ABSTRACT

The third order intermodulation distortion (IMD₃), instead of the fifth order intermodulation distortion (IMD₅), is often thought to limit the performance of the radio-over-fibre system as it tends to fall in-band. A frequency plan based on the Golomb Ruler is suggested to overcome this IMD₃ problem. However, its effect on the IMD₅ is not known. A laser model is derived based on Volterra Series with electrical parasitics included to simulate the IMD₃ and IMD₅ appearing at carrier position when the channels to be used are in accordance with the Golomb Ruler’s marks and compare them with those generated by equally spaced carriers at the maximum RF input level supported by a low and high bias current for the number of channels to be used. The IMD₃ is found to be more dominant than the IMD₅ for the equally spaced frequency plan when the low bias current was employed. The opposite was observed at high bias current. The Golomb Ruler based frequency plan led to no IMD₃ but low levels of IMD₅ were still present at the channels designated for use when both bias currents were employed but they were lower than those due to the equal frequency spaced channels.

Keywords: Fifth Order Intermodulation Distortions, Golomb Ruler and Third Order Intermodulation Distortions

1.0 INTRODUCTION

The radio-over-fibre system is an analogue fibre optic system that is used to transport free space radio signals, which can be fixed or mobile wireless. Therefore it is also known as the hybrid radio fibre system [1,2]. The configuration for this system is indicated in Figure 1.

![Figure 1: Basic configurations of the radio-over-fibre system [1]](image)

The free space radio signals from the subscribers, in the uplink, are intercepted by the antenna of the radio access point (RAP) and fed to the low noise amplifier (LNA). The output of the LNA is used to modulate the electric-to-optical converter (E/O). The light output of the E/O is transported using the optical fibre to the central station, where it is detected by the optical-to-electric converter (O/E). The recovered signal goes through further processing in the intra-networking stage [1].

When the signals from the core networks are to be simulcasted or sent to the subscribers, the downlink is referred to. Signals from the core networks can be used to direct modulate the laser if the operation at low frequency is desired. If the transmission is at higher frequencies, the dual frequency optical source must be employed, as indicated in Figure 2.

![Figure 2: Dual frequency optical source for 30 to 60GHz transmission [2,3]](image)
The core networks signals must be used to modulate the millimeter wave first. Then the modulated outcome is fed to the optical modulator together with the output of the laser diode or the output of the dual frequency optical source [3]. The light output from the optical modulator is fed to the optical fibre of the optical distribution network through the optical line termination. The other end of the optical fibre is located at the RAP, where the optical signal is converted back to electrical signal and radiated or simulcasts through the remote antenna unit to be intercepted by the antennas of the subscribers’ equipment in their premises.

One of the challenges in implementing radio-over-fibre system that serves the mobile communications is to ensure the wide dynamic range of the wireless mobile communication is met by the optical portion of this hybrid fibre radio system. This is essential especially in the uplink of the system as mobile signals intercepted can be very weak when it is at the edge of the cell or very strong when it is close to the base station. The space loss in the microcell environment depended on \( \frac{1}{\lambda^2} \) and is given by \( 20 \log \left( \frac{4\pi d}{\lambda} \right) \), where \( d \) is the distance from the base station and \( \lambda \) is the wavelength of the mobile station signal [4]. For a 300m radius microcell providing GSM (890 to 915MHz) coverage, the space loss is about 80dB. This means that the dynamic range should be larger than 80dB as the mobile station can be positioned near the base station or at the cell edge [5]. As the input to the base unit of the base station should replicate the weak and strong mobile signals intercepted at the RAP, the optical link of the radio-over-fibre system must not introduce distortions at levels that would cause the mobile signals to be below the detection threshold at the receiver side.

The dynamic range can be limited by the third order intermodulation distortions (IMD3) of type \( 2f_1-f_2 \) and \( f_1+f_2-f_3 \) as they are liable to fall in-band as compared to the second orders distortions for narrowband systems [2]. The E/O conversion stage in the radio-over-fibre system is identified as one of the contributors to this distortion [1]. The IMD3 can be reduced by rearranging the channels to be used according to the frequency plans in [6] and [7]. As the mobile communication system has to follow certain frequency management or channel assignment scheme and is subjected to interference issues such as co-channel interference [5], the frequency plan being exercised cannot be simply altered to reduce the IMD3 generated by the E/O conversion.

The signal extraction with frequency arrangement (SEFA) and signal level compression (SLC) had been recommended as means to improve the dynamic range. The SEFA technique, in Figure 3, involves extracting the desired channels from the radio signals with frequencies \( f_1 \) to \( f_n \) intercepted by the Main and Sub antenna of the RAP and reassigning them with new frequencies \( f'_1 \) and \( f'_2 \) that are no longer adjacent and could reduce the IMD3 [1], [8] before feeding them to the E/O.

