# DETERMINING APPROXIMATE STACKELBERG STRATEGIES IN CARBON CONSTRAINED ENERGY PLANNING USING A HYBRID FUZZY OPTIMISATION AND ADAPTIVE MULTI-PARTICLE SIMULATED ANNEALING TECHNIQUE

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## *ABSTRACT*

*In recent years, there has been growing international concern about climate change as a result of greenhouse gas emissions from human activity. Various process integration techniques have thus been developed to assist in determining the optimal allocation of energy sources to sectoral or regional demands under carbon footprint constraints; for example, the source-sink representation of this problem has been solved using graphical and algebraic pinch analysis techniques as well as linear programming. This work presents an extension of the original problem by incorporating a game-theoretic, two-level decision framework, which is a more accurate representation of real-life energy planning applications. The upper level decision-maker (i.e., the government) seeks to minimise total costs to society by selecting appropriate emission limits for each sector as well as subsidy levels for clean energy sources; on the other hand, the lower level decision-maker (i.e., industry) seeks to minimize total energy-related costs subject to the emission limits set by the government. This problem is a static Stackelberg game which may be formulated as a fuzzy bi-level optimisation model. A numerical example from literature is used to illustrate the modeling approach. The case study is then solved using an adaptive multi-particle simulated annealing algorithm to yield an approximate Stackelberg solution.*

**Keywords:** *Bi-level Programming; Carbon Constraints; Energy Planning; Source-sink Model; Stackelberg Game*

## 1.0 INTRODUCTION

In recent years, growing international concern about climate change has led to increased research emphasis on mitigation of emissions of greenhouse gas (GHG) emissions. Since a large portion of global GHG emissions are in the form of $CO_2$ from energy use, various approaches to achieve this goal include fuel substitution, efficiency enhancement and carbon capture. Process systems engineering (PSE) techniques have also been developed to optimise the utilisation of different fuels in energy systems with carbon emission constraints. For example, Tan and Foo (2007) developed a graphical pinch analysis approach for allocating fossil fuels to different energy sinks or demands, so as to minimise the requirement for clean or low-carbon energy sources such as renewable or nuclear power. The technique is based on approximating such energy sources as having virtually zero carbon emissions, which is justified by their low carbon footprint in comparison to conventional fossil fuels. The graphical method was also shown to be equivalent to a linear programming (LP) problem. A subsequent paper (Foo *et al.*, 2008) showed how the same problem may be solved numerically through cascade analysis.

The basic carbon pinch approach was then extended to segregated targeting problems (Lee *et al.*, 2009; Bandyopadhyay *et al.*, 2010), as well as problems involving demand growth (Atkins et al., 2010), carbon capture and storage or CCS (Tan *et al.*, 2009a) and carbon footprint visualisation for industrial plants (Tjan *et al.*, 2010). Furthermore, the methodology was also shown to be applicable to land use (Foo *et al.*, 2008), water footprint (Tan *et al.*, 2009b) and emergy (Bandyopadhyay *et al.*, 2010) constraints in energy planning. More recently, there has been a shift in emphasis towards mathematical programming, which is able to handle complex energy planning considerations that are not possible using purely pinch-based approaches. For example, Pękala et al. (2010) extended the basic LP model from Tan and Foo (2007) to cases involving biofuel production with trade considerations, and to deployment of CCS retrofits in the power sector (Tan *et al.*, 2009a).

The main limitation of these insight-based and mathematical programming-based optimisation approaches is the implicit assumption that a single decision-maker exists for the energy system. In reality, energy planning involves the complex interaction of multiple players or stakeholders. Thus, such problems are best dealt with using game theory, which accounts for the self-interested behavior of independent agents. In its simplified form, the interaction may be reduced to one that occurs between an upper level regulatory decision-maker (*i.e.*, government) on one hand, and a lower level decision-maker (*i.e.*, private industry) on the other. In general it may be assumed that the government plays the role of trying to influence industry to behave in an environmentally responsible manner (through regulations or economic incentives), while industry seeks to maximise profit. This sort of leader-follower interaction may be represented as a Stackelberg game (Stackelberg, 1951; Simaan and Cruz, 1973) where the leader's problem is to determine appropriate incentive strategies that induce the follower to react in a manner that favors the leader's interests (Salman and Cruz, 1981). This situation requires that the leader anticipates the follower's reactions. A static Stackelberg game can be formulated as a bi-level mathematical programming problem (Bard, 1998) which essentially consists of one optimisation model (representing the follower's desires) nested within an outer model (representing the leader's objective).

