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Effect of Variable Axial Force on the Vibration of a Thin Beam Subjected to Moving Concentrated Loads

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ABSTRACT

The effect of variable axial force on a loaded beam subjected to both constant and variable loads are considered herein. The beam is assumed to be uniform, thin, and has a simple support at both ends. The constant load moves with constant velocity and uniform acceleration. The Galerkin's method and the integral transformation method are employed in solving the fourth order partial differential equation describing the motion of the beam – load system. On solving, results show that, increase in the values of axial force N gives a significant reduction in the deflection profile of the vibrating beam. Results also show that the addition of the axial force N, foundation modulus K, and consideration of a damping effect in the governing equation increases the critical velocity of the dynamical system, thus, the risk of resonance is reduced.

Keywords: Axial Force; Beam; Concentrated loads, Foundation modulus, Galerkin's method.

1 INTRODUCTION

Solid bodies vibrate when loads move on them, this phenomenon has led to the study of elastic bodies' reaction under the influence of moving loads. This article considers a beam whose mass is smaller than the mass of the moving loads. In a problem of a beam under moving loads, making the position of the loads changes continuously, this effect of the load on the beam is thus of great importance. Extensive work has been done on this class of dynamical problem when the structural members have uniform cross – sections.

Euler Bernoulli beam theory shows a simplification of the linear theory of elasticity. The theory proposes a procedure for calculating the load – carrying and deflection characteristics of beams. It was first enunciated in 1750, but was not applied on a large scale until the development of the Eiffel Tower and the Ferris wheel in the late 19th century, following these successful demonstrations, it has become a cornerstone of engineering and an enabler of the second industrial revolution [8].

An extension to known analytical tools have been developed, these include; plate theory and finite element analysis. However, the simplicity of beam theory makes it an important tool in the sciences especially structural and mechanical engineering.

Several researchers have done extensively in the area of beam theory, see [1, 3, 6, 7, 8, 10, 11, 16, 17, 18, 19, 22, 23], and among the few studies about the response of elastic structures to moving distributed loads known in the literature is the work of [3], who studied the dynamic response of plates on pastermark foundation under distributed moving load.

An extensive treatment on the vibrations of beams reacting to the influence of moving loads containing a large number of similar cases have been studied in [17]. A dynamic Green function approach was used by [17] to obtain the response of a finite length of a simply supported Euler – Bernoulli beam affected by a moving load. They proposed a matrix expression for the deflection of the beam. In the case where the moving mass causes a dynamic behaviour on simply supported Euler-Bernoulli beam was studied in [8].

In the aforementioned study, several authors have neglected the damping term in the governing differential equation of motion. This study therefore investigates the transverse displacement response of axially presented thin beams when the load is constant and when it (load) varies.

In solving differential equations, several approaches have evolved, some of which include analytical approach; see [2, 4], numerical approximation; see [5, 12 – 15] and machine learning techniques; see [9, 20, 21], just to mention a few. Here in, we obtain the solution to the governing equation by using the Galerkin's method to reduce the order of the differential equation and then employ the integral transform method to obtain its solution.

2 THE GOVERNING EQUATION

The Newton's second law of motion is used to derive the equation of motion of a thin beam vibrating transversely under the effect of a moving load. The beam in consideration rests on an elastic foundation with foundation modulus K, and vibration is caused by a moving load $P(x,t)$. This beam is subjected to an axial force N and it is parallel to the x – axis. This is illustrated in Fig. 1.

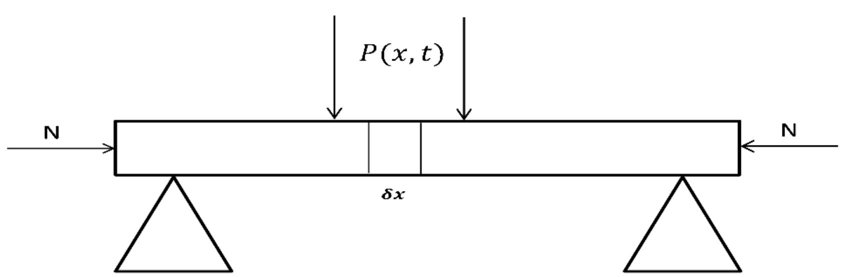

Figure 1 : A Beam resting on an elastic foundation.