The SLC technique, in Figure 4, is basically used to boost the low-level channels intercepted by the antenna system of the RAP [9] before feeding them to the E/O. The SLC has a compressor that compresses or reduces the dynamic range by elevating the minimum permissible power level to a higher level while maintaining the high-level power channels at their existing levels. Initially, the signal intercepted with the dynamic range \( \Delta x \) or between \( P_{min} \) and \( P_{max} \) will produce an output with power levels between \( P_{min} \) and \( P_{max} \). Conventionally, \( \Delta x \) is similar to \( \Delta x \) and \( P_{max} \) is the same as \( P_{max} \).

In a system without the SLC, the output of the receiver or RF amplifier would be used directly to modulate the E/O. This would cause the channel with low power levels to remain weak and be more susceptible to the noise contributed by the optical link than the channel with higher power levels. If the SLC is employed, the channel with \( P_{min} \) received by the antenna of the RAP will appear as \( P_{min} \), which is higher than the \( P_{max} \) input to E/O without the SLC [9]. Thus, elevating the carrier-to-noise ratio for the low-level channel intercepted at the O/E output if the noise or distortion levels did not increase with the raise in channel power level. However, increasing the low-level power channel would elevate the inter-modulation distortion level in other channels. Therefore, the SLC should be deployed together with the SEFA technique.

In order for the SEFA scheme to be implemented, a frequency plan by which the intercepted signals are rearranged should be provided. A frequency plan can be generated through the use of multiple insert delete algorithms where the carriers in the channel with the most number of IMD3 are deleted and shifted to channels having or leading to best IMD3 improvement [7] but the IMD3, though lowered in quantity, might still be considerably high in dBm. Hunziker suggested arranging the channels to be used in an unequal manner by employing the Golomb Ruler in [2] as it avoids IMD3 from appearing at the

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**Figure 3 : SEFA technique [8]**

**Figure 4 : SLC technique [9]**
Figure 5: The Golomb Ruler for five marks

Though arranging the channels to be used according to the Golomb Ruler’s marks can lead to zero IMDs at carrier position, other intermodulation distortions (IMD) might be present at the carrier positions when the used channels are arranged according to the Golomb Ruler’s marks. Therefore the objectives of this paper is to present a radio-over-fibre system laser model derived using Volterra Series and compute the IMDs as well as fifth orders intermodulation distortions (IMD5) appearing at carrier positions when the Golomb Ruler frequency plan (GRFP) and equal frequency spacing (EQSP) are employed for the primary-GSM carriers ranging between 890 and 915MHz with maximum RF input level supported by given bias currents. The IMDs is investigated because it is the next odd order distortion after IMD3. Individual IMDs might be insignificant as compared to the IMD3s but their levels might escalate if their quantities increase and the input magnitude intensifies as the distortions often increase with the input levels. The IMDs could rise by the power of five of its input level as compared to the IMD3s that increases by the power of three. A recent technique used to linearising the IMDs is the optoelectronic method [12] and optical injection locking [13]. The optoelectronic method is similar to predistortion method proposed in [14], where the optical signal is converted to electrical signal and uses it to combine with the transmitted RF signal to reduce the IMDs. The second method involves modulating a master (MLD) and a slave (SLD) laser diodes under injection locking with RF signals. The amplitude and phase of the RF signals are tuned before supplying it to the MLD. The MLD output is fed to the SLD, where residual amplitude modulation suppression takes place. The RF signals from MLD and SLD are added and causes the IMDs to reduce [13]. These techniques are still being investigated whereas reducing IMDS using frequency planning in SEFA with SLC permits existing or off the shelves technologies to be deployed.

2.0 VOLTERA SERIES LASER MODEL

The laser diode’s behavior can be described by the single mode laser rate equation pair as given in Equations (1) and (2) [14], [15]. This indicates the laser diode is a dynamic system, which contains energy storing elements. Thus a dynamic system is a system with memory.

\[
\frac{dN(t)}{dt} = \frac{I}{qV} - N(t) \frac{g_0}{T_s} \left( N(t) - N_0 (1 - e^{S(t)}) \right) S(t) \tag{1}
\]

\[
\frac{dS(t)}{dt} = \frac{qV}{T_p} \left( N(t) - N_0 (1 - e^{S(t)}) \right) S(t) + \Gamma \beta N(t) \frac{N(t)}{T_p} S(t) \tag{2}
\]

where \(qV\) is the product of electron charge and active region volume (Am’s), \(I\) is injected current (A), \(\tau_s\) is the photon lifetime (s), \(\beta\) is the spontaneous coupling coefficient, \(\Gamma\) is the optical confinement factor, \(\varepsilon\) is power gain compression parameter (m³), \(\tau_c\) is the carrier lifetime (s), \(N_0\) is the transparent carrier density (m³), \(g_0\) is the optical power gain (m³s⁻¹), \(N(t)\) is the carrier density (m³) and \(S(t)\) is the photon density (m³).

