FLEXURAL ANALYSIS OF COMPOSITE ONE- AND TWO-WAY SANDWICH SLABS WITH TRUSS-SHAPED CONNECTORS

A. Benayoune[1], A. A. Abang Ali[2], A. A. Abdul Samad[3] and D.N. Trikha[3]
[1]*F & J Department, University of Kuala Lumpur MFI*
[2]*Housing Research Centre, UPM*
[3] *Dept. of Civil Engineering, KUiTTHO*

ABSTRACT

As a part of a project on the development of an indigenous building system, linear and non-linear finite element analyses (FEA) were carried out to study the behaviour of composite precast concrete sandwich panel (PCSP) under flexure in order to examine feasibility of their usage as slab elements in the construction industry. The FEA results are presented and compared with experimental published data. Good correlation was found between the results obtained from the proposed finite element model and experimentally obtained results. Different aspect ratios of slabs were analysed to study their impact on the behaviour of PCSP as slabs. The effect and orientation of the shear connectors in one or two directions were also investigated. Results in the form of load-deflection curves showed that all the panels behaved linearly before development of the first cracks. The panels were found to behave in a composite manner till the first cracking load. Results also showed that PCSP with truss steel shear connectors has a potential use as one-way slab element as its behaviour was found to be very similar to those of solid slabs especially when the two concrete wythes act in a fully composite manner. However, for practical reasons, the truss shear connectors were not recommended for two-way slabs where composite behaviour is needed in both directions to increase the structural efficiency of the PCSP as two-way acting slabs. The ultimate strength and the degree of composite action desired were found to depend to a large extent upon the stiffness of the shear connector used. An expression to assess the degree of composite action is provided.

Keywords: *Defection, Finite Element Analysis, Precast Reinforced Concrete; Sandwich Slabs, Insulated Panels*

1. INTRODUCTION

Precast Concrete Sandwich Panels (PCSP) are considered as a branch of precast wall panels because of their similarity in functions but slight difference in their built up as shown in Figure 1. PCSP derive their name from their construction, since the two outer wythes of concrete have an insulating sandwiched core. These two wythes are connected through different type of concrete webs or metal connectors to ensure composite action. The complex behaviour of PCSP due to its material non-linearity, the uncertain role of the shear connectors and the interaction between its various components has led researchers to rely on experimental investigations backed by simple analytical studies. The scarcity of information on the behaviour of this important type of construction is due to the high cost of full scale testing and the extreme difficulty of fabrication of small-scale models. Furthermore, many sandwich panels in use in the North America and Europe are proprietary and the producers are thus reluctant to share information with their competitors [1] and [2].

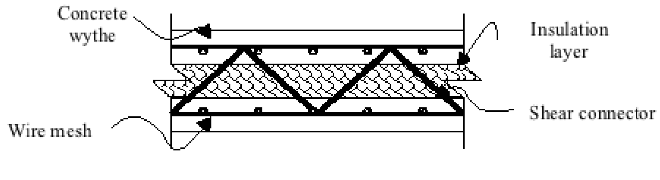

Figure 1: Precast concrete sandwich panel

PCSP generally span vertically between foundations and floors or roofs to provide an insulated outer shell to a building carrying mostly axial loads [1-7]. Their use as slab elements has rarely been attempted before [7] and [8]. Depending on their flexural behaviour PCSP can be divided into fully composite, non-composite and partially composite panels. A technical definition of the percent of composite action is not well established in the literature [1] and [2].

The aim of this paper is to investigate the flexural behaviour of one- and two-way acting slabs fabricated of precast sandwich panels with continuous truss-shaped connectors. Different aspect ratios of slabs were chosen for this study. The shear connector orientation effect in one or two direction was also investigated to study the efficacy and the influence of the placement of shear connectors to transfer shear from the upper wythe to the lower. Further, a parametric study was carried out by studying the influence of shear connector spacing on the ultimate strength and the compositeness of the PCSP acting as slab. The investigation included a study on the stress distribution, degree of composite action at the elastic and ultimate stages of the PCSP and their ultimate strengths.

2. FINITE ELEMENT MODELLING

Two models are proposed to simulate the behaviour of the PCSP under flexure. A 2-dimensional (2D) non-linear model is proposed to simulate the behaviour of one-way acting panels and a 3-dimensional (3D) non-linear model to simulate the behaviour of two-way acting panels having shear connectors spanning in both directions.

A. 2D FINITE ELEMENT MODEL

One-way panels having shear connectors spanning in one direction were modelled and analysed as a 2D problem. The concrete was idealised using 2-D isoparametric plane stress elements, whereas reinforced steel and shear connectors were idealised with 2-D bar elements. The cracking model was chosen for the 2-D plane stress elements while a Von Mises plastic material model was selected for the steel shear connectors and the reinforcement bars. The panels were considered simply supported at both ends. The FE idealisation, the applied loading and the boundary conditions are shown in Figure 2. The boundary conditions and the applied loading simulate the actual testing arrangements described in Reference [7] and [8]. Only a part 1m wide of the panel associated with one shear connector was considered.