The partial differential equation (PDE) describing the vibration of the Bernoulli – Euler beam due to the action of a moving load when the beam has constant flexural – rigidity EI and it rests on a elastic foundation K with variable axial force N is given by:

$$EI \frac{\partial^4}{\partial x^4} w(x,t) - N(4x - 3x^2 + x^3) \frac{\partial^2}{\partial x^2} w(x,t) + \varepsilon_0 \frac{\partial}{\partial t} w(x,t) + \mu \frac{\partial^2}{\partial t^2} w(x,t) + Kw(x,t) = P(x,t), \quad (1)$$

where $w(x,t)$ is the lateral deflection of the beam measured from its equilibrium position, EI is the flexural rigidity of the beam, E is the young modulus of elasticity, I is the moment of inertia, K is the foundation constant, g is the acceleration due to gravity, x is the spatial coordinate, μ is the mass per unit length of the beam, N is the axial force, t is the time coordinate, $P(x,t)$ is the variable load.

Equation (1) represents the transverse motion of the thin beam being influenced by a moving load together with the assumptions below:

i) The rotatory inertia is not considered.

ii) The thin beam has a uniform cross – section

iii) The mass of the beam is constant per unit length.

iv) The beam's axis experience extension and contraction.

v) The beam has a simple support on both edges.

Here, the beam model (1) taken to be simply supported has the boundary conditions:

$$w(x,t) = w(L,t) = 0,$$

$$\frac{\partial^2}{\partial x^2} w(0,t) = \frac{\partial^2}{\partial x^2} w(L,t) = 0,$$

with initial conditions:

$$w(x,0) = 0 = \frac{\partial}{\partial t} w(x,0) = 0.$$

3 LATERAL DEFLECTION OF THE VARIABLE AXIAL FORCE ON THE VIBRATION OF A THIN BEAM SUBJECTED TO CONSTANT LOAD

Solving the governing equation (1) for the lateral deflection, the Galerkin's method is used to reduce equation (1) to a second order partial differential equation (PDE). The second order PDE is then solved by Laplace transformation.

Here, in the case of constant load P, the variable moving load $P(x,t)$ takes the form:

$$P(x,t) = P\,\delta(x - ct), \quad (2)$$

where P is a constant and c is the constant velocity. The function $\delta(x)$ is defined as:

$$\delta(x) = \begin{cases} 0; & x \neq 0 \\ \infty; & x = 0 \end{cases}$$

And referred to as the Dirac – delta function, having following properties:

$$\int_a^b \delta(x-k) f(x) dx = \begin{cases} f(k); & a < k < b \\ 0; & a < b < k \\ 0; & k < a < b. \end{cases}$$

Assuming that the lateral deflection $w(x,t)$ has solution of the form,

$$w(x,t) = \sum_{m=1}^{\infty} Y_m(t) \frac{\sin j\pi x}{L}, \tag{3}$$

substituting equation (2) and (3) into (1), we have,

$$EI \frac{\partial^4}{\partial x^4} \sum_{m=1}^{n} Y_m(t) \frac{\sin j\pi x}{L} - N(4x - 3x^2 + x^3) \frac{\partial^2}{\partial x^2} \sum_{m=1}^{n} Y_m(t) \frac{\sin j\pi x}{L} + \varepsilon_0 \frac{\partial}{\partial t} \sum_{m=1}^{n} Y_m(t) \frac{\sin j\pi x}{L} + \mu \frac{\partial^2}{\partial t^2} \sum_{m=1}^{n} Y_m(t) \frac{\sin j\pi x}{L} + k \sum_{m=1}^{n} Y_m(t) \frac{\sin j\pi x}{L} = P\delta(x - ct). \tag{4}$$