The single mode laser rate Equations can be rearranged and combined to give an output to input relation as given in Equation (3):

\[
\frac{I_r + i(t)}{qV} = \frac{N_0}{\tau_s} + \left( \frac{d[\frac{S_r + s(t)}{\tau_s} + \frac{S_r + s(t)}{\tau_p} - \frac{N_0}{\tau_p}}{dt} \right) \tag{3}
\]

where \(I_r\) and \(i(t)\) are the laser bias and modulating currents. \(S_r\) and \(s(t)\) are the laser steady-state photon density and photon density due to the modulating current.

The terms in Equation (3) can be expanded using Taylor Series. This signifies that the system is weakly nonlinear. Volterra Series is used to model weakly nonlinear systems with memory [16]. When the laser diode is represented by Volterra Series, its output \(s(t)\) is expressed as [15]:

\[
s(t) = \sum_{u_1} \ldots \sum_{u_n} \frac{h_s(u_1, \ldots, u_n) \int i(t-u_1)du_1 \ldots du_n}{(n-1)!} \tag{4}
\]

where \(h_s(u_1, \ldots, u_n)\) is the laser diode’s \(n\)-th order Volterra Series kernel and \(i(t)\) is the modulating input signal that represents the intercepted mobile signal in terms of current density.

After performing the mentioned Taylor Series expansion, collecting the terms with \(s(t)\) leads to Equation (5) that relates the modulating input with its output [15]:

\[
N_{\tau_s} \frac{d^2s(t)}{dt^2} + N_{\tau_p} \frac{d^2s(t)}{dt^2} + R_s(t)^3 + \sigma_s(t)^2 \frac{ds(t)}{dt} + 2G_s(t) \frac{d^2s(t)}{dt^2} + G_s(t) \frac{d^2s(t)}{dt^2} + \left[ O_s(t)^3 + Q_s(t) \frac{d^2s(t)}{dt^2} + P_s(t)^3 \frac{d^2s(t)}{dt^2} + P_s(t) \frac{d^2s(t)}{dt^2} \right]
\]

This indicates that the system is a system with memory.
In order to solve the above, the exponential growth method or probing method is employed [14, 15, 16] whereby the \( i(t) \) in Equations (4) and (5) is substituted with (6):

\[
i(t) = \sum_{n=1}^{\infty} e^{j\omega_n t} + e^{j\omega_2 t} + ... + e^{j\omega_n t}
\]

(6)

Where \( n \) is the \( n \)-th order of the Volterra Series kernel seek and \( \omega_n \) is the \( n \)-th angular frequency of the input. This method permits the input to reappear after convolution is performed and the Fourier Transform of the Volterra kernel is also obtained [16], as demonstrated by:

\[
\int_{-\infty}^{\infty} h(u)e^{j\omega_2 u} du = \frac{1}{G_1(\omega)}
\]

\[
\int_{-\infty}^{\infty} h(u)e^{j\omega_2 u} du = e^{j\omega_1} H(\omega)
\]

(7)

The Fourier Transform of the first to fifth orders Volterra Series kernels are given by Equations (8) to (12):

\[
H_i(\omega_1) = \frac{1}{(D_1 + j\phi_1 - \omega_2^2 S_0)}
\]

(8)

\[
H_i(\omega_1, \omega_2) = -\frac{1}{2} [H_i(\omega_1). H_i(\omega_2)]. H_i(\omega_1 + \omega_2). G_2(\omega_1, \omega_2)
\]

(9)

\[
H_i(\omega_1, \omega_2, \omega_3) = -\frac{1}{6G_i(\omega)} \left\{ \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{2H_i(\omega)H_i(\omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega) + \sum_{k<l=m<n<p} \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \frac{6H_i(\omega, \omega, \omega)H_i(\omega, \omega, \omega) + \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \frac{4H_i(\omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega)}{\omega, \omega, \omega} + \sum_{k=l=m=n=p} \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} 6H_i(\omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega) + \sum_{k<l=m<n<p} \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \frac{4H_i(\omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega)}{\omega, \omega, \omega, \omega} + \sum_{k=0}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \frac{12H_i(\omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega)H_i(\omega, \omega, \omega, \omega, \omega) + 6H_i(\omega)}{\omega, \omega, \omega, \omega, \omega}
\]

(10)

2.1 OPTICAL MODULATION INDEX AND ELECTRICAL PARASITICS

Besides the Fourier Transform of the Volterra Series kernels, the current fed to the laser diode has to be specified to determine the IMDs. The input signal that is fed to the laser diode has to be specified to determine the range of bias current that can be employed can be determined by solving equation (19), obtained by collecting the steady-state photon density and bias current terms in Equation (3).