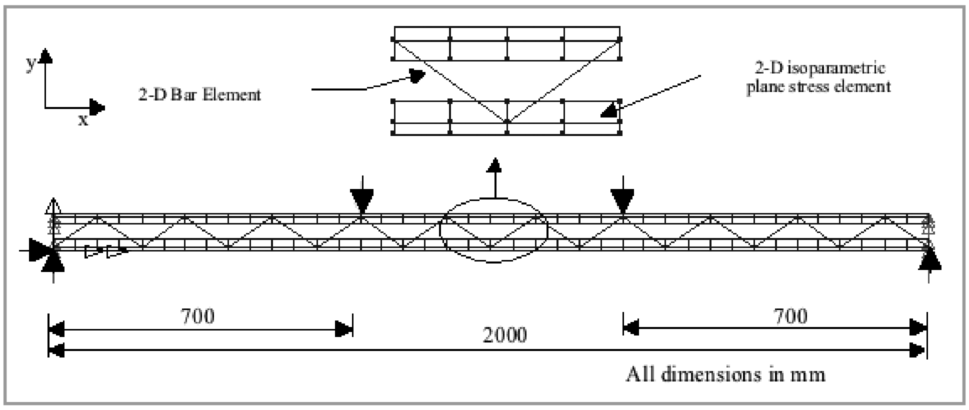

Figure 2: One-way PCSP slab idealisation, loading and boundary conditions

The following assumptions were made in developing the model:

1. No relative slip occurs between concrete and reinforcement/shear connector.
2. Effect of bond slip and dowel action is ignored.
3. Slip between steel reinforcement and shear connector is ignored.

The concrete model adopted for the current investigation is that developed by Jefferson [9]. It was incorporated into LUSAS software [10]. This model is a further development of a multi-crack plasticity approach referred to as the Multi-Crack Model developed by Carol & Bazant [11]. This model assumes that, at any point in the material, there are defined number permissible cracking directions. It further assumes that material can soften and eventually lose strength in positive loading [10].

B. 3D FINITE ELEMENT MODEL

The two-way panels were modelled and analysed as a 3-D problem as the shear connectors were placed in the longitudinal and transverse spans. The concrete wythes were modelled with four noded 3-D thin shell element having six degrees of freedom at each node. The shear connectors were idealised using 3-D bar elements. The FE idealisation, the support conditions and loading shown in Figure 3 simulated the test conditions described in references [2, 8]. The presence of the insulation layer has been ignored as it does not contribute to structural strength. More details about this model can be found in references [2, 11]

3. ONE AND TWO-WAY PCSP SLAB

In the case where the slab is supported on four sides and the ratio of the long side to the short one is equal or greater than 2,