This paper presents a bi-level optimisation model for the carbon constrained energy planning problem originally posed by Tan and Foo (2007). The approach assumes that the leader decides on the emission limits and subsidy levels for clean energy, while the follower seeks only to maximise profit (or minimise cost). The model is described in the next section. A hybrid approach is then shown for determining an approximate Stackelberg strategy, which involves relaxation of constraints in a fuzzy optimisation model, followed by determination of a highly satisfactory (*i.e.*, "satisficing") solution using an adaptive multi-particle simulated annealing (AMPSA) algorithm. A case study based on literature is then used to illustrate the methodology. Finally, conclusions are drawn and prospects for future research are identified.

## 2.0 PROBLEM STATEMENT

The idealised bi-level carbon constrained energy planning problem may be stated as follows. The problem consists of two decision makers, the first representing government and the second representing industry. Assume that there are $m$ energy sources, for which the available quantity and carbon intensities are known. In addition, there is one additional resource denoted as clean or zero-carbon energy, whose carbon intensity is so low when compared to the other fuels as to be negligible; there is no specified limit for this energy source. There are also $n$ sinks, each with a fixed energy demand and carbon emission limit imposed by the upper level decision maker. The problem is for the upper level decision maker to minimise external costs (borne by society as a result of emissions) by setting appropriate levels of subsidy for clean energy and emission limits, subject to the lower level decision maker's objective to minimise direct energy costs, and subject to system-wide energy balances. A further simplifying assumption is that all energy sources are fully interchangeable.

## 3.0 BI-LEVEL PROGRAMMING MODEL

| **Parameters** | |
|---|---|
| $B$ | External cost per unit of carbon dioxide emissions |
| $C_z$ | Unsubsidized cost per unit of clean energy |
| $C_i$ | Cost per unit of energy source $i$ |
| $D_j$ | Energy demand of sink $j$ |
| $F_{max,j}$ | Maximum allowable carbon emission intensity limit per unit energy for sink $j$ |
| $H_i$ | Carbon footprint per unit of energy source $i$ |
| $R$ | Maximum fraction of clean energy cost to be subsidized |
| $S_i$ | Maximum availability of energy source $i$ |
| | |
| **Leader's Variables** | |
| $A$ | Subsidy per unit of clean energy |
| $F_j$ | Carbon emission intensity limit per unit energy for sink $j$ |
| $G_1$ | Total cost burden to society |
| | |
| **Follower's Variables** | |
| $E_{ij}$ | Amount of energy from source $i$ allocated to sink $j$ |
| $G_2$ | Total energy cost |
| $Z_j$ | Amount of clean energy allocated to sink $j$ |
| | |

The leader's objective (Equation 1) is to minimise the cost burden to society:

$$\min G_1 \tag{1}$$

subject to:

$$G_1 = A\sum_j Z_j + B\sum_j F_j D_j \tag{2a}$$

$$F_j \leq F_{max\,j} \qquad \forall j \tag{2b}$$

The first term in Equation 2 is the cost of subsidising clean energy in the system, while the second term is the external cost of environmental impacts arising from carbon emissions. The follower's objective (Equation 3) is to minimise total energy costs:

$$\min G_2 \tag{3}$$

subject to:

$$G_2 = (C_z - A)\sum_j Z_j + \sum_i C_i \sum_j E_{ij} \tag{4}$$

The first term in Equation 4 is the cost of the subsidised clean energy, while the second term is the cost of fossil energy. A limit may be set for the fractional subsidy ($R$) of clean energy cost, as in Equation 5:

$$A \leq RC_z \tag{5}$$

The energy balance for each demand is:

$$Z_j + \sum_i E_{ij} = D_j \qquad \forall j \tag{6}$$

The first and second terms on the left hand side of Equation 6 indicate the amount of clean and fossil fuel allocated to a given sink, respectively. The carbon emission balance for each demand is:

$$\sum_i H_i E_{ij} \leq F_j D_j \qquad \forall j \tag{7}$$

Equation 7 assumes that clean energy generates negligible carbon emissions, and that whatever emissions are generated by fossil fuel use should fall within the limits imposed by the leader. The energy balance for each source is:

$$\sum_j E_{ij} \leq S_i \qquad \forall i \tag{8}$$

Equation 8 simply states that the amount of each energy source allocated to the sinks cannot exceed the available supply. In addition to these equations, all variables are non-negative. The corresponding non-negativity constraints are no longer listed here so as to save space. It can be seen that bilinear terms are present in Equations 2 and 4, and thus the model is a non-linear programming (NLP) formulation.

## 4.0 FUZZY BI-LEVEL PROGRAMMING MODEL

| Fuzzy Model Parameters | |
|---|---|
| $A'$ | Subsidy level per unit of clean energy based on follower's optimum |
| $A^*$ | Subsidy level per unit of clean energy based on leader's optimum |
| $F'_j$ | Carbon emission intensity limit per unit energy for sink $j$ based on follower's optimum |
| $F^*_j$ | Carbon emission intensity limit per unit energy for sink $j$ based on leader's optimum |
| $G'_1$ | Total cost burden to society based on follower's optimum |
| $G^*_1$ | Total cost burden to society based on leader's optimum |
| $G'_2$ | Total energy cost based on follower's optimum |
| $G^*_2$ | Total energy cost based on leader's optimum |
| | |
| **Fuzzy Model Variables** | |
| $\lambda$ | Fuzzy Degree of Satisfaction |

The fuzzy approach to solving the general bi-level mathematical programming model was first proposed by Lai (1996) and Shih *et al.* (2001), and has since been developed further, for example by Lee (2001) and Sinha (2003). The general approach is applicable to both linear and non-linear problems.

The main steps are as follows:

- The model is solved as a single-level optimisation problem using the leader's objective function (Equation 1), and disregarding the follower's objective function (Equation 3). All the constraints (Equations 2a, 2b, 4 – 8) are also used in this step. This step determines the values of the objective functions $G^*_1$, $G^*_2$, and the leader's variables $A^*$ and $F^*_j$. This is the solution if the leader is in a position to decide on the values of all the variables in the system. Inspection of the model clearly shows that finding the solution is trivial, and that the leader's preferred solution is $G^*_1 = A^* = F^*_j = 0$. Thus, the model reduces to an LP which can easily be solved to find a global optimum.
- The model is again solved as in the previous step, this time using the follower's objective function (Equation 3) and disregarding the leader's (Equation 1). As in the previous step, all the constraints (Equations 2a, 2b, 4 – 8) are considered. In this step, the values of $G'_1$, $G'_2$, and the leader's variables $A'$ and $F'_j$ are determined, which correspond to the solution if the follower was in control of all the variables in the system. It can also be seen easily that the follower will obviously prefer maximum subsidy and emission limits, which entails saturating the constraints given by Equations 2b and 5. Once these are set to the limiting values, the model again reduces to an LP for which the global optimum can be found readily.
- If the solutions in Steps 1 and 2 coincide, then they correspond to the exact Stackelberg strategy for the system. However, such cases will be rare. Thus, in general, it will be necessary to reconcile the conflict of interest between leader and follower. The procedure entails having the leader set fuzzy bounds for his objective and control variables; likewise, the follower also sets fuzzy bounds for his own objective function. Generally, these fuzzy bounds are subjectively defined (Lai, 1996; Lee, 2001; Shih **et al.**, 2001; Sinha, 2003), but the solutions to Steps 1 and 2 can be used as basis for identifying reasonable values. The leader then relinquishes control to the follower, on the condition that his fuzzy bounds are used as constraints, in addition to all other constraints listed in the previous section. For simplicity, the fuzzy bounds are assumed to be defined by linear membership functions (Lai, 1996; Lee, 2001; Shih **et al.**, 2001; Sinha, 2003) as follows:

$$G_1 - G'_1 \geq \lambda\,(G^*_1 - G'_1) \tag{9}$$

$$A - A' \geq \lambda\,(A^* - A') \tag{10}$$

$$F_j - F'_j \geq \lambda\,(F^*_j - F'_j) \qquad \forall j \tag{11}$$

$$G_2 - G^*_2 \geq \lambda\,(G'_2 - G^*_2) \qquad \forall j \tag{12}$$

- The variable $\lambda$ which ranges from 0 to 1 is introduced as an index of fuzzy degree of satisfaction of the constraints. A solution is partially satisfactory in the fuzzy sense when $0 < \lambda < 1$. Note that when $\lambda \to 1$, $G_1$, $A$ and $F_j$ approach the optimal values from Step 1 while $G_2$ approaches the optimum from

Step 2. Hence, it is in effect possible to seek a compromise that may be considered as an approximate Stackelberg strategy (Tan *et al.*, 2010) by maximising the value of λ:

$$\boldsymbol{max}\ \lambda \qquad (13)$$

Thus, the original bi-level problem has been translated into an equivalent single-level fuzzy optimisation problem. The non-linear programming (NLP) model may then be solved using an appropriate optimisation algorithm. The next section describes the stochastic algorithm used in this work.

## 5.0 ADAPTIVE MULTI-PARTICLE SIMULATED ANNEALING (AMPSA)

| AMPSA Nomenclature | |
|---|---|
| $\alpha$ | Random walk coefficient |
| $\beta$ | Particle interaction coefficient |
| $\Delta_{k,t+1}$ | Change in fitness value of the ***k***th solution between successive iterations |
| $f(\boldsymbol{x}_{k,t+1})$ | Fitness value of the ***k***th candidate solution in iteration ***t*** + 1 |
| $f(\boldsymbol{x}_{k,t})$ | Fitness value of the ***k***th solution in iteration ***t*** |
| $\boldsymbol{P}_{k,t+1}$ | Probability of updating the ***k***th solution |
| $\boldsymbol{r}_{k,t+1}$ | Vector of random numbers in the interval [–0.5, 0.5] |
| $\boldsymbol{R}_{k,t+1}$ | Matrix with diagonals comprised of random numbers in the interval [0, 1] and with all non-diagonal elements at 0 |
| $\rho$ | Adaptive cooling coefficient |
| $\boldsymbol{s}_t$ | Standard deviation of all fitness values in iteration ***t*** |
| $\boldsymbol{T}_t$ | Temperature in iteration ***t*** |
| $\boldsymbol{x}_{k,t+1}$ | Vector of decision variables corresponding to the ***k***th candidate solution in iteration ***t* + 1** |
| $\boldsymbol{x}_{1,t}$ | Vector of decision variables corresponding to best solution in iteration ***t*** |
| $\boldsymbol{x}_{k,t}$ | Vector of decision variables corresponding to ***k***th solution in iteration ***t*** |

This section describes the simulated annealing (SA) based stochastic algorithm used to solve the fuzzy NLP derived from the original bi-level model. SA was originally developed by Kirkpatrick *et al.* (1983) based on a mathematical analog of metallurgical annealing. In the latter, the crystalline lattice of a heated metallic substance is allowed to reach the lowest possible energy levels by means of slow cooling. The enhanced algorithm is described below.

The adaptive multi-particle simulated annealing (AMPSA) technique was developed by Tan (2008) for general optimisation problems encountered in process systems engineering applications. The algorithm combines features of SA and particle swarm optimisation (PSO); the latter optimisation method was developed by Kennedy and Eberhart (1995). Unlike conventional SA, it involves parallel search using multiple particles that interact among themselves in order to accelerate the search for near-optimal solutions. In every iteration, a candidate solution is determined through a random perturbation of each of the multiple particles or solutions currently present in the algorithm:

$$\boldsymbol{x}_{k,t+1} = \boldsymbol{x}_{k,t} + \alpha\boldsymbol{r}_{k,t+1} + \beta\boldsymbol{R}_{k,t+1}(\boldsymbol{x}_{1,k} + \boldsymbol{x}_{k,t}) \qquad \forall \boldsymbol{k,t} \qquad (14)$$

