Modified Logistic Model with Migration Factor to Predict the Population of Purwanegara Village in Jawa Tengah, Indonesia

by

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LIST OF SYMBOLS

$P(t)$	The number of population at the time t
R	Intrinsic rate coefficient
M	Migration factor
P_0	Population at time $t = 0$
r	Intrinsic growth rate
b	Positive constant
K	Carrying capacity of environment
q	Migration factor by a constant value
p	The number of population divided carrying capacity of environment
y	Migration factor by a constant value divided carrying capacity of environment and intrinsic growth rate
s	Intrinsic growth rate multiplied time
α	Immigration rate
β	Emigration rate
U	Intrinsic growth rate minus immigration rate plus emigration rate
C*	Any constant
y^*	Equilibrium point of the autonomus differential equation
$G(t)$	Estimated population of Purwanegara village at time t
$\varepsilon t\%$	Absolute error percentage
$i(t)$	The number of in-migration or immigration at time t
$e(t)$	The number of out-migration or emigration at time t
$n(t)$	The number of immigration minus emigration at time t

Model Logistik Terubahsuai dengan Faktor Migrasi Untuk Meramal Populasi di Kampung Purwanegara Negeri Jawa Tengah, Indonesia.

ABSTRAK

Model pertumbuhan populasi ialah satu model yang digunakan secara meluas untuk meramal dan membuat jangkaan populasi bagi manusia, haiwan, bakteria dan bahkan ianya juga digunakan dalam model pertumbuhan ekonomi. Banyak kajian telah dijalankan oleh para penyelidik bagi model pertumbuhan populasi dengan mengambil kira faktor-faktor kelahiran baru, kematian dan juga kapasiti mampu untuk membuat ramalan bilangan populasi manusia di sesebuah kawasan. Daripada kajian-kajian sebelum ini hanya terdapat satu kajian yang melibatkan faktor migrasi sebagai satu input di dalam model logistik. Nilai bagi faktor migrasi tersebut adalah pemalar. Oleh itu berlainan dengan model logistik terubahsuai seperti di atas, kajian ini mengubahsuai model logistik dengan menambah faktor migrasi sebagai satu fungsi untuk populasi tersebut. Fungsi ini mengambil kira migrasi dan juga interaksi antara orang-orang yang terbatas oleh kapasiti mampu bagi persekitaran. Model ini boleh diselesaikan secara kualitatif dengan menggunakan analisis titik keseimbangan dan secara kuantitatif dengan menggunakan kaedah pembolehubah boleh pisah. Kedua-dua kaedah akan menghasilkan keputusan yang sama. Model logistik terubahsuai telah digunakan bagi data populasi penduduk di Kampung Purwanegara di Negeri Jawa Tengah, Indonesia. Dengan menggunakan data, melalui kaedah graf dan analisis titik keseimbangan keputusan telah terhasil yang mana bagi tempoh masa yang lama populasi bagi kampung Purwanegara akan menghampiri kepada nilai maksimum 68,495 orang. Model logistik terubahsuai menghasilkan purata peratusan kesilapan tentu yang mana 0.61376782 % untuk peramalan bagi populasi daripada Januari, 2012 sehingga Oktober, 2016. Kemudian, berdasarkan kepada model validasi, peratusan kesilapan tentu yang mana 2.79737591 % untuk peramalan bagi populasi bulan November, 2016. Berdasarkan kepada kaedah graf sebelumnya, jangkaan bagi populasi di Kampung Purwanegara daripada Disember 2016 sehingga Disember 2018 boleh dilakukan. Melalui keputusan yang diperolehi melalui model logistik terubahsuai dengan faktor migrasi sebagai satu fungsi bagi populasi telah memberikan keputusan yang lebih baik untuk meramalkan populasi di Kampung Purwanegara di Negeri Jawa Tengah, Indonesia apabila dibandingkan dengan model logistik biasa.