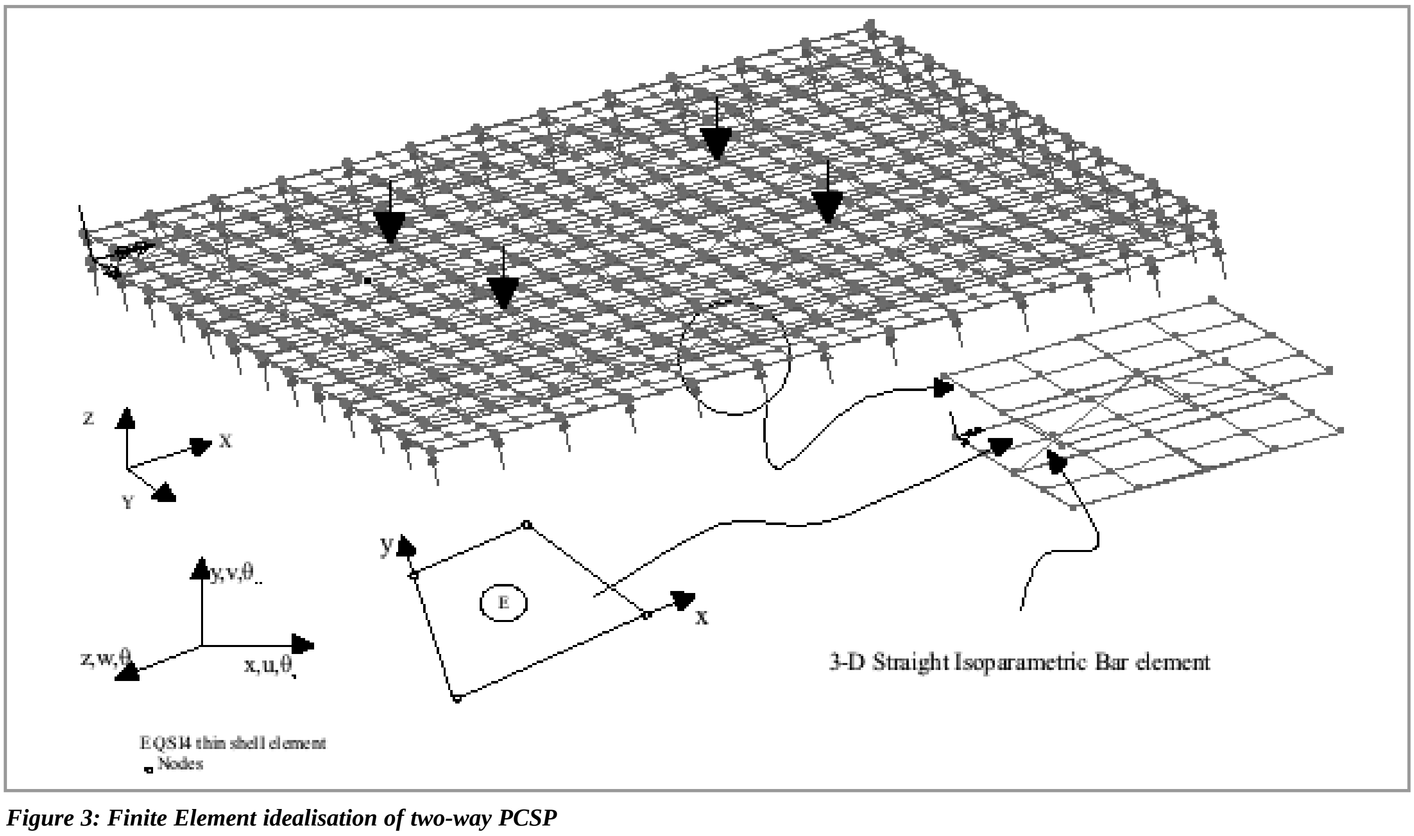

Figure 3: Finite Element idealisation of two-way PCSP

most of the loads (about 95%) are known to be carried in the short direction, and one way action is considered for all practical cases. If the ratio, however, is less than 2, the two-way action of the slab comes into play and the loads on the slab are transferred to all four supports. It is proposed to investigate typical PCSP acting as two-way with different aspect ratios provided with one-way and two-way shear connectors to assess the efficacy and the influence of the placement of the shear connectors to transfer the horizontal shear form the upper wythe to the lower under transverse loading. The 3-D FE model described earlier will be first validated and then adopted for the current study.

A. CHOICE OF PANEL DIMENSIONS

The panels were subjected to uniformly distributed lateral loads of 5.42 kN/m2 and 8 kN/m2 (typical service load, and design loads respectively). The sizes of the slab panels along with their respective aspect ratios are presented in Table 1. The aspect ratio of the slab varies in the range of 1 to 6. The series were chosen on the basis of the known behaviour of the solid slabs. PCSP having aspect ratio more than 2 were expected to behave as one-way slabs, whereas the panels with an aspect ratio of less than 2 were expected to behave as two-way slabs.

Table 1: Sandwich slab sizes and aspect ratios

Panel		a (mm)	b (mm)	a/b ratio
P1-1	P1-2	2000	2000	1.00
P2-1	P2-2	2000	1250	1.60
P3-1	P3-2	2000	1000	2.00
P4-1	P4-2	2000	750	2.67
P5-1	P5-2	2000	500	4.00
P6-1	P6-2	3000	500	6.00
P7	-	3200	3200	1.00

The first indices indicate the panel number, whereas the second indicate whether the shear connectors are placed in one- or two-directions. Throughout the next sections P-1 is used for all panels with one-way shear connectors (shear connectors in one direction) and P-2 for panels provided with shear connectors in both directions.

The deflections of the two concrete wythes of each panel were plotted separately to study whether the wythes deflect together or otherwise and the influence of shear connectors emplacement on the behaviour of the slab panels. This gives an indication of the composite behaviour of the panels and the role of shear connectors in distributing the service loads to the supports.

In order to validate the proposed models a comparison of the theoretical results with the experimental data [4] was made as detailed below. The theoretical results were obtained by adopting FEM models as described earlier. The sizes of the panels used for the FEM validation are given in Table 3. More details about test specimens and test setup can be found in [4] and [5].

Table 3: Details of text panels used for FEM valiation

Panels	Size (m x m)	Aspect Ratio	Number of truss connector	
			Longitudinal	Transverse
P11 One-way	2 x 0.75	2.67	3	-
P22 Two-way	1.5 x 1.5	1	6	6

4. DISCUSSION OF RESULTS

A. VALIDATION OF FINITE ELEMENT MODELS

a. One-Way PCSP Slab - 2D Model

Figure 4 shows the experimental load–mid span deflection curves for panels P11 under different load stages. Also shown in the same figure are the FEA load deflection curves as well as the theoretical extremes of fully composite and non-composite panels using conventional elastic theory. It is seen that the panel exhibits highly composite behaviour at the linear stage while near the first cracking, the panel tends to behave as partial composite till the failure load. However, after the cracking occurs, the FE model becomes significantly stiffer than the actual tested specimen. This is because the FE model assumes a perfect bong between the concrete and the steel throughout. The ultimate failure load by FEA (20.70kN) is found to be in good agreement with the experimental ultimate load (21.4kN) where the difference is less than 4%. The full panel was modelled by taking the full width as a thickness of the panel and the areas of shear connectors being added so that to simulate axial stiffness. Therefore the 2-D proposed model predicted with an acceptable accuracy the deflection especially in the elastic stage and the ultimate failure load under lateral loading of PCSP acting as one- way slabs. It can be also concluded that the modelling of the number of the shear connectors by adding their corresponding areas (stiffness) gives a very acceptable results.

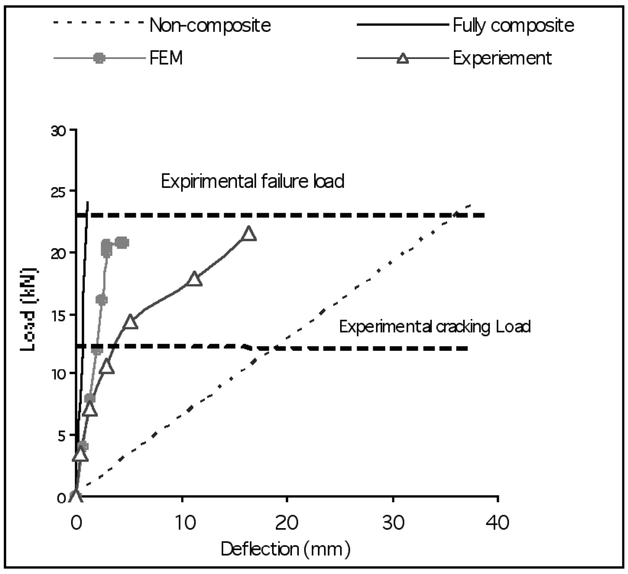

Figure 4: Load-deflection profiles at mid-span for panel P11

b. Two-Way acting PCSP Slab

Figure 5 shows the load deflection curves for panel P22 at different load increments. It is seen that the finite element model predicted deflections correlated very well with experimental deflections. The deflections at cracking loads as obtained using FEM were found 1.2% higher than those obtained experimentally. While the FEM predicted ultimate load (135.2kN) was found higher by around 16% than the experimentally obtained value (117.3kN). It can be concluded that the two results are in good agreement.