Obtaining the first, second, third and fourth derivative of (3) gives

$$\begin{aligned} \frac{\partial}{\partial x} w(x,t) &= \sum_{m=1}^{\infty} Y_m(t) \frac{j\pi}{L} \cos \frac{j\pi x}{L}, \\ \frac{\partial^2}{\partial x^2} w(x,t) &= -\sum_{m=1}^{\infty} Y_m(t) \left(\frac{j\pi}{L}\right)^2 \sin \frac{j\pi x}{L}, \\ \frac{\partial^3}{\partial x^3} w(x,t) &= -\sum_{m=1}^{\infty} Y_m(t) \left(\frac{j\pi}{L}\right)^3 \cos \frac{j\pi x}{L}, \\ \frac{\partial^4}{\partial x^4} w(x,t) &= \sum_{m=1}^{\infty} Y_m(t) \left(\frac{j\pi}{L}\right)^4 \sin \frac{j\pi x}{L}, \end{aligned} \tag{5}$$

and substituting (5) into (1) gives

$$EIY_m(t) \left(\frac{j\pi}{L}\right)^4 \frac{\sin j\pi x}{L} + N(4x - 3x^2 + x^3) Y_m(t) \left(\frac{j\pi}{L}\right)^2 \frac{\sin j\pi x}{L} + \varepsilon_0 \frac{\partial}{\partial t} Y_m(t) \frac{\sin j\pi x}{L} + \mu \frac{\partial^2}{\partial t^2} Y_m(t) \frac{\sin j\pi x}{L} + kY_m(t) \frac{\sin j\pi x}{L} = P\delta(x - ct). \tag{6}$$

Galerkin's method requires that the right hand size of equation (6) be orthogonal to the function $\frac{\sin j\pi x}{L}$, we therefore multiply the equation (6) by $\frac{\sin m\pi x}{L}$ and integrate from 0 to L, yielding

$$EIY_m(t)\left(\frac{j\pi}{L}\right)^4\int_0^L \frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}dx \ + NY_m(t)\left(\frac{j\pi}{L}\right)^2\int_0^L (4x-3x^2+x^3)\frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}\ dx$$
$$+\varepsilon_0\frac{\partial}{\partial t}Y_m(t)\int_0^L \frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}dx \ +\mu\frac{\partial^2}{\partial t^2}Y_m(t)\int_0^L \frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}dx$$
$$+kY_m(t)\int_0^L \frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}dx$$
$$=P\int_0^L \delta(x-ct)\frac{\sin m\pi x}{L}dx. \tag{7}$$

Equation (7) resolves to;

$$\left[EIY_m(t)\left(\frac{m\pi}{L}\right)^4+\varepsilon_0\frac{\partial}{\partial t}Y_m(t)+\mu\frac{\partial^2}{\partial t^2}Y_m(t)\left(\frac{j\pi}{L}\right)^2+kY_m(t)\right]I_1+NY_m(t)\left(\frac{j\pi}{L}\right)^2 I_2$$
$$=PI_3, \tag{8}$$

where

$$I_1=\int_0^L \frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}dx\,,$$
$$I_2=\int_0^L (4x-3x^2+x^3)\frac{\sin j\pi x}{L}\frac{\sin m\pi x}{L}dx$$
$$I_3=\int_0^L \delta(x-ct)\frac{\sin m\pi x}{L}dx.$$

Evaluating I_1, I_2, and I_3 (by integration by parts), when $j=m=1$ yields

$$I_1=\frac{L}{2}, \tag{9}$$

$$I_2=L^2-\frac{L^3}{2}+\frac{L^4}{8}+\frac{3L^3}{4\pi^2}-\frac{3L^4}{8\pi^2}, \tag{10}$$

$$I_3=\sin\left(\frac{c\pi t}{L}\right). \tag{11}$$

Substituting I_1, I_2, and I_3 into equation (8) yields,

$$EIY_m(t)\left(\frac{\pi}{L}\right)^4\frac{L}{2} + NY_m(t)\left(\frac{\pi}{L}\right)^2\left[L^2 - \frac{L^3}{2} + \frac{L^4}{8} + \frac{3L^3}{4\pi^2} - \frac{3L^4}{8\pi^2}\right] + \varepsilon_0\frac{\partial}{\partial t}Y_m(t)\frac{L}{2} + \mu\frac{\partial^2}{\partial t^2}Y_m(t)\frac{L}{2} + kY_m(t)\frac{L}{2} = P\sin\left(\frac{c\pi t}{L}\right), \quad (12)$$