\[
I_0 = qV \left[ N_0 \tau_s + \frac{S_0}{\tau_p} - \Gamma \beta N_0 \right] \left[ \frac{1}{\tau_s} + \frac{1}{\tau_p} \left( \frac{1 - \beta}{\Gamma G_1(1-\epsilon S)} \right) \right]
\]

(19)
When the input to the laser diode is made up of several carriers, the total modulation index \( m_{\text{tot}} \) needs to be considered. The \( m_{\text{tot}} \) is related to \( m_{\text{opt}} \) by \[14\]:

\[
m_{\text{tot}} = \sum_{n=1}^{N} m_{\text{opt}, n}^2
\]

(20)

The \( m_{\text{opt}} \) should be maintained at approximately 1 to avoid overmodulation, which can lead to higher levels of distortions \[18\]. As the \( m_{\text{tot}} \) total modulation index is limited to a maximum of 1, the \( m_{\text{opt}} \) might depend on the number of carriers to be used, which in turn is related to the frequency reuse factor implemented.

The actual current at the active region can be different from anticipated as the current has to pass through the electrical parasitics of the laser diode \[19\]. Therefore, the electrical parasitics should be accounted for in the computation of the intermodulation distortions. The IMD are related to the Fourier Transform of the Volterra Series kernels, the electrical parasitics \( P_{\text{elec}}(w) \) and the modulating signal magnitude using Equation (21) assuming the carriers possess the same magnitude or optical modulation index:

\[
IMD\left( \sum_{x=1}^{N} a_x \omega_x \right) = \frac{1}{\Delta t} \frac{2}{\sum_{x=1}^{N} |a_x|} \left( \prod_{x=1}^{N} P_{\text{elec}}(\omega_x)^{m_{\text{opt}}^2} \right) \left( \prod_{x=1}^{N} H(\pm \omega_1, \ldots, \pm \omega_n) \right) \left\| a_x \right\|_{\infty}^{m_{\text{opt}}}.
\]

(21)

where \( a_x \) is the coefficient of the \( x \)-th tone that generates the required intermodulation distortion (IMD), \( \prod_{x=1}^{N} |a_x| \) represents the order of the IMD, and \( N \) is the number of tones that make up the IMD.

### 3.0 FREQUENCY PLANS, SIMULATION ASSUMPTIONS AND PARAMETERS

For this paper, the modulating signal is assumed to be made up of carriers with the same magnitude in the primary-GSM frequencies, which is from 890 to 915 MHz with a 200 kHz spacing. This means that 125 carriers can be supported. The channel numbering begins from 1 to 125. According to Wake in [2], the frequency reuse factor greater than 15 or a cluster of more than 15 cells are needed to meet the interference requirement. This means that the 125 carriers will be divided among the cells. For the frequency reuse factor of 20, each cell might be assigned six or seven carriers. Six carriers were employed in the IMD, and IMD appearing at carrier positions simulation in this paper. The channel number \( (\text{ch}_{\text{no}}) \) and carrier frequency \( (f_c) \) are related using Equation (21).

\[
f_c = 890 MHz + ch_{\text{no}} \times 200 kHz
\]

(22)

The Golomb Ruler for six marks in [11] is used to determine the channel used for the GRFP. Table 1 indicates the channel number \( (\text{ch}_{\text{no}}) \) and the carrier frequencies for both the GRFP and EQSP. The Golomb Ruler frequency plan took up about a fifth of the bandwidth occupied by the equally spaced carriers.

<table>
<thead>
<tr>
<th>Carrier Number</th>
<th>EQSP</th>
<th>GRFP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Channel Number</td>
<td>Carrier frequency</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>890.2MHz</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>894.2MHz</td>
</tr>
<tr>
<td>3</td>
<td>41</td>
<td>898.2MHz</td>
</tr>
<tr>
<td>4</td>
<td>61</td>
<td>902.2MHz</td>
</tr>
<tr>
<td>5</td>
<td>81</td>
<td>906.2MHz</td>
</tr>
<tr>
<td>6</td>
<td>101</td>
<td>910.2MHz</td>
</tr>
</tbody>
</table>