Modified Logistic Model with Migration Factor to Predict the Population of Purwanegara Village in Jawa Tengah Province, Indonesia

ABSTRACT

Population growth model is a model that has widely been used in order to do the estimation and forecasting the population of peoples, animals, bacteria and even in economics growth. Many studies have been carried out by the researchers on population growth model by considering the factors of birth, death and carrying capacity in order to predict the number of population at certain area. From previous the studies there was only one study that involved the factor of migration as an input in the logistic model. The value of the migration is a constant value. Therefore in contrast with the above modified logistic model, in this study logistic model is modified by adding a migration factor as a function of population. This function takes into account the migration and the interaction between peoples that is limited to the carrying capacity of the environment. This model can be solved qualitatively by using the analysis of equilibrium point and quantitatively by using the separable variables method. Both of these methods produce the same result. Modified logistic model has been applied in the population data of Purwanegara village in Jawa Tengah Province, Indonesia. By using the data, through the graph method and analysis of equilibrium point the same results are produced, where for the long term behavior the population of Purwanegara village will approach to the maximum value of 68,495 persons. Modified logistic model gives average absolute error percentage which is 0.61376782 % for the estimation of the population from January, 2012 until October, 2016. Then, based on validation model, the absolute error percentage which is 2.79737591 % for the estimation of the population in November, 2016. Hence, based on the graph method, prediction for the population in Purwanegara village from December 2016 until December 2018 can be done. From the results, the modified logistic model with migration factor as a function of population gives a better result for predicting the population of Purwanegara village in Jawa Tengah Province, Indonesia compared to logistic model.

CHAPTER 1

INTRODUCTION

1.1 Overview of the Study

According to Statistics of Jawa Tengah Province (2016), the population of Indonesia are all residents of the entire territory of Indonesia who have stayed for six months or longer, and those who intended to stay more than six months even though their length of stay is less than six months. In addition, population is one of the important assets for development since the population is the subject and object of development (Salladien, 2003). Since, the population is the subject and object of development, it means the population act as development activist who aims for the benefit of the population. In the development, the population quality can speed up the development process. However, the population lacking in quality will burden the development oppositely. According to Mantra (2003), the development plan is necessary for the formulation of policies. For the planning of development it requires a prediction about the population. A prediction of the population can be done by using mathematical model.

Mathematical model that can be used to predict the population is population growth model. Generally, the population growth model can be written as

$$P'(t) = RP(t) + M \tag{1.1}$$

where, R is intrinsic rate coefficient, $P(t)$ is the number of population, and M is migration rate (immigration rate (α) - emigration rate (β)), if $\alpha \geq \beta$, then $M \geq 0$, else $M < 0$.

The migration factor that is considered can be either a constant value or a specific function. The specific function in migration factor can be a function of time or a function of population (Borrelli and Coleman, 1987).

Population growth model consists of simple and complex models of population growth. The simple models of population growth are Malthus and logistic models (Borrelli and Coleman, 1987). The Malthusian principle of explosive growth of human populations has become one of the classic laws of population change. Formula to be considered in Malthus model are natural birth rate and natural death rate (Edwards and Penney, 2008). The principle follows directly from Equation (1.1) if we set $M = 0$ and R a positive constant, say r. Thus Equation (1.1) becomes

$$P'(t) = rP(t) \tag{1.2}$$

In this case, Equation (1.2) is linear and has the exponential solution $P(t) = P_0 e^{rt}$ where P_0 is the population at the time $t = 0$ (Borrelli and Coleman, 1987). Malthus model is a model of population growth that will keep increasing indefinitely for long term behavior. Logistic model is the extension of Malthus model. Logistic model is a restrict growth model within the context of rate, Equation (1.1) with $M = 0$ and assume that the rate coefficient R has the form $r - bP(t)$, where r and b are positive constants (Borrelli and Coleman, 1987). A negative coefficient b represents a restraint on the growth rate, whereas b positive would lead to accelerated growth. Thus, Equation (1.1) becomes

$$P'(t) = \left[r - bP(t)\right]P(t). \tag{1.3}$$

It is normal to write the rate coefficient as $r\,(1 - P(t)/K)$, where r and K are positive constants, rather than as $r - bP(t)$. Thus, the logistic model with initial condition is

$$
\begin{aligned}
&P'(t) = r\left(1 - \frac{P(t)}{K}\right)P(t) \\
&P(0) = P_0.
\end{aligned} \tag{1.4}
$$