and simplifying to

$$\left[EI\frac{\pi^4}{2L^4} + N\left(\frac{\pi^2L^2}{L^2} - \frac{\pi^2L^3}{2L^2} + \frac{\pi^2L^4}{8L^2} + \frac{3\pi^2L^3}{4L^2\pi^2} - \frac{3\pi^2L^4}{8L^2\pi^2}\right) + \frac{kL}{2}\right]Y_m(t) + \frac{\varepsilon_0 L}{2}\frac{\partial}{\partial t}Y_m(t) + \frac{\mu L}{2}\frac{\partial^2}{\partial t^2}Y_m(t) = P\sin\left(\frac{c\pi t}{L}\right). \quad (13)$$

Dividing equation (13) through by $\frac{\mu L}{2}$ yields

$$\frac{2}{\mu L}\left[EI\frac{\pi^4}{2L^4} + \frac{N\pi^2L^2}{L^2} - \frac{N\pi^2L^3}{2L^2} + \frac{N\pi^2L^4}{8L^2} + \frac{3N\pi^2L^3}{4L^2\pi^2} - \frac{3N\pi^2L^4}{8L^2\pi^2} + \frac{kL}{2}\right]Y_m(t) + \frac{\varepsilon_0}{\mu}\frac{\partial}{\partial t}Y_m(t) + \frac{\partial^2}{\partial t^2}Y_m(t) = P\sin\left(\frac{c\pi t}{L}\right), \quad (14)$$

Choosing

$P_1 = \frac{2P}{\mu L}$,

$m_1 = \dfrac{\varepsilon_0}{\mu}$,

$$m_2 = \frac{2}{\mu L}\left[EI\frac{\pi^4}{2L^4} + \frac{N\pi^2L^2}{L^2} - \frac{N\pi^2L^3}{2L^2} + \frac{N\pi^2L^4}{8L^2} + \frac{3N\pi^2L^3}{4L^2\pi^2} - \frac{3N\pi^2L^4}{8L^2\pi^2} + \frac{kL}{2}\right],$$

equation (14) then becomes

$$m_2Y_m(t) + m_1\frac{\partial}{\partial t}Y_m(t) + \frac{\partial^2}{\partial t^2}Y_m(t) = P_1\sin\left(\frac{c\pi t}{L}\right). \quad (15)$$

Taking the Laplace transform of equation (15) with $t = 0$ and considering the boundary condition, we have;

$\mathcal{L}(Y_m''(t)) = s^2\mathcal{L}[Y_m(t)] - sY_m(t) - Y_m'(t)$,

$\mathcal{L}(Y_m'(t)) = s\mathcal{L}[Y_m(t)] - Y_m(t)$,

and $Y_m(0) = Y_m'(0)$.

Therefore equation (15) becomes

$$m_2\mathcal{L}[Y_m(t)] + m_1 s\mathcal{L}[Y_m(t)] + s^2\mathcal{L}[Y_m(t)] = P_1\,\mathcal{L}[\sin\left(\frac{c\pi t}{L}\right)], \quad (16)$$

which can be written as

$$\mathcal{L}[Y_m(t)] \ = P_1\,\mathcal{L}[\sin\left(\frac{c\pi t}{L}\right)] \times \frac{1}{[m_2 + m_1 s + s^2]}. \tag{17}$$

Let the roots of $m_2 + m_1 s + s^2$ be s_1 and s_2 where

$$s_1 = \frac{-m_1 + \sqrt{{m_1}^2 - 4m_2}}{2}, \qquad s_2 = \frac{-m_1 - \sqrt{{m_1}^2 - 4m_2}}{2}.$$

Then equation (15) becomes

$$\mathcal{L}[Y_m(t)] \ = P_1\,\mathcal{L}[\sin\left(\frac{c\pi t}{L}\right)] \times \frac{1}{(s - s_1)(s - s_2)}.$$