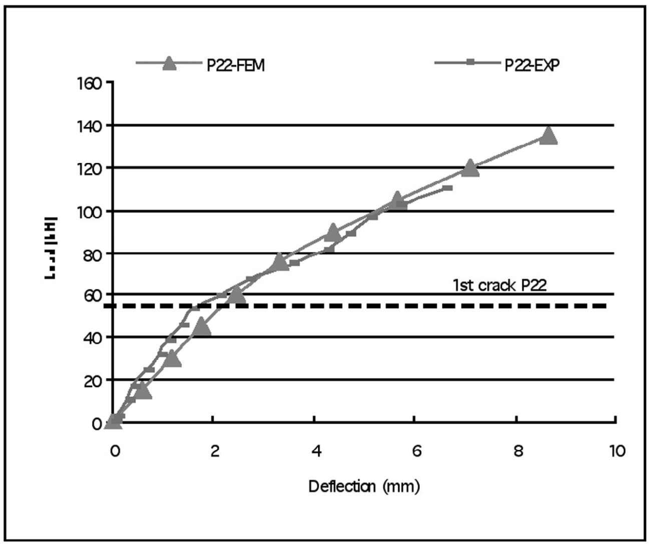

Figure 5: Load-deflection profiles at mid-span for panel P22

B. EFFECT OF ORIENTATION OF SHEAR CONNECTOR

Figures 6 to 8 show the deflection curves along the mid span in x- and y- directions for panels with one-way and two-way oriented shear connectors. It is seen that the deflection value for panel having aspect ratio a/b = 1 (two-way acting slab) was 37% higher than that of the same panel provided with tow-way shear connectors. This difference decreased to 14% when the aspect ratio a/b increases to 1.6. Further decrease was noticed where the difference became insignificant when a/b increases to 2. This is further shown in Figure 9 where the deflection profiles at mid-span of P1-1 and P1-2 were plotted. Also shown in the same figure the deflection profiles of a solid slab having the same dimensions and a thickness equal to the sum of the two concrete wythes. It is seen that the deflection of the panel P1-2 provided with shear connectors in both directions is 12% higher that that of the solid slab, whereas the deflection for the same panel with shear connectors in one direction is found 80% higher than that of the solid slab. This shows that P1-2 behave more likely as solid slab as it achieved a high composite action through the orientation of shear connectors in both directions. The provision of two-way shear connectors enhanced greatly the structural perform-ance of PCSP acting as two-way slabs.

C. EFFECT OF ASPECT RATIO

The deflections for the upper and the lower wythes of each panel at service load against the aspect ratios of the slabs are shown in Figures 10 and 11. It can be noticed that when the shear connectors are placed in two directions the difference in deflections of the upper wythes to the lower wythes were less than 6%. The same observation could be made for panels P-1.

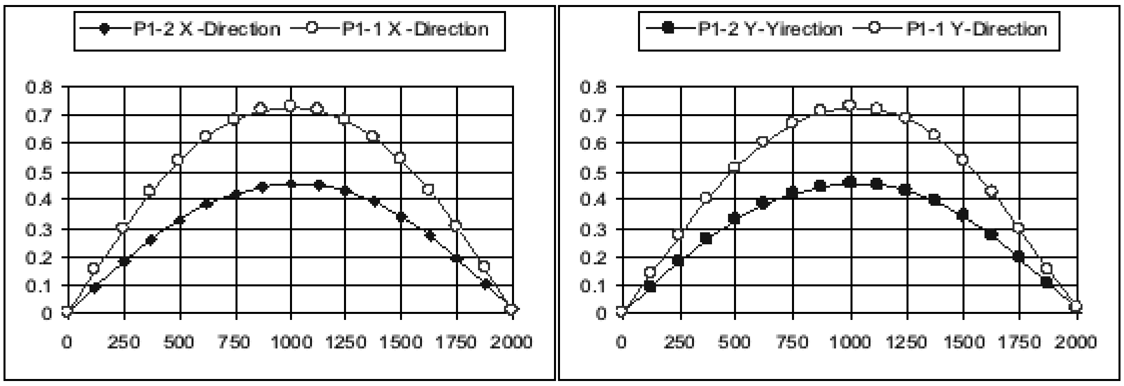

Figure 6: Panel P1-1 & P1-2 mid span deflections in x- and y-directions (a/b=1)

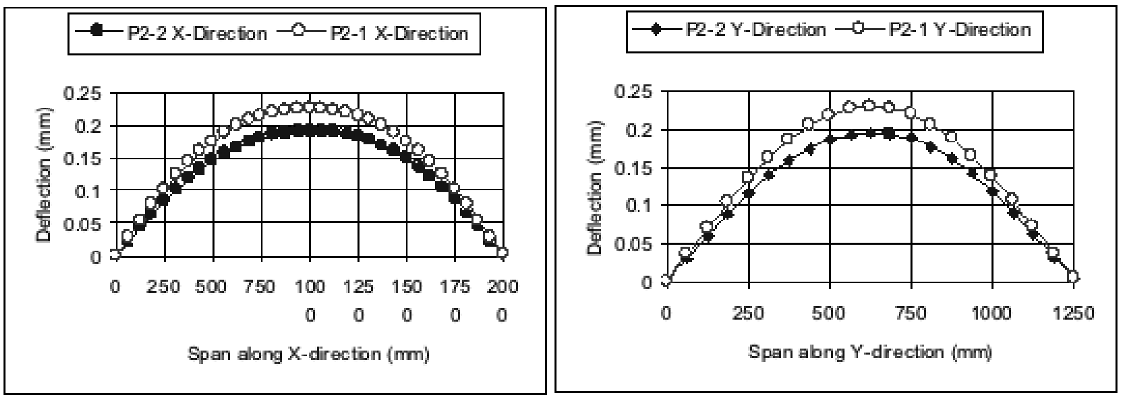

Figure 7: Panel P2-1 & P2-2 mid span deflections in x- and y-directions (a/b=1.6)