It is observed that $P = 0$ and $P = K$ are the solutions of the logistic model, where K is the carrying capacity. In other words, logistic model assumes that for long term behavior the population will not keep increasing indefinitely but will be limited to the value of carrying capacity of the environment (Edwards and Penney, 2008). Thus, the logistic model is more realistic than Malthus model to predict the population in the future. Several studies by Peleg et al. (2007), Ramos (2013) and Sari (2014) have revealed the usefulness of logistic equations in estimating the events in the field of biology, economics and demography.

The complex population growth model of Malthus and logistic models are modified by considering many factors that affect the growth of population. Because the population is not only affected by birth and death factors, it can also be affected by migration factor so logistic model can be modified by taking into account the migration factor (M) (Borrelli and Coleman, 1987). Meyer and Ausubel (1999) and Safuan et al. (2012) have modified logistic model by replacing the constant K in Equation (1.4) with a function $K(t)$ but the model does not include the migration factor. Laham et al. (2012) have modified logistic model with periodic harvesting. Logistic model with the migration factor of a constant value has been discussed by Rakhim (2015).

Purwanegara village is one of the important villages in Purwokerto Utara sub district, Banyumas regency, Jawa Tengah province, Indonesia. Because the village has a good economy growth and location near the central of city with many universities, then the village provides temporary location to live. Prediction of the population in Purwanegara village has been studied by Sari (2014) using the logistic model and Rakhim

(2015) using the logistic model with migration factor of a constant value. Because migration factor can be either a constant value or a specific function (a function of population or a function of time) (Borrelli and Coleman, 1987), this study is proposed to modify the logistic model with the migration factor as a function of population and to apply the model for predicting the population of Purwanegara village, Indonesia.

1.2 Problem Statement

Population growth rate is not only affected by birth rate and death rate, but also migration rate (Borelli and Coleman, 1987; Murray, 2002). Logistic model has been widely used to predict the population in an area. Sari (2014) has studied the logistic model to predict the population of Purwanegara village, in Jawa Tengah province, Indonesia, but the model did not include the migration factor. From previous study, there was only one study involved the factor of migration as an input in the logistic model. Rakhim (2015) extended the logistic model where the logistic model was considered with migration factor to predict the population in Purwanegara village, but she considered migration factor as a constant value only. For long term behavior, not only the logistic model will be limited in carrying the capacity of environment but also the migration factor. Hence, the migration factor that is considered must not be a constant value but as a function of population.

1.3 Objectives of the Study

The main objectives of this study are:

1. To extend the modified logistic model with the migration factor as a function of population

2. To apply the modified logistic model with the migration factor as a function of population for predicting the population of Purwanegara village, in Jawa Tengah province, Indonesia.
3. To validate the modified logistic model with the migration factor as a function of the population through simulation approach.

1.4 Research Scope

The scopes of this study are listed as follows:

1. This research concentrates on modified logistic model with the migration factor as a function of population.
2. This research concerns with the finding of the solution of modified logistic model with the migration factor as a function of population using the quantitative method (separable variables method) and qualitative method (analysis stability of equilibrium point).
3. Carrying capacity of environment and intrinsic growth rate in this research are assumed as constant value.
4. This research will compare the logistic model and modified logistic model with migration factor as a function of population in applying the model to predict the population of Purwanegara village, in Jawa Tengah province, Indonesia.

1.5 Significant of the Research

The significances of doing this research are it can enrich the literatures and devoted the new insight to develop population growth model, especially in logistic model with the migration factor. Furthermore, this research can assist the government of Purwanegara village in predicting the population that is believed to be more accurate in the future.

1.6 Organization of the Thesis

The second chapter of this thesis is the literature review. This chapter consists of two subtopics. The first subtopic is the introduction of Chapter 2. Then, the next subtopic is modified logistic model.

The third chapter of this thesis is the methodology of the research. In this chapter, there are two subtopics. The first subtopic is the introduction of the chapter. The second subtopic is the framework of the research methodology.

