# INVESTIGATION ON AT-SITE FLOOD FREQUENCY ANALYSIS IN SOUTH-EAST AUSTRALIA

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# ABSTRACT

Flood frequency analysis is frequently adopted in flood risk assessment for catchments where recorded streamflow data of reasonable length are available. In the Australian Rainfall and Runoff (ARR) 1987, the national guideline for design flow estimation, Log Pearson Type 3 (LP3) distribution fitted with method of product moments (MPM) is recommended for at-site flood frequency analysis. This paper uses annual maximum flood data from 18 gauged sites across South-east Australia to assess the relative performances of the currently recommended method of at-site flood frequency analysis (LP3-MPM) and some of the techniques developed in recent years e.g. Generalised Extreme Value (GEV)-LH moments, LP3-Bayesian maximum likelihood (BML) and Generalised Pareto (GPA)–L moments. Several measures are adopted to compare the performances of various flood frequency analysis methods in the range of average recurrence intervals of 2 years to 100 years including statistical hypothesis testing and comparison of the quantile estimates obtained from the fitted distributions with graphical estimates. It is found that the GPA-L moments and GEV-LH moments methods provide the best fit to the observed flood data of the selected stations followed by the LP3-BML method. The currently recommended ARR method (LP3-MPM) does not perform as well as the above three methods.

Keywords: Design Floods, Flood Frequency Analysis, Probability Distributions, LP3 Distribution

# **1.0 INTRODUCTION**

Flood frequency analysis is a statistical technique that uses historical streamflow data to select and/or fit a probability distribution at a given location within a catchment. Selection of an appropriate probability distribution is an important step in flood frequency analysis and has been widely researched [1, 2, 3]. The selection of an appropriate probability distribution for a particular application cannot normally be made on a physical basis. Cunnane [2] argues that empirical suitability plays a much larger role in selection of distribution. Many different distributions have been recommended across different countries in the world. For example, Cunnane [2] mentioned that Extreme Value Type 1 (EV1) distribution was recommended for 10 countries, Generalised Extreme Value (GEV) distribution for 2 countries and Log Pearson Type 3 (LP3) distribution for 7 countries. Lim and Lye [4] found that GEV and Generalised Logistic distribution could well approximate the observed extreme floods in Sarawak, Malaysia. In Australian Rainfall and Runoff (ARR), the national guideline for design flow estimation, LP3 distribution coupled with method of product moments (MPM) was recommended for general use similar to USA [5, 6].

In recent years, there has been a greater interest in Australia on Generalised Extreme Value (GEV) distribution coupled with L moments and LH moments [3, 7, 8]. One of the limitations of the ARR method (LP3-MPM) is that the product moments of the logarithms of a data series are equally influenced by small values that do not constitute floods, as they are by the larger observations. Moreover, the higher moments (e.g. coefficient of variation and skewness) are much affected by extremes in the data series. In contrast, L moments are less affected by extremes in the data series [9]. Due to theoretical advantages of L moments, it would be expected that the resulting flood estimates using this procedure would be more accurate (*e.g.* smaller standard error of estimate). The *LH* moments also provide more weightage to the larger values in the flood series and hence are expected to provide better fits to the upper tail of the distribution [7, 10].

## 2.0 ASSUMPTIONS

Flood frequency analysis should satisfy some basic assumptions. The available flood peak data series is regarded as a sample of a population, which extends into the future and into the past, beyond the period of record. Inferences about the population are made based on the sample. The sample should be 'representative' of the population, which is possible if the population is 'homogeneous' and the sample is 'unbiased'. A 'representative' sample is one whose statistics (such as mean, standard deviation, etc.) are equal to those of the population. A population is 'homogeneous' if all of its members occurred under the same conditions. Examples of factors causing non-homogeneity are land use changes such as the clearing of forest to grow pasture, the urbanisation of rural catchments, the construction of a dam in a catchment, the construction of levee banks along a river, etc. A sample is unbiased if the expected values of its parameters are equal to those of the population. For most of the flood frequency analysis methods, the items of the sample should be independent of each other both temporally and spatially. The methods of estimating the population parameters from the sample must also be unbiased.