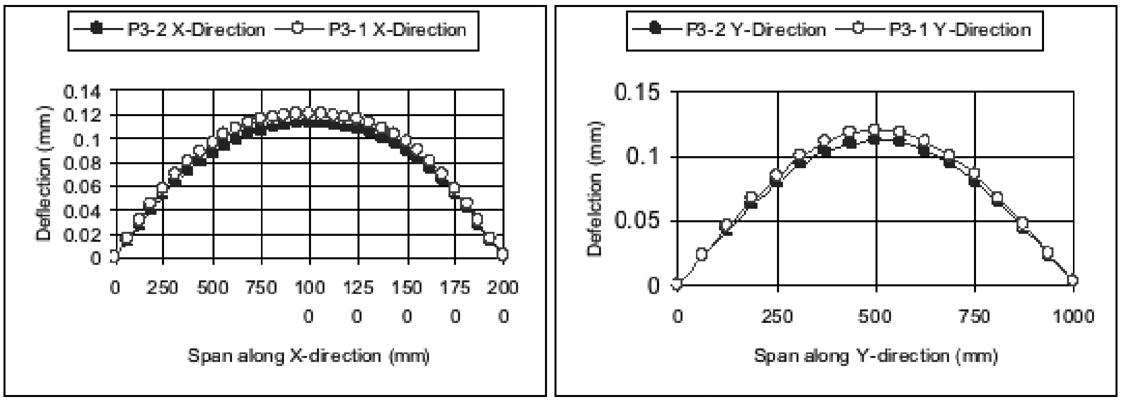

Figure 8: Panel P3-1 & P3-2 mid span deflections in x- and y-directions (a/b=2)

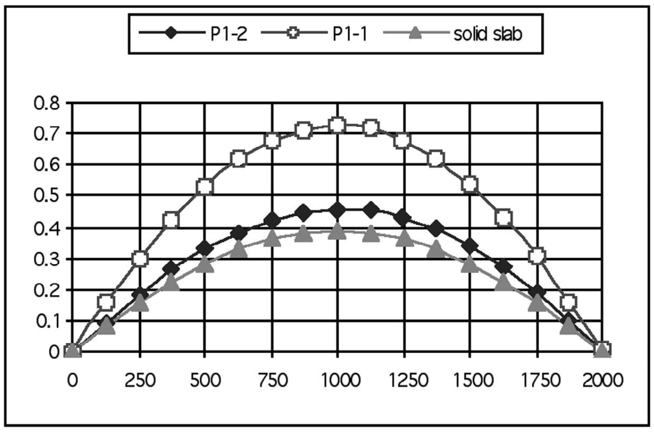

Figure 9: Comparison of defection profiles along the spans of panels P1-2, P1-1 and solid slab

Figure 12 shows a comparison of deflections of P-1 and P-2 against their aspect ratio. It can be noted that the difference in deflection values at mid-span was less than 6% when the aspect ratio was greater or equal 2 because most of the loads was carried in the short direction, therefore one-way action is considered and one-way shear connector in the shorter span to tie the two concrete wythes is sufficient. However, when the aspect ratio is less than 2 the difference in deflection varied from 17% to 60% as shown in the same figure. This confirmed the conclusion drawn earlier.

From practical point of view, the one-way shear connectors are favoured over the 2 directions ones in case of continuous truss-shaped connectors. This is due to the difficulty encountered in practice when inserting the

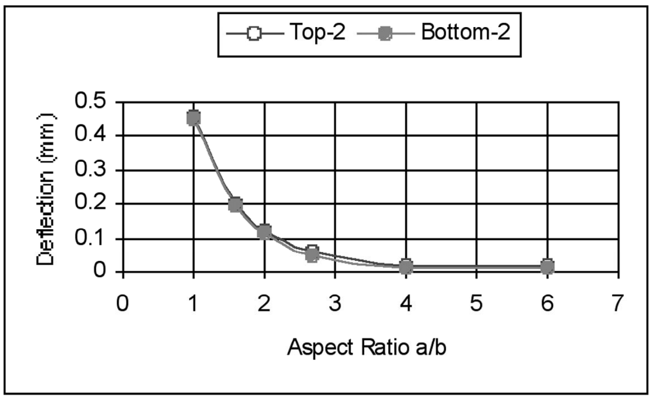

Figure 10: Deflections at the centre of the upper and the lower wythes against aspect ratio for Panel P-2

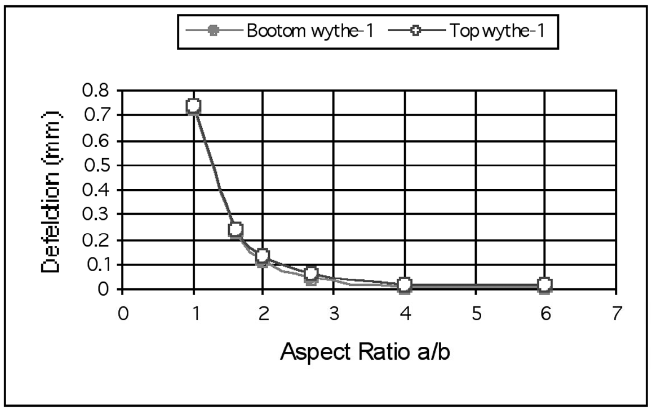

Figure 11: Deflections at the centre of the upper and the lower wythes against aspect ratio for Panel P-1