The fourth chapter is the result and discussion. This chapter consists of four subtopics, and the first subtopic is the introduction of the chapter. Second subtopic is the logistic model with migration factor as a function of population. Third subtopic is about applying the modified logistic model as a function of population to predict the population of Purwanegara village, in Jawa Tengah province, Indonesia. Fourth subtopic is about the simulation of the modified logistic model with migration factor as a function of population.

The fifth chapter of this thesis is the conclusion. This chapter consists of the overall conclusion of all the findings in this study. This chapter also discusses the suggestions for future research related to this study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter contains two subtopics. Subtopic 2.2 contains literature review related to modified logistic growth models that have been studied in the field of biology, ecology, business, economic and etc.

2.2 Modified Logistic Model

The first population growth model was introduced in 1798 by Thomas Robert Malthus. This model was called Malthus model or exponential growth model. Factors that are considered in Malthus model is only natural birth rate and natural death rate (Edwards and Penney, 2008). Malthus model is not very realistic for the populations for long term behavior, however it has been used due to some justification for the growth's estimation of certain bacteria at early stages (Murray, 2002). Hence, in 1840 the Belgian statistician and astronomer, Pierre Francois Verhulst, extended the Malthus model (Edward and Penney, 2008). Population growth model by Pieree Francois Verhulst is called a logistic model. Factors that are considered are not only natural birth rate and natural birth rate, but also carrying the capacity of environment. Thus, logistic model is more realistic than Malthus model to the purpose of prediction of the population.

The logistic model is increasingly being used to describe microbial growth. Logistic model has been studied for describing the growth pattern of pseudomonas putida (Annadurai et al., 2000). Logistic model of subsurface fungal growth with application to biomediation has been studied by Classen et al. (2000). Comparison of maximum specific growth rates and lag times estimated from absorbance and viable count data by using logistic model has been studied by Dalgaard and Koutsoumanis (2001). Different with Dalgaard and Koutsoumanis (2001), comparison of analysis methods for determination of the kinetic parameters in batch cultures has been studied by Tobajas and Garcia-Calvo (2000). Logistic model for predicting Yersinia enterocolitica inactivation in Turkish feta cheese during storage has been studied by Bozkurt and Erkmen (2001). Then, logistic model for predicting inactivation of microbial growth has been studied by Zhao et al., (2001). The logistic function model was derived specifically to describe batch growth of microorganisms, as a function of nutrient depletion, based on substituting potential cell density for the concentrations of nutrients. Batch cultures of nine microorganisms were analyzed statistically, using logistic function model, its transformations, and the graphic methods (Wachemhein et al., 2003). The logistic model for sigmoid microbial growth curves revisited has been used. The logistic model based on the notion that the isothermal momentary growth rate is proportional to the momentary population's size and the fraction of resources that are still available in the habitat (Peleg et al., 2007). In other words, the logistic function model describes the growth of microbial populations as a function of initial population density, time, growth rate, and final population density (Annadurai et al., 2000; Classen et al., 2000; Dalgaard and Koutsoumanis, 2001; Tobajas and Garcia-Calvo, 2000; Bozkurt and Erkmen, 2001; Zhao et al., 2001; Wachemhein et al., 2003; Peleg et al., 2007).

Analysis of logistic model growth model has been studied by Tsolaris and Wallace (2002), if the generalized logistic form is to be adopted as a viable modelling tool then detailed curve-fitting of real data has to be undertaken.