## 3.0 OBJECTIVES OF THE STUDY

The principal objective of this study is to assess the relative performances of the currently recommended flood frequency analysis method in Australia (LP3-MPM) with some of the recently developed techniques: GEV-*LH* moments, LP3-Bayesian maximum likelihood (BML), Generalised Pareto (GPA)–*L* moments and GEV-*L* moments. This paper focuses on the range of average recurrence intervals (ARI) of 2 years to 100 years

and considers 12 distributions/methods altogether: LP3-MPM, Normal-MPM, Lognormal (LN)-MPM, Extreme Value Type 1 (EV1)-MPM, Generalised Extreme Value (GEV)-L moments, GEV-LH moments, GPA-L moments, LP3-BML, LN-BML, EV1-BML, GEV-BML and GPA-BML. This study includes two most commonly adopted two-parameter distributions by many other countries, which are EV1 and LN; as noted by Cunnane [2] that EV1 and LN are used as standard distributions in 10 and 8 countries, respectively. Although, normal distribution is not a good descriptor of observed annual maximum flood series, it has been included to see how it compares with the other frequently adopted probability distributions. It may be noted here that Matalas and Wallis [11] found that the use of the normal distribution for flood frequency analysis can, in certain circumstances, minimise expected overdesign costs. The list includes GPA distribution as study by Vogel et al. [3] found that GPA distribution performed significantly better in Australia.

## 4.0 STUDY AREA AND DATA

Annual maximum flood data from 18 gauged stations across south-east Australia are used as shown in Figure 1 and listed in Table 1. These catchments are mainly rural with no major land use changes over the periods of streamflow records. The catchment area ranges from 15 to 621 km<sup>2</sup> with a median value of 268 km<sup>2</sup>. The streamflow record lengths are in the range of 40 to 59 years, with a mean value of 46 years.

## 5.0 METHODS

Cunnane [2] identified various techniques for evaluating the suitability of distributions into two groups: (a) tests of descriptive ability which seek from among known distributions that one which fits observed data best judged according to methods such as graphical, goodness-of-fit tests and test based on skewness and (b) tests of predictive ability which examine the statistical behaviour, especially the sampling distribution of coefficient of variation and coefficient of skewness and standardised largest sample values, of candidate distributions to determine whether they are capable of producing random samples having the same statistical characteristics as observed flood series. This is done by methods such as split sample and robustness tests.

There are however potential problems with the above methods. For example, goodness-of-fit tests are not conclusive when seeking a flood distribution. They can reject some distributions but are not necessarily good discriminators between accepted ones. Behaviour analysis indicates that real annual maximum flood data samples behave differently from random samples drawn from the parent distributions conventionally used in flood frequency analysis. Robustness studies indicate that quantile estimates using two-parameter distributions suffer more from bias than those based on multi-parameter ones, while the latter suffer from large standard error [2].

This paper uses two approaches to assess the suitability of selected distributions. Firstly, a traditional method is used in that the quantile estimates obtained from the fitted probability distributions are compared with graphical estimates. This is referred to as graphical method. In the second approach, a number of statistical tests are applied to assess the goodness–of–fit for a particular distribution. Each of the 12 selected distributions is fitted to the on-site annual maximum flood data of each of the 18 stations. For each of the distributions, flood quantiles  $Q_{ARI}$ 

are obtained for ARIs of 2, 5, 10, 20, 50 and 100 years. These estimates are referred to as 'distributional estimates'  $(X_D)$ . For *LH* moments method, a shift of 2 is adopted, which is referred to as *LH*2 method.