Let

$$F(s) = P_1\,\mathcal{L}[\sin\left(\frac{c\pi t}{L}\right)],$$

$$G(s) = \frac{1}{(s - s_1)(s - s_2)},$$

then

$$f(t) = \mathcal{L}^{-1}[F(s)] = P_1 \mathcal{L}^{-1}\,\mathcal{L}[\sin\left(\frac{c\pi t}{L}\right)] = P_1\,[\sin\left(\frac{c\pi t}{L}\right)]\,,$$

$$g(t) = \mathcal{L}^{-1}[G(s)] = \mathcal{L}^{-1}\left[\frac{1}{(s - s_1)(s - s_2)}\right] = \frac{\left[e^{s_1(t)} - e^{s_2(t)}\right]}{s_1 - s_2}.$$

Thus,

$$f(t) = P_1\,[\sin\left(\frac{c\pi t}{L}\right)]\,, \qquad g(t) = \frac{\left[e^{s_1(t)} - e^{s_2(t)}\right]}{s_1 - s_2}.$$

Using the convolution theorem;

$$f * g = \int_0^t f(u) g(t - u) du \tag{18}$$

where, $Y_m(t) = f * g$

$$f * g = \int_0^t \frac{\left[e^{s_1(t-u)} - e^{s_2(t-u)}\right]}{s_1 - s_2} P_1\,[\sin\left(\frac{c\pi u}{L}\right)] du$$

$$= \frac{P_1}{s_1 - s_2}\left[e^{s_1(t)}\int_0^t e^{-s_1 u}\,[\sin\left(\frac{c\pi u}{L}\right)]du - e^{s_2(t)}\int_0^t e^{-s_2 u}\,[\sin\left(\frac{c\pi u}{L}\right)]du\right] \tag{19}$$

Denote $\emptyset = \frac{c\pi}{L}$, then (19) becomes

$$f * g = \frac{P_1}{s_1 - s_2}\left[e^{s_1(t)}\int_0^t e^{-s_1 u}\sin\emptyset u du - e^{s_2(t)}\int_0^t e^{-s_2 u}\sin\emptyset du\right]$$

$$= \frac{P_1}{s_1 - s_2}\left[e^{s_1(t)} I_a e^{s_2(t)} I_b\right] \tag{20}$$

where,

$$I_a = \int_0^t e^{-s_1 u}\sin\emptyset u du\,,$$

$$I_b = \int_0^t e^{-s_2 u}\sin\emptyset du\,.$$

Evaluating I_a and I_b by integration by part gives

$$I_a = \left[\frac{-\emptyset e^{-s_1 t}\cos\emptyset t + \emptyset - s_1\, e^{-s_1 t}\sin\emptyset t}{\emptyset^2 + {s_1}^2}\right], \tag{21}$$

and

$$I_b = \left[\frac{-\emptyset e^{-s_2 t}\cos\emptyset t + \emptyset - s_2\, e^{-s_2 t}\sin\emptyset t}{\emptyset^2 + {s_2}^2}\right]. \tag{22}$$

substituting (21) and (22) into (20) yields

$$Y_m(t) = \frac{P_G\emptyset}{\emptyset^2 + {s_1}^2}\left(\cos\emptyset t + e^{s_1 t} - \frac{s_1\sin\emptyset t}{\emptyset}\right) - \frac{P_G\emptyset}{\emptyset^2 + {s_2}^2}\left(\cos\emptyset t + e^{s_2 t} - \frac{s_2\sin\emptyset t}{\emptyset}\right) \tag{23}$$

Substituting equation (23) into equation (3)gives the solution

$$w(x,t) = \sum_{m=1}^{n}\left\{\frac{P_G\emptyset}{\emptyset^2 + s_1^2}\left(\cos\emptyset t + e^{s_1 t} - \frac{s_1\sin\emptyset t}{\emptyset}\right) - \frac{P_G\emptyset}{\emptyset^2 + s_1^2}\left(\cos\emptyset t + e^{s_2 t} - \frac{s_2\sin\emptyset t}{\emptyset}\right)\right\}\frac{\sin j\pi x}{L} \tag{24}$$

where,

$$P_G = \frac{P_1}{s_1 - s_2}, \quad P_1 = \frac{2P}{\mu L}, \quad \emptyset = \frac{\pi c}{L},$$

and s_1, s_2 are the roots of the quadratic expression $m_2 + m_1 s + s^2$, which are given as

$$s_1 = \frac{-m_1 + \sqrt{m_1^2 - 4m_2}}{2}, \quad s_2 = \frac{-m_1 - \sqrt{m_1^2 - 4m_2}}{2}$$