Equation 14 shows that the random perturbations consist of a random walk component (given by the second term on the right side of the equation) plus an interaction component (given by the last term) which biases the search of each particle towards the direction of the best solution currently available (*i.e.*, ***k*** = 1). Note that the final term disappears for the "lead particle" corresponding to the best solution. The next step is to determine whether the candidate solution determined by Equation 14 is to be accepted or not. The probability of acceptance is given by the Metropolis criterion:

$$\boldsymbol{P}_{k,t+1} = \boldsymbol{min}\left[1, e^{-\left(\frac{\Delta_{k,t+1}}{\boldsymbol{T}_k}\right)}\right] \qquad \forall \boldsymbol{k,t} \qquad (15)$$

where:

$$\Delta_{k,t+1} = f(\boldsymbol{x}_{k,t+1}) - f(\boldsymbol{x}_{k,t}) \qquad \forall \boldsymbol{k,t} \qquad (16)$$

Equations 15 and 16 assume that the problem is one of function minimisation. If the new solution is better than the current one, then a "greedy" heuristic is employed and the new solution automatically replaces the old one. It thus becomes the starting point for the next iteration. If, however, the new candidate solution is worse than the previous one, it is not rejected outright. Instead, it is accepted with a probability defined by the exponential distribution in Equation 15. It can be easily seen that the probability of accepting such solutions declines as the extent of fitness deterioration increases. The probability distribution is modulated by the temperature parameter, which in AMPSA is a linear function of the standard deviation (or degree of diversity) of the currently available multiple solutions:

$$\boldsymbol{T}_t = \rho \boldsymbol{s}_t \qquad \forall \boldsymbol{t} \qquad (17)$$

In general, the diversity of the different solutions being computed in parallel declines as the algorithm progresses; as a result, the value of the temperature parameter falls as well. This achieves the cooling effect that is normally accomplished by geometric progression in conventional SA. In this case, the temperature is said to be adaptive since it responds to the quality of the solutions found by the algorithm to date.

It should be emphasized that, in practice, AMPSA, as with most stochastic algorithms, does not exhibit true convergence in the strict mathematical sense. Solutions found by such techniques are said to be "satisficing," *i.e.*, not strictly optimal, but highly satisfactory (or near-optimal) from the application standpoint. Furthermore, a stochastic algorithm will not necessarily give exactly the same solution each time when it is used to solve the same optimization problem repeatedly. In this work, the algorithm was implemented in a program coded in Visual Basic for Applications (VBA) (Tan, 2008).

## 6.0 CASE STUDY

This illustrative case study is based on the example from Tan and Foo (2007), which involves the static allocation of clean or zero-carbon energy, along with coal, oil and natural gas, across three regions. Each region has a specified energy demand and a carbon emissions limit to be set by the leader. The data for the sources and sinks are shown in Table 1. The maximum allowable emissions limits in the final column correspond to carbon intensity limits of 20, 50 and 100 *kt/EJ* for Regions I, II, and III, respectively. However, the leader's objective is to minimise total costs to society, which consists of the cost to subsidise clean energy plus the external costs associated with carbon emissions.

***Table 1: Energy sources and sinks (Tan and Foo, 2007) [16]***

| Source | Quantity (EJ) | Carbon footprint (kt/EJ) | Sink | Requirement (EJ) | Emissions limit (kt) |
|---|---|---|---|---|---|
| ***Coal*** | 600 | 105 | Region I | 1000 | 20,000 |
| ***Oil*** | 800 | 75 | Region II | 400 | 20,000 |
| ***Natural gas*** | 200 | 55 | Region III | 600 | 60,000 |
| ***Zero-carbon*** | No limit | 0 | Total | 2000 | 100,000 |