#### 5.1 Graphical Method

The observed annual maximum flood data of each of the selected stations is plotted on a probability paper; the annual exceedance probability (AEP) is then computed using Cunnane's unbiased plotting position formula [12]. A 'best-fit' flood frequency curve is then drawn subjectively through points and  $Q_{ARI}$  is estimated for the 6 ARIs where ARI is taken as the inverse of AEP. These are referred to as 'non-parametric estimates' ( $X_{NP}$ ). The percentage deviation (relative error, RE) of the  $X_D$  from the  $X_{NP}$  for a given distribution and ARI is obtained using the following equation:

$$RE = \frac{X_D - X_{NP}}{X_{NP}} \times 100 \tag{1}$$

A positive value of RE indicates that a psarticular probability distribution overestimates the observed flood quantile, while a negative value indicates an underestimation of the observed flood quantile.

The RE values estimated by Equation 1 are by no means 'true error' associated with flood quantile estimates obtained by a probability distribution. This can possibly be taken as an estimate of 'most likely error' in practical flood frequency analysis. It should be noted here that since the streamflow record lengths of the study stations are in the range of 40 to 59 years, the non-parametric flood quantile estimates for ARI of 100 years are subject to larger estimation error, thus, the results for this ARI should be used with 'caution'.

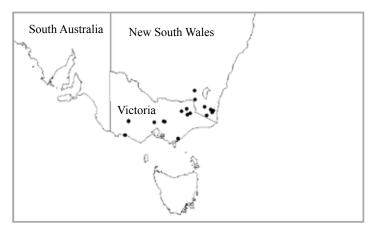


Figure 1: Locations of the selected stream gauging stations across South-east Australia

#### 5.2 Goodness-of-fit tests

To test the statistical hypothesis whether a particular distribution provides an adequate fit to the observed annual maximum flood series data, five goodness of fit tests are applied: Chi-squared (C-S) test, Kolmogorov–Smirnov (K-S), Anderson–Darling (A-D) test, Fillben Correlation Coefficient (FCC) test and Cramer Von-Mises (CVM) test [12, 13]. The reason for selecting five different tests is that there is no single test that can give conclusive results and a particular test emphasises on a particular aspect of the goodness-of-fit [12, 13]. All the tests are carried out at 5% significance level.

## 6.0 RESULTS

The RE values are examined in two ways: initially the absolute values are considered (*i.e.* the magnitude of REs without considering its sign); secondly, the sign of REs is also considered to indicate whether a particular distribution underestimates or overestimates the observed flood quantiles, which is a measure of the degree of bias of a particular distribution.

The statistics of the REs (ignoring the sign) for the 18 stations over the 6 ARIs are provided in Table 2, which shows that the GPA-L moments and GEV-LH2 methods have the lowest median RE (less than 5%) followed by LP3-BML (7.4%) and GPA-BML (7.7%). The normal-MPM and LN-MPM distributions show about 3.5 times higher RE values as compared to the GPA-L and GEV-LH2 moments methods. The BML method of parameter estimation does not perform as well as the L and LH moments methods with respect to median error values. For the EV1, LN, GEV and GPA distributions, the median RE values for the BML method are higher by 3.7%, 0.8%, 2.1% and 3%, respectively as compared to the MPM/L and LH2 moments methods. In terms of 95% percentile of the RE values, GEV-LH2 moments method performs the best (17.3%) followed by GPA-L moments (20.7%) and LP3-BML (33.3%).

It is then assessed which of the distributions performs best at individual station levels in that the distributions are ranked based on median RE values for each of the 18 stations (Table 3). Rank 1 implies that a distribution has the lowest median RE value for a station. GPA-*L* moments method receives Rank 1 for 5 out of the 18 stations (*i.e.* 28%). The GEV-*LH*2 moments method also receives Rank 1 for 5 stations followed by the LP3-BML, which receives 3 stations as Rank 1. Considering Ranks 1, 2 and 3 together, GPA-*L* moments scores 13 stations (72%) followed by GEV-*LH*2 moments method which scores 11 stations (61%) followed by LP3-BML (39%). The LP3-MPM scores 4 stations (22%).

The median RE values over different ARIs are then examined (Table 4), which shows that RE values generally increase with ARIs. Overall, GPA-*L* moments method shows the lowest RE values for all the ARIs followed by GEV-*LH*2 moments method and GPA-BML method. The LP3-MPM and LP3-BML do not perform as well as GPA-*L* moments and GEV-*LH*2 moments methods.