Application of logistic model has been studied for electrochemical dynamics and the logistic equation has a continuous solution without bifurcation with a sole stable equilibrium at saturation (Sadkowski, 2000). Then, an application of logistic model has been studied for predicting of maize yield under water and nitrogen management (Sepaskhah et al., 2011). Logistic equation is used to quantify the influence of seasonal water and nitrogen application on maize biomass accumulation and grain yield and to develop empirical models for prediction of maize biomass and grain yield. It is concluded that logistic equation along with the presented empirical models for prediction of constants in logistic equation and harvest index are appropriate for accurate prediction of dry matter and grain yield of maize at the study region. Application of logistic model in medicine using the logistic model is used to model the growth of tumors. This application can be considered an extension of the above mentioned use in the framework of ecology. Denoting with $P(t)$ the size of the tumor at time t (Sweilam et al., 2012). Application of logistic model as a mathematical model in explaining and forecasting different areas in business and economics such as product diffusion and market acceptance, inflation rate of goods, purchasing power of peso, and employment and unemployment in the Philippines. Results show that logistic equation can be used to predict or describe about some business and economic systems as well as describing the effects of the measures employed by the different sectors that related to the problems. (Ramos, 2013). Application of logistic model for looking at the exact solutions of nonlinear differential equations has been studied by Kudryashov (2015). Logistic function is shown to be solution of the Riccati equation, some second-order nonlinear ordinary differential equations and many third-order nonlinear ordinary differential equations. Then, application of logistic model is employed for estimating the population in the United Nations (Cohen, 1995), Manokwari, Papua, Indonesia (Iswadi, 2009), United States and China (Hsieh, 2014), India (Kulkarin et al., 2014), and Purwanegara village, Indonesia (Sari, 2014) have been discussed. Historical chart of logistic model is given in Figure 2.1.