***Table 2: Solution to case study***

| Variable | Leader's model | Follower's model | Fuzzy limits | Fuzzy model solved by AMPSA |
|---|---|---|---|---|
| $\lambda$ | n/a | n/a | n/a | 0.11 ± 0.02 |
| $G_1$($10^3$ cost units) | 0 | 390 | 0 – 390 | 315 ± 10 |
| $G_2$ ($10^3$ cost units) | 3,200 | 2,353 | 2,353 – 2,700 | 2,662 ± 8 |
| $A$ (cost units/*EJ*) | 0 | 320 | 0 – 200 | 180 ± 4 |
| $F_1$ (*kt/EJ*) | 0 | 20 | 0 – 12 | 9.0 ± 1.5 |
| $F_2$ (*kt/EJ*) | 0 | 50 | 0 – 40 | 29.2 ± 5.7 |
| $F_3$ (*kt/EJ*) | 0 | 100 | 0 – 50 | 39.9 ± 5.8 |
| $Z_1$ (*EJ*) | 1,000 | 773 | n/a | 902 ± 15 |
| $Z_2$ (*EJ*) | 400 | 133 | n/a | 274 ± 32 |
| $Z_3$ (*EJ*) | 600 | 0 | n/a | 344 ± 44 |
| $E_{11}$ (*EJ*) | 0 | 100 | n/a | 70 ± 24 |
| $E_{12}$ (*EJ*) | 0 | 0 | n/a | 81 ± 34 |
| $E_{13}$ (*EJ*) | 0 | 500 | n/a | 159 ± 57 |
| $E_{21}$ (*EJ*) | 0 | 127 | n/a | 22 ± 21 |
| $E_{22}$ (*EJ*) | 0 | 267 | n/a | 41 ± 44 |
| $E_{23}$ (*EJ*) | 0 | 100 | n/a | 97 ± 77 |
| $E_{31}$ (*EJ*) | 0 | 0 | 0 | 0 |
| $E_{32}$ (*EJ*) | 0 | 0 | 0 | 0 |
| $E_{33}$ (*EJ*) | 0 | 0 | 0 | 0 |

*Table 3: Example of a satisficing energy allocation network (values given in EJ)*

| | Zero-carbon | Coal | Oil | Natural gas |
|---|---|---|---|---|
| ***Region I*** | 933 | 24 | 43 | 0 |
| ***Region II*** | 275 | 110 | 16 | 0 |
| ***Region III*** | 327 | 219 | 54 | 0 |
| ***Excess*** | n/a | 248 | 687 | 200 |

Here it is assumed that the maximum acceptable value for the subsidy (***A***) is 320 cost units per *EJ*, equivalent to 20% of the cost of clean energy, while the coefficient for environmental costs is B = 1 unit per kiloton (***kt***) of $CO_2$. The follower seeks to minimize the cost of supplying energy to the different sinks, assuming that the costs of clean energy, coal, oil and natural gas are 1,600, 1,000, 1,200 and 1,250 units per *EJ*, respectively. In this case study, fictitious cost units are used but the relative magnitudes of the coefficients are based on realistic assumptions. A similar approach to costing is used by Pękala *et al.* (2010).

Solving the model using the leader's objective function while disregarding the follower's objective (Step 1) gives the result in the second column of Table 2. Note that this step assumes that the leader is able to dictate all decisions within the system. Since this step reduces the model to *LP*, the solution can be easily found using any optimization software; in this case, Lingo 11.0 was used. The optimal level of subsidy (***A***) as well as the emission limits ($F_1$, $F_2$ and $F_3$) are all zero, which leads to the external costs borne by society ($G_1$) to be zero as well. These limits force the follower to use only zero-carbon energy sources to meet the demands of the three regions, such that fossil energy use ($E_{ij}$ for all ***i*** and ***j***) becomes zero throughout the system. The corresponding cost for the follower ($G_2$) becomes 3,200 × $10^3$ units.