The parameters of the Ortel-1510B and its electrical parasitics in [19] were referred to for the simulation. A summary of the laser diode parameters is given in Table 2. The equivalent circuit and parameters for the electrical parasitics are indicated in Figure 6 and Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>product of electron charge and active region volume, ( qV )</td>
<td>( 1.28 \times 10^{-15} ) (Am’s)</td>
</tr>
<tr>
<td>Laser threshold current, ( I_{\text{th}} )</td>
<td>( 15.1 \times 10^{-17} ) (A)</td>
</tr>
<tr>
<td>photon lifetime, ( \tau_p )</td>
<td>( 1 \times 10^{-12} ) (s)</td>
</tr>
<tr>
<td>carrier lifetime, ( \tau_s )</td>
<td>( 1.6 \times 10^{-9} ) (s)</td>
</tr>
<tr>
<td>transparent carrier density, ( N_0 )</td>
<td>( 1 \times 10^{24} ) m(^{-3})</td>
</tr>
<tr>
<td>optical power gain, ( g_0 )</td>
<td>( 3.3 \times 10^{-12} ) (m^3s^-1)</td>
</tr>
<tr>
<td>spontaneous coupling coefficient, ( \beta )</td>
<td>( 2 \times 10^{-3} )</td>
</tr>
<tr>
<td>optical confinement factor, ( \Gamma )</td>
<td>( 0.35 )</td>
</tr>
<tr>
<td>power gain compression parameter, ( \epsilon )</td>
<td>( 3.2 \times 10^{-23} ) (m^3)</td>
</tr>
</tbody>
</table>

![Figure 6](image-url)
Table 3: Parasitics elements employed in simulation [19]

<table>
<thead>
<tr>
<th>Elements</th>
<th>Value</th>
<th>Elements</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_{p1}</td>
<td>3.0pF</td>
<td>R_{p2}</td>
<td>0.5Ω</td>
</tr>
<tr>
<td>L_{p1}</td>
<td>0.14nH</td>
<td>C_{p3}</td>
<td>0.46pF</td>
</tr>
<tr>
<td>R_{p1}</td>
<td>1.5Ω</td>
<td>Z_{01}</td>
<td>48.4Ω</td>
</tr>
<tr>
<td>R_{stub}</td>
<td>1.5Ω</td>
<td>f_{01}</td>
<td>3.8GHz</td>
</tr>
<tr>
<td>C_s</td>
<td>5.6pF</td>
<td>Z_{02}</td>
<td>50.5Ω</td>
</tr>
<tr>
<td>R_s</td>
<td>6Ω</td>
<td>f_{02}</td>
<td>5.13GHz</td>
</tr>
<tr>
<td>C_{p2}</td>
<td>0.2pF</td>
<td>R_m</td>
<td>58.5Ω</td>
</tr>
<tr>
<td>L_{p2}</td>
<td>1.3nH</td>
<td>C_m</td>
<td>1800pF</td>
</tr>
</tbody>
</table>

Two different simulations were performed. The first assumed a low $I_0$ of 25mA with $m_{tot}$ of 1, which translated to -3.889 dBm per carrier. The second simulation was based on the maximum permissible $I_0$ of 0.44A with $m_{tot}$ of 1, which translated to 28.786dBm per carrier. The $m_{tot}$ of 1 is used to represent maximum dynamic compression in the SLC. In both cases, the $m_{opt}$ is 0.408. The maximum $I_0$ that can be used with the Ortel-1510B is determined by solving Equation (19) and indicated in Figure 7.

The steady-state photon density is linear for the bias current from approximately 15mA until 0.88A. Therefore, $I_0$ ranges between 15 to 0.44A (based on mid-point biasing). $\Delta I$ must not exceed $0-I_0$ to avoid static non-linear distortion, due to clipping at threshold and limiting at saturation [20]. From Equations (18) and (20), $I_0$ has to be 1.564A to support six carriers of 40dBm. As such, the Ortel-1510B cannot support modulating input with RF level of 40dBm with $I_0$ of 0.44A. The maximum RF level that can be supported by this device is 28.786dBm.

4.0 RESULTS AND DISCUSSIONS

4.1 IMD$_3$ AND IMD$_5$ QUANTITIES

The IMD$_3$ and IMD$_5$ that appeared at carrier position based on simulation are listed in Table 4. Therefore, they were employed in determining the composite IMD$_3$ and IMD$_5$ quantity and levels appearing at each channel or carrier used.

The total number of IMD$_8$ and IMD$_9$ at carrier positions are indicated in Figure 8. Table 5 indicates the types of IMD$_3$ and IMD$_5$ that appeared at carrier position for both frequency plans. Both the IMD$_3$ and IMD$_5$ appeared at the EQSP’s used channels. The total number of IMD$_3$ maximises at the middle used channels following the total number to IMD$_3$ of type $f_1 + f_2 - f_3$. This type of IMD$_3$ contributes 66.7 to 77.8% of the total number of IMD$_3$ for the EQSP. Only two $2f_1 - f_2$ type IMD$_3$ appeared at all EQSP used channels.