For each of the probability distributions, there are number of cases where a distribution underestimates the observed flood quantiles as summarised in Table 5. There are 108 cases (6 ARIs and 18 stations). This shows that LP3-MPM and GEV-*L* moments method perform best as these have similar proportion of cases with underestimation (45%) and over-estimation (55%), followed by EV1-MPM, GPA-*L* moments and GEV-*LH*2 moments, which respectively show that about 60% cases have under-estimations and 40% cases have overestimations.

Box plot of the RE values (considering sign) is presented in Figure 2. A negative value of RE in the box plot indicates that a fitted distribution underestimates the observed flood. Box edges mark the first and third quartiles, the horizontal line in each box depicts the median, and the lower and upper whiskers indicate 5<sup>th</sup> and 95<sup>th</sup> percentiles, respectively.

Figure 2 shows that GPA-*L* moments method has smallest error band followed by the GEV-*LH*2 moments, LP3-BML and GPA-BML methods. The zero line is located very close to the median values for the GPA-*L* moments, GEV-*LH*2 moments and LP3-MPM methods indicating lowest degree of bias for these methods. It should be noted here that Figure 2 does not show the full range of RE values to get a better view of the box plots (the range of RE values are provided in Table 2).

Table 1: List	of the selected	stream gauging	stations
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Station ID	Name	Record length (years)	Catchment area (km <sup>2</sup> )	
227200	Tarra River at Yarram	41	215	
237200	Moyne River at Toolong	45	570	
230204	Riddells creek at Criddells Creek	45	79.1	
238207	Wannon River at Jimmy Creek	43	40.3	
238208	Jimmy Creek at Jimmy Creek	40	22.5	
401210	Snowy Creek at Below Granite	54	407	
401215	Morras Creek at Uplands	55	471	
401216	Big River at Joker Creek	59	356	
403205	Ovens River at Bright	48	495	
405205	Murrindindi River at Murrindindi	42	108	
405217	Yea River at Devline	40	360	
219001	Rutherford Creek at Brown Mountain	45	15	
219003	Bemboka River at Morarns Crossing	50	316	
219006	Tanawargalo Tant MT	42	88	
222004	Little Plains River at Wellesley	51	621	
222007	Wullwye Creek at Woolway	43	520	
401009	Maragle Creek at Maragle	43	220	
410061	Adelong Creek	45	155	

#### 6.1 Goodness-of-fit test: Visual Assessment

Each of the fitted distributions are plotted as shown in Figure 3 for visual assessment of the goodness-of-fit and the fitting is rated as either 'good', 'medium' or 'poor'. The results of this procedure is summarised in Table 6, which shows that GPA-BML method has the highest number of stations having 'good' fit (67%), followed by GPA-*L* moments (56%), LP3-BML (50%) and GEV-*LH*2 (44%). The LP3-MPM shows only 11% stations as 'good' fit.

### 6.2 Goodness-of-fit test: Statistical

The FCC test rejects the hypothesised distribution for maximum number of stations (6 stations out of 18 *i.e.* 33%). The C-S, CVM, A-D and K-S tests reject 22%, 21%, 11% and 9% of the stations, respectively. Considering all the tests, the normal-MPM distribution shows the poorest fit in that the null

hypothesis of a normal-MPM distribution is accepted for only 44% of the stations. The LN-MPM, EV1-MPM, LP3-MPM, GEV-*LH*2, GPA-*L* moments and GEV-*L* moments methods are accepted for 74%, 76%, 92%, 92%, 96%, 97% of the stations. No

hypothesis test is conducted for the BML method. The statistical test does not provide any meaningful results as the tests cannot identify which of the accepted candidate distributions best fit the observed flood data.