4 LATERAL DEFLECTION OF THE VARIABLE AXIAL FORCE ON THE VIBRATION OF A THIN BEAM SUBJECTED TO VARIABLE LOAD

The PDE describing the vibration of the Bernoulli – Euler beam being influenced by the action of a variable moving load

$$P(x,t) = Pe^t \delta(x - ct),$$

with the beam having a constant flexural – rigidity EI and the beam resting on a simply support elastic foundation K with variable axial force N is defined by,

$$\begin{aligned} EI\frac{\partial^4}{\partial x^4}w(x,t) - N(4x - 3x^2 + x^3)\frac{\partial^2}{\partial x^2}w(x,t) + \varepsilon_0\frac{\partial}{\partial t}w(x,t) + \mu\frac{\partial^2}{\partial t^2}w(x,t) + Kw(x,t) \\ = Pe^t\delta(x - ct). \end{aligned} \quad (25)$$

We assume that

$$w(x,t) = \sum_{m=1}^{\infty} Y_m(t)\frac{\sin j\pi x}{L} \quad (26)$$

Then equation (25) becomes

$$\begin{aligned} EIY_m(t)\left(\frac{j\pi}{L}\right)^4 \frac{\sin j\pi x}{L} + N(4x - 3x^2 + x^3)Y_m(t)\left(\frac{j\pi}{L}\right)^2 \frac{\sin j\pi x}{L} + \varepsilon_0\frac{\partial}{\partial t}Y_m(t)\frac{\sin j\pi x}{L} \\ + \mu\frac{\partial^2}{\partial t^2}Y_m(t)\frac{\sin j\pi x}{L} + kY_m(t)\frac{\sin j\pi x}{L} \\ = Pe^t\delta(x - ct). \end{aligned} \quad (27)$$

Following the approach discussed in section 3, the Galerkin's method is used to reduce (27) into a second order PDE

$$m_2Y_m(t) + m_1\frac{\partial}{\partial t}Y_m(t) + \frac{\partial^2}{\partial t^2}Y_m(t) = P_1e^t\int_0^L \frac{\sin m\pi x}{L}\delta(x - ct)dx, \quad (28)$$

where m_1 , m_2 and P_1 is properly defined in the equation of the analysis of the constant load model. The right hand side of (28) implies

$$P_1 e^t \int_0^L \frac{\sin m\pi x}{L} \delta(x - ct)dx \;=\; P_1 e^t \frac{\sin m\pi ct}{L}.$$

Taking the Laplace of equation (27) in conjunction with the initial condition yields

$$\left[s^2 \overline{Y}_m(s) - sY_m(0) - Y'(0)\right] + m_1\left[s\overline{Y}_m - \overline{Y}_m(0)\right] + m_2 \overline{Y}_m = \mathcal{L}\left[P_1 e^t \frac{\sin m\pi ct}{L}\right]. \tag{29}$$

Applying the initial condition and rearranging, we have

$$\overline{Y}_m(t) = P_1 \mathcal{L}\left[e^t \frac{\sin m\pi ct}{L}\right] \frac{1}{m_2 + m_1 + s^2}. \tag{30}$$

Let the roots of $m_2 + m_1 + s^2$ be s_1 and s_2, where

$$s_1 = \frac{-m_1 + \sqrt{{m_1}^2 - 4m_2}}{2}, \qquad s_2 = \frac{-m_1 - \sqrt{{m_1}^2 - 4m_2}}{2}.$$

Thus, equation (30) can be expressed as

$$\overline{Y}_m(t) = P_1 \mathcal{L}\left[e^t \frac{\sin m\pi c}{L}\right] \times \frac{1}{(s - s_1)(s - s_2)}. \tag{31}$$

Resolving by the inverse Laplace transform as discussed in section 3, yields

$$\begin{aligned} Y_m(t) = {} & \frac{P_p e^{s_1 t}}{\theta^2 + {s_3}^2}\left[-\theta e^{s_3 t}\cos\theta t + \theta + s_3 e^{s_3 t}\sin\theta t\right] \\ & - \frac{P_p}{\theta^2 + {s_4}^2}\left[\theta e^{s_4 t}\cos\theta t + \theta + s_4 e^{s_4 t}\sin\theta t\right] \end{aligned} \tag{32}$$

where $P_p = \frac{P_1}{(s_1 - s_2)}$, $\quad s_3 = (1 - s_1)$, and $s_4 = (1 - s_2)$.