![Figure 8: Total number of IMD$_3$ and IMD$_5$ at carrier positions](image-url)
The total number of IMD\(_3\)s at EQSP used channels is 7 to 12 times higher than that of the IMD\(_3\)s. The total number of IMD\(_3\) maximised at the fourth channel used in the EQSP because the number of \(2f_1 + f_2 - 2f_3\), \(2f_1 + f_2 - f_3 - f_4\) and \(f_1 + f_2 + f_3 - 2f_4\) types IMD\(_3\) maximised at this channel. The total number of IMD\(_3\) decreases slightly at the last two channels used as the number of \(2f_1 + f_2 - f_3 - f_4\) and \(f_1 + f_2 + f_3 - f_4 - f_5\) types IMD\(_3\) are at or near maximum. These types of IMD\(_3\) made up 66 to 78% of total number of IMD\(_3\)s in the EQSP.

The GRFP led to zero IMD\(_3\) from falling onto the channel used but some IMD\(_5\)s were still present. The \(2f_1 + f_2 - f_3 - f_4\) type IMD\(_5\) made up 38 to 54% of the total IMD\(_5\) appearing at the GRFP used channels. This is followed by IMD\(_5\) of type \(2f_1 + f_2 - 2f_3\) and \(f_1 + f_2 + f_3 - f_4 - f_5\), which contributed 4 to 27.3% and 4 to 27.3%. The total number of IMD\(_5\)s for the EQSP are three to five times higher than the GRFP at all carrier positions. This implies that lower IMD\(_5\) levels might be generated by the GRFP carriers than the EQSP carriers.

4.2 COMPOSITE IMD\(_3\) AND IMD\(_5\) LEVELS

4.2.1 LOW BIAS CURRENT

The composite IMD\(_3\) and IMD\(_5\) levels at carrier positions for both frequency plans at \(I_0\) of 25\(\text{mA}\) are given in Figure 9 and, Tables 6 and 7. The GRFP’s composite IMD\(_3\) is not indicated as zero IMD\(_3\) appears at carrier position. EQSP’s composite IMD\(_3\) levels maximise at the middle channels. This is contributed mainly by the IMD\(_3\) of type \(f_1 + f_2 - f_3\), which makes up 79 to 87.5% of the composite IMD\(_3\) observed. Though there are similar total numbers of IMD\(_3\) at end used channels, the composite IMD\(_3\) levels are higher at higher frequencies channels. The individual IMD\(_3\) level is a function of the resultant frequency by which it appears. Therefore, individual IMD\(_3\) levels are greater at higher frequencies.

Figure 9: The carrier positions third and fifth orders IMDs levels at 25mA bias current

For the first three used channels, the \(2f_1 + f_2 - f_3 - f_4\) type IMD\(_5\) contributed 46.2 to 53.2% of the composite IMD\(_5\) levels for the GRFP. This is because the total number of \(2f_1 + f_2 - f_3 - f_4\) type IMD\(_5\) is twice that of the \(f_1 + f_2 + f_3 - f_4 - f_5\) type IMD\(_5\). The \(f_1 + f_2 + f_3 - f_4 - f_5\) type IMD\(_5\) contributed 48.3 to 65% of the composite IMD\(_5\) levels in the last three used channels of the GRFP. These individual IMD\(_5\) levels are higher than that of the \(2f_1 + f_2 - f_3 - f_4\) type as they result from carriers of higher frequencies.
The composite IMD₃ levels for the GRFP are between 1.16 to 2.24 times lower than those of the EQSP IMD₃, for the first five used channels. However, the opposite is observed in the last channel. This is because there are four times more IMD₅ appearing at that channel as compared to IMD₃. Thus, signifying though the GRFP led to zero IMD₃, the IMD₅ at carrier positions can still be comparable to the IMD₅ of EQSP at high mtot and low bias current. The composite IMD₅ levels due to the EQSP are 2.66 to 5.53 times higher than those of the GRFP. This indicates that the GRFP is a better frequency plan than EQSP to be implemented with SEFA and SLC with maximum dynamic compression at high mtot and low bias current.

### 4.2.2 HIGH BIAS CURRENT

The composite IMD₃ and IMD₅ levels in Figure 10 and Table 8 had increased more than 200 times and 77 to 118 times as compared to when Iₒ was 0.44A. The Fourier Transform of the third and fifth order Volterra kernels generally decreased by 200 and 560000 times at 0.44A as compared to 25mA. However, the 28.786dBm RF level supported by 0.44A translated to DI₃ of 0.1735A, which is 43 times greater than the DI₃ supported by 25mA. As such, ΔF at 0.44A is 6.6 × 10⁴⁰ times greater than that at 25mA. ΔF at 0.44A is 1.46 × 10⁴⁰ times greater than that at 25mA. This causes the individual IMD levels to increase, which in turn increases the composite IMD levels.