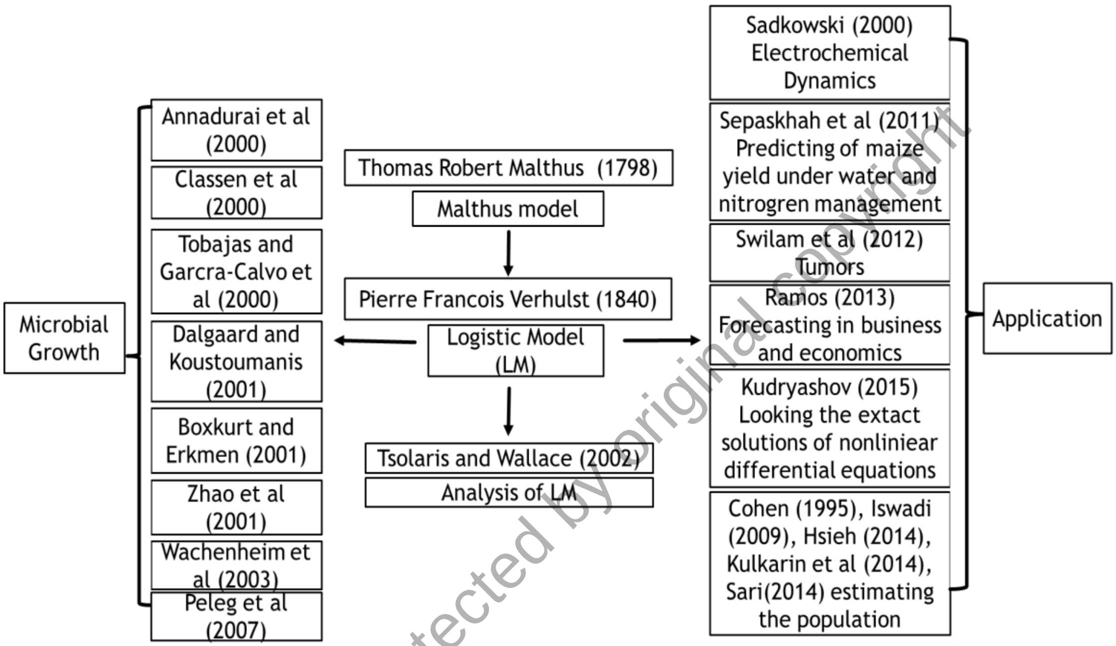

Figure 2.1 Historical chart of logistic model

Logistic model can be modified by taking into accounts of many factors that affect the growth of population. Meyer and Ausubel (1999), Hazimura and Matsuyama (1999), Thornley and France (2005), Rakhmawati and Sutimin (2006), Sakanoue (2007), and Safuan et al. (2012) have modified logistic model by replacing the constant *K* in Equation (1.4) with a function *K(t)*. Holmes et al. (1994) and Acevedo et al. (2012) have modified the logistic model with brownian random dispersal (a diffusive logistic model). Here, a diffusive logistic growth model to quantify forest recovery is described and a system in which forest diffuses through a non-forest matrix is defined. The model consists of a diffusion term that describes the spread of forest in continuous space and time, and a logistic growth reaction that describes change in the proportion of forest. Barnes and Fulford (2002), Forys and Czochra (2003), Ariano (2005), Toaha (2008), Wahyullah (2009) have modified logistic model with delayed carrying capacity. In other words expression for a delayed logistic equation, assuming that the rate of change of the population depends on three components: growth, death, and intraspecific competition, with the delay in the growth component.

Laham et al. (2012) have modified logistic model for purpose of periodic harvesting strategies for tilapia fish farming. Two logistic growth models have been used namely constant harvesting and periodic harvesting. Even though tilapia fish farming has been commercialized, the use of mathematical models in determining harvesting strategies has not been widely applied in Malaysia. Logistic growth model is appropriate for population growth of animal when overcrowding and competition resources are taken into consideration. Doust and Saraj (2015) have modified logistic model with harvesting factor, which is studied in two cases constant and non-constant. In fact, the nature of equilibrium points and solutions behavior has been analyzed for both of the above cases by finding the first integral. They have used modified logistic model but the models did not include the migration factor. Then, modified logistic model with migration factor has been studied by Rakhim (2015) but migration factor that is considered used a constant value only. Historical chart of modified logistic model is given in Figure 2.2.

According to Rakhim (2015), logistic model can be modified with migration factor by a constant value as follows,

$$P'(t) = r\left(1 - \frac{P(t)}{K}\right)P(t) + q \tag{2.1}$$

where q is a constant value of migration factor with the description of the variables and parameters in Table 2.1.

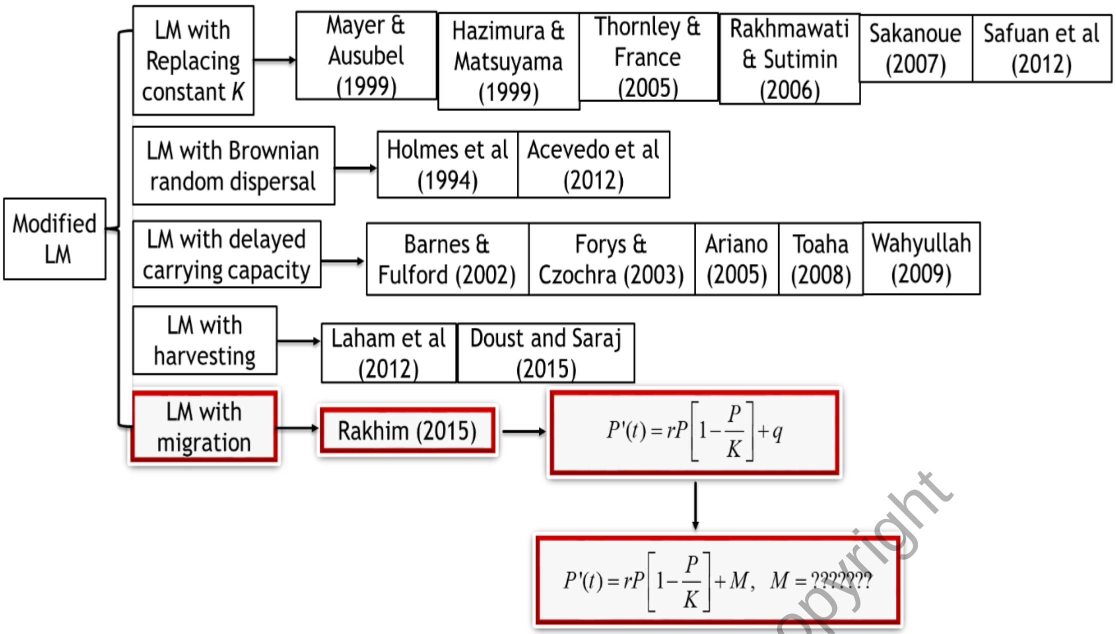

Figure 2.2 Historical chart of modified logistic model

Table 2.1: Variables and parameters of logistic model with migration factor by a constant value.

