635     | 0.01039      | 0.001653     |
| im2twi | 0            | 0            | 0.569924     | 0.00741      | 0.005256     | 0.000815     |
| error  | 0            | 0            | 0.043461     | 0.002225     | 0.005134     | 0.000838     |
| mean error | 0.012915   |              |              |              |              |              |
In order to view the sensitivity of G.M and Z.M against various image orientations for different image categories graphs of mean error versus image variants for G.M and Z.M are plotted to further highlight the major differences, as depicted in Figure 2 and 3.

**Figure 2:** Graph of Mean Error vs. Image Variants of Different Categories for G.M

**Figure 3:** Graph of Mean Error vs. Image Variants of Different Categories for Z.M
Further analysis is done to determine intraclass invariances of each image category to different perturbations both for G.M and Z.M. Results are tabulated in Table 5 and 6.

**Table 5: Intraclass invariances of G.M**

<table>
<thead>
<tr>
<th>Cat.</th>
<th>Shape</th>
<th>im_h</th>
<th>im_v</th>
<th>imo_90</th>
<th>im_b</th>
<th>im_s</th>
<th>im_rpl</th>
<th>im_twl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Circle</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>2</td>
<td>Rectangle</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>3</td>
<td>Word-embedded</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>4</td>
<td>Miscellaneous</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

where

- Cat. represents Category
- im_h the original image placed horizontally
- im_v the original image placed vertically
- imo_90 the original image rotated to 90 degrees
- im_b the original image is enlarged by 50%
- im_s the original image is reduced by 50%
- im_rpl the original image is rippled to −160 degrees
- im_twl the original image twirled to −72 degrees

- × less sensitive (mean error below 1)
- ✓ slightly sensitive (1 =< mean error <= 2)
- ✓ ✓ very sensitive (mean error > 3)

Similarly, in order to determine intraclass invariances of set of features produced by Z.M for each image category to each perturbation, analysis is further carried out on Figure 3 and the results are tabulated as shown in Table 6. Please note again here that the mean error range for Z.M falls below 0.1, very much lower than G.M.

**Table 6: Intraclass invariances of Z.M**

<table>
<thead>
<tr>
<th>Cat.</th>
<th>Shape</th>
<th>im_h</th>
<th>im_v</th>
<th>imo_90</th>
<th>im_b</th>
<th>im_s</th>
<th>im_rpl</th>
<th>im_twl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Circle</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>2</td>
<td>Rectangle</td>
<td>×</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>3</td>
<td>Word-embedded</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>4</td>
<td>Miscellaneous</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

where

- × less sensitive (mean error < 0.01)
- ✓ moderately sensitive (0.01 =< mean error <= 0.02)
- ✓ ✓ sensitive (0.02 =< mean error < 0.8)

Based on the sensitivity results portrayed in Table 5, the findings obtained regarding the set of G.M of trademark image features are as follows:

- It is invariant to horizontal, vertical, rippled and twirled perturbations for all trademark image categories.
• It is sensitive to image enlargement for all categories.
• 3 out of 4 categories are sensitive to image orientation.
• Category 2 (rectangle) is sensitive to orientation, size enlargement and size reduction.
• Category 1 (circle) is only sensitive to image enlargement.

Based on the sensitivity results portrayed in Table 6, it is found that the set of features produced by Z.M:

• all categories of trademark images are invariance to horizontal and twirled perturbations.
• trademark images belong to Category 2 (rectangle) are only sensitive to vertical perturbation.
• trademark images belong to Category 1 (circle) are sensitive to vertical and 90 angular orientation. They are moderately sensitive to image enlargement, image reduction and rippled perturbation.
• Trademark images belong to Category 3 and 4 are invariance to all perturbations.

It is noticed that set of G.M features for Category 2 (rectangle) is very sensitive to every image perturbations, however its Z.M features are invariant almost to all perturbations. On the other hand set of G.M features for Category 1 (circle) is almost invariant to all perturbations, however its Z.M features is sensitive to almost all perturbations.