The composite IMD₃ levels of the EQSP are higher than its composite IMD₅ levels when Iₒ is 0.44A. This is contributed by the greater decrease the Fourier Transform of the fifth order Volterra kernels encountered and smaller increase in ΔF₅ than that of the third order. As a result, each individual IMD₅ level becomes minute as compared to the IMD₃. Though the total number of IMD₅ is seven to 12 times that of IMD₃, the composite IMD₅ levels are less than half the composite IMD₃ levels.

The composite IMD₅ levels for the GRFP at 0.44A in Figure 10 and Table 9 increased 20 times as compared to those at 25mA. This increase is mainly due to the increase in DI₅ term. The 2f₁+f₂-f₃ and f₁+f₂+f₃-f₄-f₅ types of IMD₅ still contributed 82.2 to 94.6% of the GRFP composite IMD₅ levels. The composite IMD₅ levels for this frequency plan are 2.63 to 5.49 times lower than those of the EQSP. This indicates that the GRFP is more suitable to be implemented with the SEFA and SLC when high mtot and high bias current are employed.

### Figure 10 : The carrier positions third and fifth orders IMDS levels at 0.44A bias current

The f₁+f₂-f₃ type IMD₃ still contributed highly to the EQSP composite IMD₃ levels, between 80 to 91%. The f₁+f₂+f₃-2f₄, 2f₁+f₂-f₃, and f₁+f₂+f₃-f₄类型 IMD₅ still contributed 91 to 95% of the EQSP composite IMD₅ levels.

### Table 8 : Distortion levels (10¹⁵/m³) at EQSP carriers at 0.44A

<table>
<thead>
<tr>
<th>Distortion Type</th>
<th>Carrier Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2f₁-f₁</td>
<td>1642.644</td>
</tr>
<tr>
<td>f₁+f₂-f₃</td>
<td>6562.613</td>
</tr>
<tr>
<td>Total IMD₃</td>
<td>8205.26</td>
</tr>
<tr>
<td>3f₁-2f₁</td>
<td>7.578033</td>
</tr>
<tr>
<td>3f₁+f₁-f₁</td>
<td>76.809</td>
</tr>
<tr>
<td>Total IMD₅</td>
<td>2733.76</td>
</tr>
<tr>
<td>Total IMDs</td>
<td>10939.02</td>
</tr>
</tbody>
</table>

### Table 9 : Distortion levels (10¹³/m³) at GRFP carriers at 0.44A

<table>
<thead>
<tr>
<th>Distortion Type</th>
<th>Carrier Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2f₁-f₁</td>
<td>0</td>
</tr>
<tr>
<td>f₁+f₂-f₃</td>
<td>0</td>
</tr>
<tr>
<td>Total IMD₃</td>
<td>0</td>
</tr>
<tr>
<td>3f₁-2f₁</td>
<td>0</td>
</tr>
<tr>
<td>3f₁+f₁-f₁</td>
<td>15.4077</td>
</tr>
<tr>
<td>Total IMD₅</td>
<td>500.651</td>
</tr>
</tbody>
</table>

The composite IMD₃ levels for the GRFP at 0.44A in Figure 10 and Table 9 increased 20 times as compared to those at 25mA. This increase is mainly due to the increase in ΔF₃ term. The 2f₁+f₂-f₃ and f₁+f₂+f₃-f₄-f₅ types of IMD₅ still contributed 82.2 to 94.6% of the GRFP composite IMD₅ levels. The composite IMD₅ levels for this frequency plan are 2.63 to 5.49 times lower than those of the EQSP. This indicates that the GRFP is more suitable to be implemented with the SEFA and SLC when high mtot and high bias current are employed.
4.3 EFFECTS OF IMD$_3$ AND IMD$_5$ ON CARRIERS

The undistorted carriers' magnitudes for both frequency plans are indicated in Figures 11 and 12.

These undistorted carriers decrease with increasing carrier number at both $I_0$ though their magnitudes were the same at the E/O input. The decrease is because the Fourier Transform of the first order Volterra series kernel decreases for frequencies between 890 to 915 MHz. As the EQSP used channels are at higher frequencies than those of the GRFP, its undistorted carriers decrease more drastically than those of GRFP. The EQSP carriers' decreases are 5.33 and 5.92 times greater than the GRFP for $I_0$ of 25 mA 0.44 A.

The distorted carriers for both $I_0$ are indicated in Figures 13 and 14. The distorted carriers are generally lower than those undistorted.