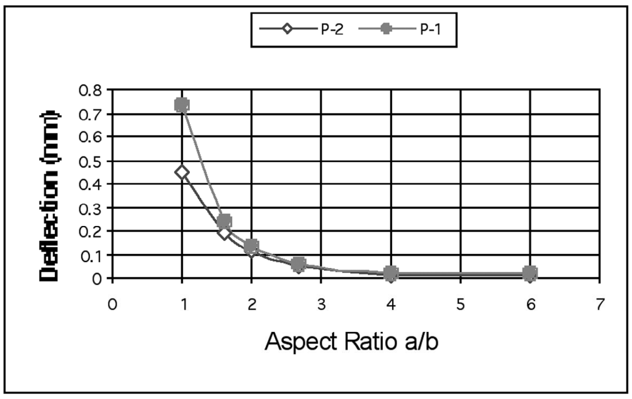

Figure 12: Deflections at the centre versus the aspect ratio for panel P-1 and P-2

insulating material, i.e. it must be cut into small pieces to fit between the connectors in both directions. If a composite action is needed in both directions to increase the structural efficiency of the PCSP panels acting as two-way slab, the truss shear connector are not recommended. Shear concrete webs could be used in such cases.

D. EFFECT OF SHEAR CONNECTOR STIFFNESS

The influence of the of shear connectors stiffness as measured by its diameters on the ultimate strength and the compositeness of the PCSP was investigated. Panel P11 was chosen for this study as it was already validated earlier. Non-linear analysis was carried out on the panel with different number of shear connectors (P11-4, P11-3, P11-2). The second index indicates the number of shear connectors. The number of shear connectors was introduced in the 2-D model by increasing the stiffness of the shear connector as measured by its bar diameter by 4, 3 and 2 times to have a panel with 4, 3 and 2 shear connectors respectively. This means, the areas of shear connectors were added so that axial stiffness is properly simulated. The loads were gradually increased till failure of the panel in each case.

The load-deflection profiles for the three panels, at mid span, at different load increments are illustrated in Figure 13. Also shown in the same Figure are the theoretical extremes of fully composite and non-composite panels using the classical elastic theory. It can be noticed that the panels with 4 and 3 shear connectors exhibited a high composite behaviour though the panel P11-4 was slightly

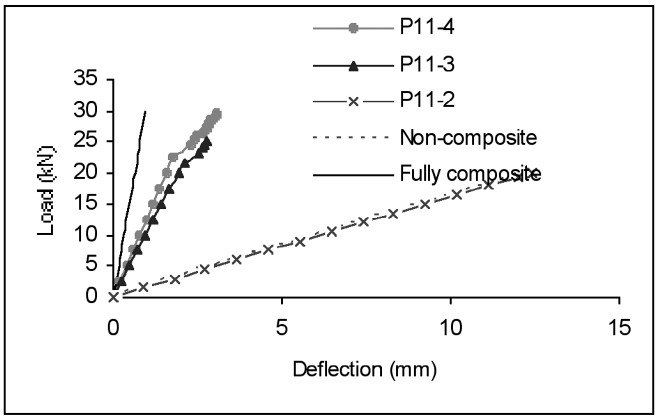

Figure 13: Load-deflection profiles

stiffer than the panel P11-3. It can be observed that the ultimate load increases with the increase of the shear connectors' number being respectively 20kN, 25.16kN and 29.75kN for panels P11-2, P11-3, and P11-4 respectively. It is observed that the ultimate loads of the panel P11-2 and Panel P11-3 were less by about 32.7% and 15.5% as compared to the ultimate load for panel P11-4. It is seen that the deflection profile of panel P11-2 is very similar to non-composite deflection calculated assuming non-composite action. This shows that the two concrete wythes acted almost independently in resisting loads, when the number of shear connectors was 2 in the present case.

The strain distributions across the thickness of the panels at mid span for the three PCSP at different load stages are shown in Figures 14, 15, and 16. Discontinuity of strain across the depth for panels P11-4 and P11-3 was found to be relatively small at the initial load stages. However the discontinuity became larger with the loading approaching the failure load. For Panel P11-2, the discontinuity of strain across the panel depth was significant even at initial stage of loading. This panel tended to behave more like non-composite than fully composite panel as each wythe of concrete had its own neutral axis. This confirms the conclusion drawn earlier for all the panels, that composite action of the panel is governed by the stiffness provided by shear connectors.

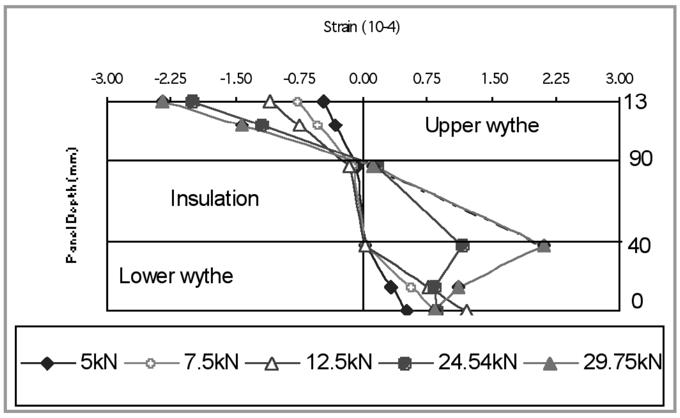

Figure 14: Strain variation across the panel P11-4 at mid-span at different load stages