Table 2: Statistics of the relative error values (ignoring the sign of relative errors)

Statistics	LP3- MPM	EV1- MPM	Normal- MPM	LN- MPM	GEV-L moments	GPA-L moments	GEV- LH2	LP3- BML	EV1- BML	LN- BML	GEV- BML	GPA- BML
Min	0.00	0.56	0.41	0.15	0.18	0.01	0.01	0.06	0.19	0.15	0.16	0.15
Max	64.84	61.54	105.14	354.21	729.35	43.73	34.97	52.86	60.66	377.32	373.61	109.95
Median	9.52	7.95	15.92	14.23	9.12	4.71	4.83	7.42	11.62	15.04	11.24	7.71
Mean	12.63	11.89	21.71	38.33	34.69	7.46	6.84	11.06	15.57	40.47	27.84	12.34
20 <sup>th</sup> percentile	3.24	3.07	7.51	4.68	3.66	1.46	1.92	2.60	5.80	4.60	3.57	2.44
40 <sup>th</sup> percentile	6.23	5.81	14.30	9.85	7.30	3.53	3.58	4.83	9.20	9.40	8.42	5.76
60 <sup>th</sup> percentile	12.70	9.66	19.79	20.28	11.40	6.16	6.95	9.44	13.00	22.33	14.43	10.00
80 <sup>th</sup> percentile	19.43	15.56	31.30	47.66	22.23	11.86	10.79	18.58	23.84	51.40	38.12	18.47
95 <sup>th</sup> percentile	34.00	38.83	56.48	164.69	128.09	20.70	17.32	33.31	47.73	176.87	102.74	36.09

 Table 3: Ranks of distributions based on median relative error (ignoring the sign of relative errors).

 The entry for a rank and distribution indicates the number of stations corresponding to the rank

Distribution	Rank											
	1	2	3	4	5	6	7	8	9	10	11	12
LP3-MPM	1	2	1	2	3	0	4	2	1	1	1	0
EV1-MPM	1	1	2	4	2	2	2	1	1	1	1	0
Normal-MPM	0	0	0	0	0	2	0	2	2	2	3	7
LN-MPM	0	1	0	4	1	2	0	3	0	1	6	0
GEV-L moments	0	3	1	2	0	3	0	2	1	1	1	4
GPA-L moments	5	2	6	1	2	0	1	1	0	0	0	0
GEV-LH2	5	4	2	1	3	0	3	0	0	0	0	0
LP3-BML	3	3	1	4	0	3	2	0	1	1	0	0
EV1-BML	0	0	0	1	3	0	2	2	3	1	4	2
LN-BML	0	1	2	0	0	2	3	0	2	1	1	6
GEV-BML	2	1	1	1	2	2	0	0	5	3	1	0
GPA-BML	2	1	1	1	1	3	1	2	3	3	0	0

Table 4: Median relative error (%) over ARIs (ignoring the sign of relative errors)

Distribution	ARI (years)								
	2	5	10	20	50	100			
LP3-MPM	6.2	5.8	9.6	7.2	14.1	21.3			
EV1-MPM	7.5	7.2	10.2	7.1	5.2	9.4			
Normal-MPM	25.5	8.3	11.1	13.8	18.1	23.1			
LN-MPM	14.1	6.5	14.1	20.2	38.4	59.3			
GEV-L moments	3.3	9.6	11.2	7.7	8.7	21.1			
GPA-L moments	5.1	3.5	4.2	3.6	3.9	8.7			
GEV-LH2	2.7	3.9	6.7	5.9	4.1	8.2			
LP3-BML	5.5	3.8	7.5	5.6	12.9	19.5			
EV1-BML	8.5	7.4	10.8	12.9	16.9	20.5			
LN-BML	13.9	5.6	14.2	22.5	14.9	61.2			
GEV-BML	7.5	5.8	6.6	11.3	33.3	55.1			
GPA-BML	6.6	5.0	6.5	8.1	11.5	9.9			