The composite IMD$_3$ and IMD$_5$ levels at carrier position lowered the undistorted EQSP carriers by 38 to 61.6% and 1.15 to 2.36% at 25 mA and at 0.44 A. The carriers are less distorted at 0.44 A as the increase in the carrier is 30 times greater than the increase of composite IMD$_3$ and IMD$_5$ levels. This signifies higher bias current is preferred to low bias current when implementing SLC with maximum dynamic range compression. The composite IMD$_3$ levels contributed 76.6 to 82.9% of the carrier position IMD levels in the EQSP at 25 mA. On the other hand, the composite IMD$_5$ levels contributed 67 to 81% of the carrier position IMD levels in EQSP at 0.44 A. The middle used channels are the most distorted channels in the EQSP at 25 mA. The fourth used channel is most distorted in the EQSP at 0.44 A. This is because the composite IMD$_3$ and IMD$_5$ levels maximise in these channels.

The GRFP distorted carriers are merely 0.083 to 0.26% and 0.053 to 0.165% lower than its undistorted carriers at 25 mA and 0.44 A. The sixth used channel in the GRFP is the most distorted channel for both bias current cases. The GRFP carriers are generally less distorted as compared to the EQSP mainly because it led to zero IMD$_3$ and less IMD$_5$ at carrier positions.
Another way of evaluating the IMDs’ effect on the carriers is to use the carrier-to-IMD (CIMD) ratio, given in Figures 15 and 16. The CIMD ratio trends for the both frequency plans are very different. The CIMD ratio for the GRFP decreases with the carrier numbers. The CIMD ratio for the EQSP minimises at the fourth carrier for both bias current. On the whole, the CIMD ratios were improved or increased when 0.44A was employed. The CIMD ratios of both frequency plans are also compared to the composite triple beat (CTB) and composite fifth order (CFO) in Figures 14 and 15. The composite triple beat (CTB) is the carrier to total IMD$_3$ in a channel [18]. The composite fifth order (CFO) refers to the carrier to total IMD$_5$ in a channel for this paper.

The GRFP’s CIMD ratios are higher than the EQSP for both bias current cases as it led to only IMD$_5$ at the carrier positions. The GRFP’s CIMD ratios from 51.66 to 61.59dB and 55.63 to 65.56dB when 25mA and 0.44A bias current were employed. The CIMD ratio differences between both frequency plans ranged from 10.5 to 18.02dB at 25mA. The EQSP’s CIMD ratios were 18.04 and 27.8dB lower than those of the GRFP. The CIMD ratios for the GRFP exceeded the EQSP’s CTB for all carrier number at 25mA bias current, except at the fourth. This is because the composite IMD$_5$ level at the fourth carrier of the GRFP was higher than the composite IMD$_3$ level of the EQSP.

The EQSP’s CIMD ratio curve resembled and appeared nearer to its CFO than its CTB as its composite IMD$_3$ levels were higher than its composite IMD$_5$ levels. In addition, this also caused the EQSP CFOs to be lower than its CTBs. The EQSP’s CFO curve was below the GRFP’s CIMD trend at 0.44A. The EQSP’s CIMD ratio curve resembled and appeared nearer to its CTB than its CFO as its composite IMD$_3$ levels were higher than its composite IMD$_5$ levels.

High CIMD ratio is desired as they indicate that the carriers are less suppressed by the distortions. The CIMD is important especially when the signal strength is used in mobile communications to decide whether calls are to be handoff or avoidance of call drop. One of the criterias for handoff to take place in the mobile communications is the carrier to interference ratio of 18dB [5]. The sources of interference can come from co-channel interference. However the situation can be worsened when the co-channel interference is coupled with the distortions from the optical link. Therefore the distortions that cannot be filtered from the E/O should be minimised and this can be done using the GRFP and higher bias current. As such, the GRFP would be a better candidate than the equal frequency spacing to be incorporated into the SEFA.
5.0 CONCLUSION

The GRFP led to no IMD₁ and less IMD₃ levels to appear at carrier positions as compared to the equally spaced carriers. This in turn resulted in high CIMP ratios. As such, the GRFP can be used together with the SEFA and SLC schemes before the E/O conversion stage of the radio-over-fibre system. However, one problem might arise from deploying the GRFP. As the number of channels to be used increases, the bandwidth occupied by the GRFP can exceed that of the equal frequency spacing and lead to interference with signal designated in other bands for the radio-over-fibre system providing multi-band services. An alternative to this is to employ the optimum Golomb Ruler (OGR) that can lead to slightly reduced bandwidth consumption. Another option is to utilise a sub-optimal Golomb Ruler with a higher bias current, if the OGR is not available. This might lower the carrier position IMD levels and consume less bandwidth.

REFERENCES


PROFILE

Lisa Yong received her BEng. (Hon.) in Electronics and Telecommunications Engineering and Master of Engineering in Electronics and Telecommunication Engineering from Universiti Malaysia Sarawak. She currently a lecturer at the School of Engineering and Science, Swinburne University of Technology (Sarawak Campus), Kuching. Her research interests include radio-over-fibre systems, non-linear distortion modelling and linearization and renewable energy.