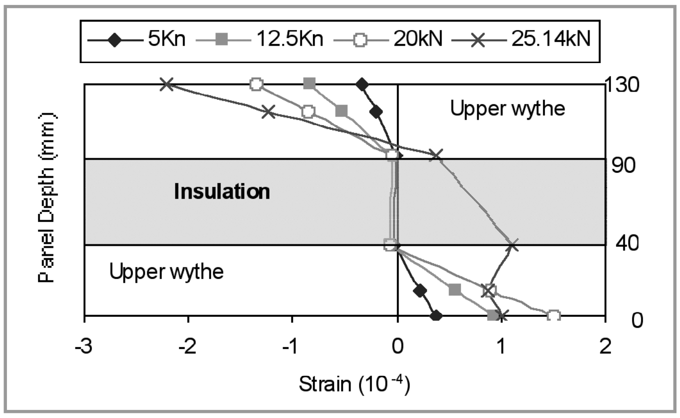

Figure 15: Strain variation across the panel P11-3 at mid span at different load stages

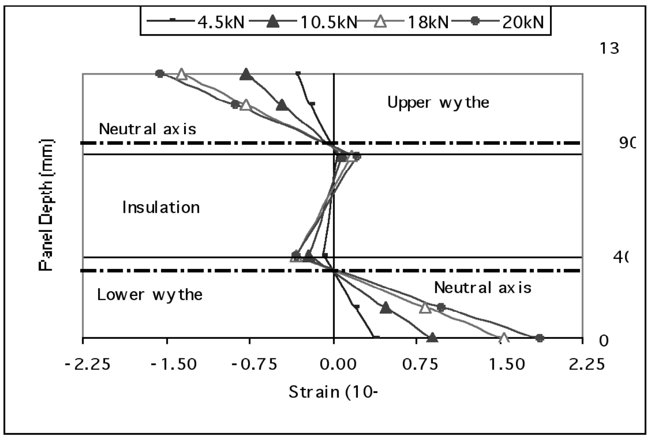

Figure 16: Strain variation across the panel P11-2 at mid span at different load stages

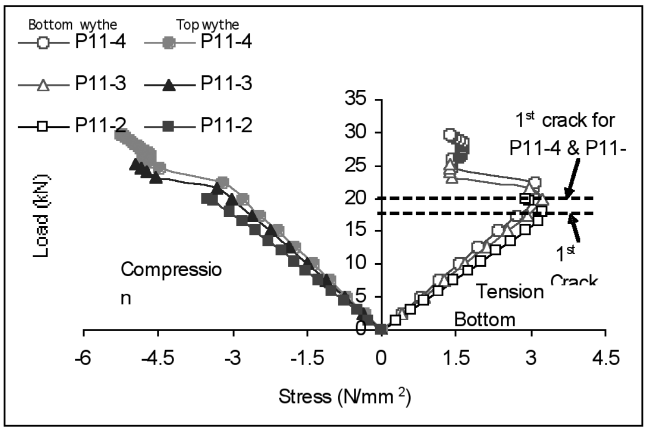

Figure 17: Load-stress relationships at different load stages

E. DEGREE OF COMPOSITE ACTION AT ELASTIC STAGE

Figure 17 shows the stress variation for the bottom and top wythe of concrete at mid-span under increasing applied loadings. It was observed that the cracking load of the P11-2 (18 kN) was lower compared to the panels P11-3 and P11-4 (20kN).

At the linear stage, this model can also be used to evaluate the amount of composite behaviour provided by the panel. The distribution of stresses across the panel can be used to assess the effective moment of inertia Ie given by the following expression:

$$I_e = \frac{Mh}{\sigma_b - \sigma_t}$$

in which σ_b, σ_t are the stresses at the bottom and the top face of the panel respectively, M is the applied bending moment and h is the depth of the panel.

The ratio I_e/I_g gives the degree of composite action achieved of the panel. Table 4 presents the degree of composite action of the three analysed panels at a load of 15kN (elastic stage), M=5.25kN and h=130mm.

Table 4: Efficiency of PCSP

Panels	σ_t(N/mm^2)	σ_b(N/mm^2)	I_e(10^6mm^4)	I_e/I_g%
P11-4	-2.1	2.33	154.06	89.22
P11-3	-2.24	2.52	143.38	83.03
P11-2	-2.53	2.88	126.16	73.06

where, I_g is the moment of inertia of PCSP section, calculated assuming fully composite action for the three panels, I_g =172.67 106 mm^4. From Table 4, it can be noticed that the panel P11-4 is 89% composite, panel P11-3 is 83%, whereas panel P11-2 is 73% composite. Therefore, panels P11-4 and P11-3 can be considered highly composite panels. While panels P11-2 can be regarded as partially composite.