## Table 5: Proportion of under and overestimation by a distribution

Distribution	% of cases with underestimation	% of cases with overestimation
LP3-MPM	45	55
EV1-MPM	58	42
Normal-MPM	64	36
LN-MPM	39	61
GEV-L moments	54	46
GPA-L moments	60	40
GEV-LH2	59	41
LP3-BML	35	65
EV1-BML	71	29
LN-BML	38	62
GEV-BML	37	63
GPA-BML	29	71

Table 6: Summary of goodness-of-fit results: visual assessment

Distribution	Visual assessment	% of stations
	Good	11
LP3-MPM	Medium	39
	Poor	50
	Good	10
EV1-MPM	Medium	50
	Poor	40
	Good	0
Normal-MPM	Medium	0
	Poor	100
	Good	11
LN-MPM	Medium	33
	Poor	56
	Good	28
GEV-L moments	Medium	44
	Poor	28
	Good	56
GPA-L moments	Medium	44
	Poor	0
	Good	44
GEV-LH2	Medium	50
	Poor	6
	Good	0
LP3-BML	Medium	50
	Poor	28
	Good	28
EV1-BML	Medium	0
	Poor	72
	Good	22
LN-BML	Medium	22
	Poor	56
	Good	39
GEV-BML	Medium	28
	Poor	33
	Good	67
GPA-BML	Medium	11
	Poor	22

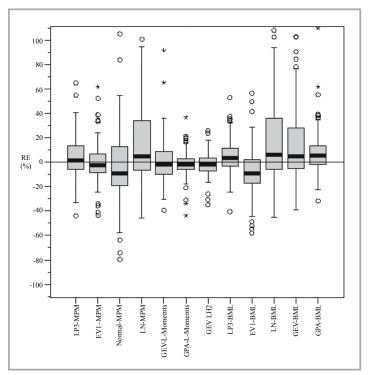


Figure 2: Box plot of the relative error (RE) values in the range of  $\pm 100\%$  for various distributions.

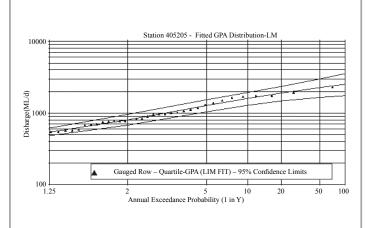


Figure 3a: Fitted GPA-L moments distribution for Station 405205 (rated as good fit)

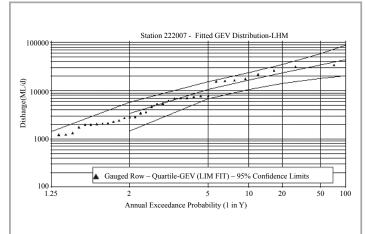


Figure 3b: Fitted GEV-LH2 distribution for Station 222007 (rated as medium fit)

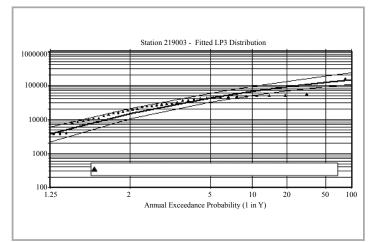


Figure 3c: Fitted LP3-BML distribution for Station 219003 (rated as poor fit)

# 7.0 CONCLUSIONS

This paper investigates the performances of some most commonly used probability distributions for at-site flood frequency analysis at 18 selected stations in South-east Australia. Following conclusions can be made from this study:

- The GPA-*L* moments and GEV-*LH*2 moments methods provide the best fit to the data in that the median, mean and 95% percentile of the relative error values are about 5%, 7% and 20%, respectively. The next best result is shown by the LP3-BML method having the median, mean and 95% percentile of the relative error values are about 7%, 11% and 33%, respectively. The currently recommended ARR method (LP3-MPM) does not perform as well as the above three methods.
- The *L* and *LH* moments methods of parameter estimation of the hypothesised probability distribution appear to provide better results than the Bayesian maximum likelihood and product moment methods.
- From the visual assessment, it appears that the three-parameter distributions generally provide a better fit to the observed annual maximum flood series than the two-parameter distributions. However, no single distribution is found to fit the observed data for all the stations satisfactorily.

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