On the other hand, Step 2 involves solving the model using the follower's objective function, while disregarding the leader's, which then gives the result in the third column of Table 2. Here it is assumed that the follower dictates the decisions; the model again reduces to a simple *LP* which is solved here with Lingo 11.0. Thus, subsidy level for clean energy (***A***) as well as regional emission limits $F_1$, $F_2$ and $F_3$) reach their maximum levels, which leads to an external cost ($G_1$) of 390 × $10^3$ units, while the total of energy costs ($G_2$) is at the minimal level of 2,353 × $10^3$ units. Note that the energy sources used are 906 *EJ* (773 + 133 + 0 *EJ*) of clean energy, 600 *EJ* (100 + 0 + 500 *EJ*) of coal and 494 *EJ* (127 + 267 + 100 *EJ*) of oil. Interestingly, there is no usage of natural gas at all in either the leader's or the follower's solution. Thus, the use of this fuel may be excluded from the fuzzy model as well.

The next step is to determine the fuzzy bounds for the leader's control variables and objective function, relative to his previously determined optimum. The fuzzy range for the follower's objective function is also determined. It should be noted that, in general, these limits may be determined subjectively (Lai, 1996; Shih *et al.*, 2001, Lee, 2001 and Sinha, 2003); however, the solutions to Steps 1 and 2 provide some indication of reasonable numerical values to be used so that a partially acceptable solution can be found. These ranges are given in the fourth column of Table 2 and are then integrated into a single, unified fuzzy NLP model, as described previously. The model is then solved for a near-optimal or satisficing solution using an AMPSA program coded in VBA (Tan 2008). This program has twenty particles and terminates after 1,000 iterations, which gives a total of 20,000 function evaluations (20 × 1,000) per run. As with most stochastic algorithms, AMPSA does not exhibit convergence towards an optimum in the strict mathematical sense, but in general will yield satisficing solutions if properly tuned. The final column of Table 2 shows the range of values (average ± standard deviation) found for ten repeated runs using AMPSA. It can be seen that the results for λ as well as the leader's and follower's objectives ($G_1$ and $G_2$) are fairly consistent. On the other hand, there is considerable variation in the final values of the decision variables, with the exception of the level of subsidy (***A***). This result indicates that the problem exhibits degeneracy – *i.e.*, there may be multiple solutions that exhibit the same optimal or near-optimal objective function values.

Table 3 shows an example of an allocation network for which λ = 0.10, ***A*** = 180 cost units/*EJ*, $F_1$ = 5.8 *kt/EJ*, $F_2$ = 31.7 *kt/EJ* and $F_3$ = 45.1 *kt/EJ*. This solution is randomly selected from the ten runs implemented for the case study; it must be emphasized that it is only one of many possible schemes that achieves an approximate Stackelberg strategy. In this case, the resulting objective function values are $G_1$ = 322 × $10^3$ cost units for the leader and $G_2$ = 2,666 × $10^3$ cost units for the follower; it may be seen that both of these values fall within the statistical range of variation given in Table 2. Note that, in this solution, the three region use a combined 1,535 *EJ* (933 + 275 + 327 *EJ*) of clean energy, as compared to only 906 *EJ* without the leader's intervention in the form of carbon emission limits and subsidies. Thus, by using an approximate Stackelberg strategy, the leader is able to induce environment-friendly behavior even when the decision of the follower is motivated purely by cost considerations.

## 7.0 CONCLUSION

A hybrid approach to the determination of approximate Stackelberg solutions in bi-level carbon-constrained energy planning problems has been developed. This technique assumes that the upper level decision-maker, or leader, controls the emission limits to be imposed on the lower-level decision-maker, or follower, as well as subsidy levels for clean energy sources. The leader (*i.e.*, government) seeks to minimise total costs to society as a result of environmental impacts as well as the direct expense of the clean energy subsidy. On the other hand, the follower (*i.e.*, industry) is assumed to control the actual allocation of energy sources to different demands in order to minimise energy costs while meeting the emission limits imposed by the leader. The solution procedure involves relaxing the conflicting solutions of the leader and follower to yield a fuzzy non-linear program (NLP); a satisficing or near-optimal solution can then be found using an adaptive, multi-particle simulated annealing algorithm (AMPSA). As demonstrated by a numerical case study, the model makes it is possible for the leader to identify appropriate emission limits and subsidy levels to induce the follower to allocate energy sources in an environmentally responsible manner, even when the latter's principal goal is to minimize costs. Further research can still be done to extend the model to more realistic cases by relaxing some of the simplifying assumptions used in this paper.

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## PROFILE

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