F. DEGREE OF COMPOSITE ACTION AT ULTIMATE STAGE

The following calculations were performed at the ultimate strength for the analysed panels to estimate the composite action of each panel. It is difficult to assess the ultimate flexural strength of the PCSP by classical method, as it is not possible to either know the degree of composite action between the two wythes or to incorporate its influence on the transverse load carrying capacity of PCSP. However, at the two extremes of composite action namely, fully composite and non-composite action can be carried out. The degree of composite action at ultimate stage is being determined by using the method described below:

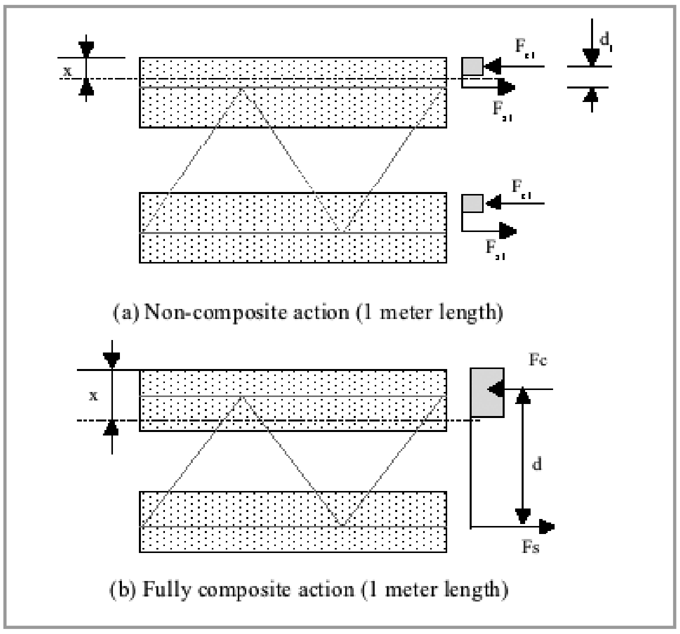

Figure 18: Non-composite and fully composite panels

When no composite action is assumed at ultimate strength (Figure 18a), the ultimate flexural capacity of the panel would be calculated as follows:

Each wythe is reinforced with 10 of 6mm diameter bars, A_s = 282.7 mm^2.

Steel yield stress $f_y = 250\ N/mm^2$, concrete strength $f_{cu} = 25\ N/mm^2$, the panel span $l = 2m$.

$F_{s1} = A_s f_y = 282.7 \times 250 = 70675\ N$
$F_{c1} = 0.85 f_{cu} b s_1 = 0.85 \times 25 \times 1000s = 21250s$

Where

A_s Area of tension reinforcement
b Per meter length of wall section or the connectors spacing.
F_{c1} Compressive force in concrete (non-composite)
F_{s1} Force in tension reinforcement (non-composite)
f_y Yield stress of steel
s_1 0.9x, depth of neutral axis measured from the more highly compressed face for one wythe

At equilibrium, $F_{s1} = F_{c1}$,
$s_1 = F_{s1} / F_{c1} = 3.33mm.$ (Depth of the neutral axis)
d_1 = 18 mm (each wythe separately, Figure 18a)

The ultimate moment would be:
$M_{u\,(one\ wythe)} = F_{s1}\,(d_1 - s_1/2) = 1.15\ kNm.$
$M_u = 2 \times 1.15 = 2.3kN$ *(for both wythe)*

Hence, the ultimate load carrying capacity of the slab per meter length = $\frac{8M}{l^2} = 4.6kN/m.$

The total load resisted by the panel as **non-composite** is $P_u = 9.2kN.$

When the panel is assumed **fully composite at ultimate strength** (Figure 18b), the ultimate flexural capacity of the panel would be calculated as follows,
$F_s = 70675\ N$
$s = 3.33mm$
$d = 130 - 18 = 112\ mm$
$M_u = T\,(d - s/2) = 7.8\ kNm$

Hence, the ultimate load carrying capacity of the slab per meter length = $\frac{8M}{l^2} = 15.6kN/m.$

The total load resisted by the panel as fully composite is $P_u = 31.2kN.$

where
d Depth of the reinforcement as sown in Figure 18.
Fc Compressive force in concrete
Mu Ultimate moment capacity under flexure
s 0.9x, depth of neutral axis measured from the more highly compressed face
Fs Force in tension reinforcement

Table 5: FEA and hand calculated (elastic theory) ultimate strength

FEM ultimate strength (kN)			Theoretical calculated ultimate strength kN)	
F11-4	F11-3	F11-2	100% composite	0% composite
29.75	25.14	20	31.2	9.2

Table 5 presents the ultimate load capacity of the three analysed panels along with the two extreme of composite action. It can be observed that the ultimate loads obtained by FEA for panels P11-4, P11-3 and P11-2 are less by around 5%, 19% and 36% respectively as compared to the ultimate load obtained by the classical analysis assuming fully composite action. If we consider the percentage of composite action at ultimate strength is the ratio of the theoretical calculated ultimate strength assuming fully composite action to the ultimate strength obtained by FEA, then the panel P11-4 would be 95% composite, panel P11-

3 is 81%, whereas panel P11-2 is 64% composite. This results correlate well with the previous results in Section 5.5, where the degree of composite action was assessed within the elastic stage (89%, 83% and 73 for panels P11-4, P11-6, and P11-2 respectively).

CONCLUSION

This paper presented the structural behaviour of the PCSP acting as one- and two-way slab with continuous truss-shaped connectors under flexure. FEA results were compared to experimental data. The finite element model matched the experimentally obtained results. The validated models were used to further study the behaviour of the panels acting as one-way and two-way slabs. It was found that the placement of shear connectors in both directions gives better load distribution and PCSP in this case behave more likely as solid slabs. In the case where PCSP acts as one-way slab, it was found that the provision of the shear connectors in the shorter span was sufficient to tie the two concrete wythes so that they act as a single unit. It was also found that the ultimate strength and the degree of composite action desired depend to a large extent upon the stiffness of the shear connector. The ultimate load was found to increase with the increase of the shear connectors' number being respectively 20 kN, 25.16 kN, 29.75 kN for connector numbers 2, 3 and 4 respectively. At both linear and non-linear stages, the proposed 2-D model can also be used to evaluate the amount of composite behaviour provided by the panel. An expression to calculate the degree of composite action was proposed for this purpose.■

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