The total moment functions of G.M obtained from central moments up to order three is seven. However it is observed that the values for higher-order moment functions from Moment Function 5 to Moment Function 7 are less significant when compared to lower-order moments. This is due to the intrinsic nature of trademark images that do not have a uniform distribution of black pixels due to occlusion, unlike hand-written numerals. In handwritten numerals, higher-order moment functions are significant [18]. On the other hand, Z.M has a total moment functions of six. Conversely, the lower-order moment functions of Z.M do not contribute at all to represent an image. Moments of order three is very significant and it becomes less significant as the moments orders are greater than three. This is due to computation of Z.M that is based on polar coordinates that constrained the image to a unit circle.

Studies are then performed to study the sensitivity of various image perturbations to both G.M and Z.M features. It is noticed that G.M is sensitive to image scaling with regard to size enlargement and 90 degree angular orientation. As for other perturbations like horizontal and vertical displacement, rippled and twirled, the G.M features are invariant. Images belong to Category 2 (rectangle) display the highest mean error in terms of size enlargement, size reduction and 90 degree angular orientation when compared to other shapes. The fact that the image is sensitive to 90 degrees angular orientation is because the G.M features are not rotation invariant, except for circle shape. Image of size circle is invariant to 90 degrees angular orientation is due to the nature of shape circle that maintain its original shape regardless of any rotation. With regards to scaling, when the trademark image undergoes perturbations, its global shape changes dramatically thus the G.M features differ too, since the G.M technique captures global shape. This finding falsifies the claim made by [8] in deriving the equations for a set of 7 moment functions.
The introduction of a parameter

\[ \eta_{pq} = \frac{\mu_{pq}}{\mu_{q0}}, \]

is insufficient to make the G.M features invariant to size for the case of trademark images. Hu tested the G.M invariance property on character recognition. Hence the size invariance is dependent on types of images used.

The mean errors of all image categories for Z.M features lie below 0.1, very much lower when compared to G.M features. Within the (0 to 0.08) range trademark images of shape circle and rectangle are sensitive to perturbations. Almost all perturbations affect trademark images of shape circle except horizontal displacement. This is due to the Z.M that are defined in terms of polar coordinates (r,θ), the Zernike polynomials will have to be evaluated at each pixel position. The polar form suggests a square-to-circular image transformation, so that the Zernike polynomials need to be computed only once for all pixels mapped to the same circle [7]. As for the image of type circle, it is possible the pixel coordinates (represented by the radius and position index of the pixel) of the transformed circular image differ from the original image that causes the variances observed.

Z.M image features belonging Category 3 and 4 are invariant to all perturbations. Category 3 is word-embedded mark. From the literature, it is found that Z.M has been successfully applied to extract features from handwritten numerals and alphabets [9][10][11][12][13]. However this is the first attempt of using Z.M function orders computed from G.M to extract features from trademark images. The results for intraclass variation are far better than G.M image features.

CONCLUSION

This paper presents G.M and Z.M as feature extraction techniques for trademark images, their implementations, results and evaluations of the resulting features in terms of scale, angular, position and distortion invariances. The findings obtained are that the set of trademark image features produced by G.M are invariant to position and distortion. It is sensitive to scale and 90 degrees angular orientation. However the Z.M features are invariant to all perturbations in this experiment, except for circle shape, however the mean error is acceptable (i.e. 0.08). Thus we can conclude that the set of features produced by Zernike Moment is of better quality than that of Geometric Invariant Moment technique.

REFERENCES


APPENDIX A

FOUR CATEGORIES OF TRADEMARK IMAGE SAMPLES

Category 1

im23

im24

censtra

im50

im51

ICI

im61

im63

im64

im69

im71

im75

Category 2

im2

im3

Marlboro

APS

SRE
MALAYSIA

im4

im7

im8

FOGA

im9

im12

im40

im46

im16

POLARIS

POP

LUBEXPRESS

MEGA

ESTIQUE
APPENDIX B

Category 3  WORD-EMBEDDED
im43  im41  im80  im85  im72

Hoteles  PERIDUA  SOL  3M
im42  im47  im81  im77  im70

Category 4  COMPLEX
im17  im79  im26  im66  im88

im73  im41  im27  im28  im38