Symbol	Definition	Dimension
P(t)	The population at the time *t*	Person
K	Carrying capacity	Person
r	Intrinsic growth rate	Per unit time
q	Migration rate by constant value	Person per unit time

Logistic model with migration factor with a constant value can be solved qualitatively. As for the first step to solve this modified logistic model with migration factor of a constant value is by converting the Equation (2.1) by using the non-dimension method. Non-dimension is one of the applied differential equations methods that use the procedure of changing the dimension of variables into dimensionless, before the dimensionless variables are grouped in order to obtain new fewer variables (Ledder, 2005). Since *P(t)* and *K* have the same dimension of person, then it can be formed into a dimensionless, namely p so, $p = \frac{P(t)}{K}$ and also dimensionless of $s = r\,t$. Hence, Equation (2.1) can be written based on the definition of new variables as follows

$$\frac{d(Kp)}{d\left(\frac{s}{r}\right)} = rKp(1-p) + q$$

$$\frac{d(p)}{d(s)} = p(1-p) + \frac{q}{rK} \qquad (2.2)$$

Suppose, $\frac{q}{rK} = y$ with $y > 0$, then

$$\frac{dp}{ds} = p(1-p) + y. \qquad (2.3)$$

Equation (2.3) is a logistic model with migration factor with a constant value in the form of a model with non-dimension. The solution of Equation (2.3) is obtained by using the analysis stability of equilibrium point, $p = 0.5 + 0.5\sqrt{1+4y}$ with $y = q/rK$, where $y > -0.25$. In contrast with the model proposed by Rakhim (2015), this research introduces modified logistic model with migration factor as a function of the population.

CHAPTER 3

METHODOLOGY

3.1 Introduction

In this chapter 3, the framework of the research methodology is explained. Then, some chosen methods to solve the modified logistic model as a function of population and to predict the population of Purwanegara village using modified logistic model with migration factor as a function of population are described.

3.2 Research Methodology

Methodologies for this study are as follows:

i. Extension of the model

This research is based on the logistic model. Logistic model is the expansion of Malthus model. In addition to the natural birth rate and natural death rate, logistic model also assumes the carrying capacity of environment. In other words, the population for long term behavior will not keep increasing indefinitely but will be limited to the value of carrying capacity of environment (Edwards and Penney, 2008). Since the population is not only affected by natural birth and death factors, but it also affected by migration factor, hence the logistic model can be modified by taking into account the migration factor (*M*),

$$P'(t) = rP(t)\left(1 - \frac{P(t)}{K}\right) + M. \tag{3.1}$$

The considered migration factor in logistic model can be either a constant value or a specific function. The specific function in migration factor can be a function of time or a function of population (Borrelli and Coleman, 1987). Modified logistic model with migration factor by a constant value has been studied (Rakhim, 2015). This research will focus on the modified logistic model with migration factor as a function of population. It can be solved either by quantitative or qualitative method.

Quantitative method is a solution that can be solved analytically. Analytic solution is either in explicit function or implicit function form (Edwards and Penney, 2008). In this study, logistic model with migration factor as a function of population consists of two cases, (i) case $r-(\alpha-\beta)>0$ and (ii) case $r-(\alpha-\beta)<0$. Each case can obtain the general and specific solution. Logistic model with migration factor as a function of population can be solved quantitatively by using separable variables method. Then, for plotting the solution graph of logistic model with the migration factor as a function of population, we need to perform the first derivative relatively to monotonicity and second derivative relatively to concavity. A function is called pure monotone in an interval if the function is going up (increasing) or going down (decreasing) in a certain interval. The first derivative is interpreted as a slope of graph of the function so that the first derivative can be employed to determine the increase or decrease of the graph. To determine whether the graph is going up or going down, $P'(t)=0$ is taken (Repka, 1994). Furthermore, in order to determine the concavity graph for the solution of the model, the differential