Substituting (32) into (26) yields the solution

$$\begin{aligned} w(x,t) = \sum_{m=1}^{\infty} & \left\{ \frac{P_p e^{s_1 t}}{\theta^2 + {s_3}^2}\left[-\theta e^{s_3 t}\cos\theta t + \theta + s_3 e^{s_3 t}\sin\theta t\right] \right. \\ & \left. - \frac{P_p}{\theta^2 + {s_4}^2}\left[\theta e^{s_4 t}\cos\theta t + \theta + s_4 e^{s_4 t}\sin\theta t\right] \right\} \frac{\sin j\pi x}{L} \end{aligned} \tag{33}$$

5 NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the analytical result, the length of the thin beam $l = 12.192m$, other values used are velocity c of concentrated loads $8.128m/s$, Young modulus, $E = 2.10924 \times 10^9$, and moment of inertia $I = 2876 \times 10^{-3}$. The values of the foundation modulus are chosen as $0N/m^3$and $4 \times 10^7 N/m^3$ the values for axial force N chosen as $0\ N/m^3$ and $2 \times 10^7 N/m^3$, the transverse displacement of a supported beam under the influence of a concentrated moving loads are computed and plotted against time t. This has been done for constant and variable loads with the chosen values for foundation modulus, circular frequency and axial force.

Fig. 2 displays how the axial force N affects the transverse deflection of the beam resting on an elastic foundation reacting to the moving distributed loads for both constant and variable loads with a fixed value of foundation modulus. The figure 4.2 reveals that the responses of amplitude decrease as axial force N increases.

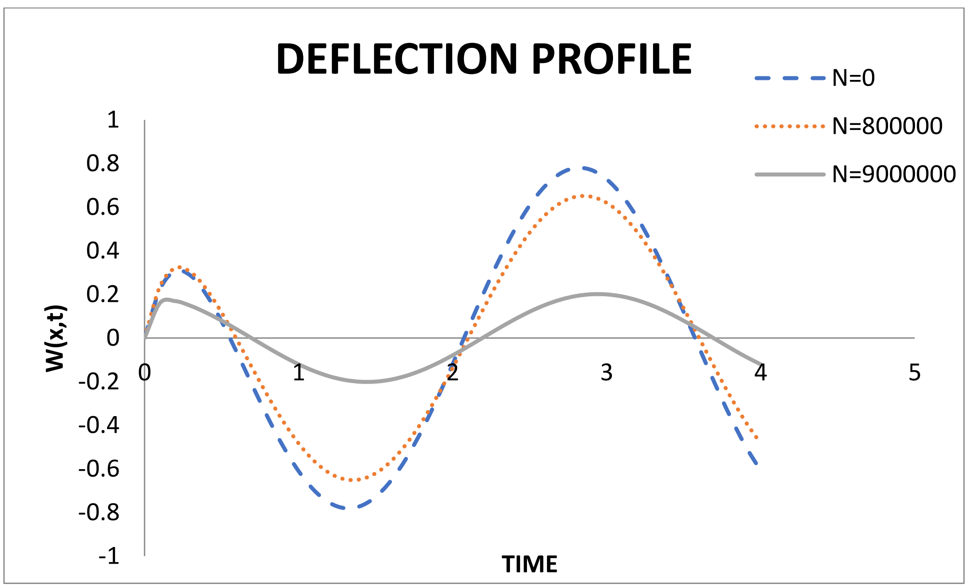

Figure 2: Displacement responses of a simply supported beam resting on an elastic foundation and subjected to an action of constant moving loads for various axial force, N and fixed value of foundation modulus K = 200,000.

Fig. 3 displaces the response of amplitude of a supported beam under moving loads for both constant and variable loads. It is observed that as the value of foundation modulus K, increases for a fixed value of axial force N, then the response amplitude decreases.

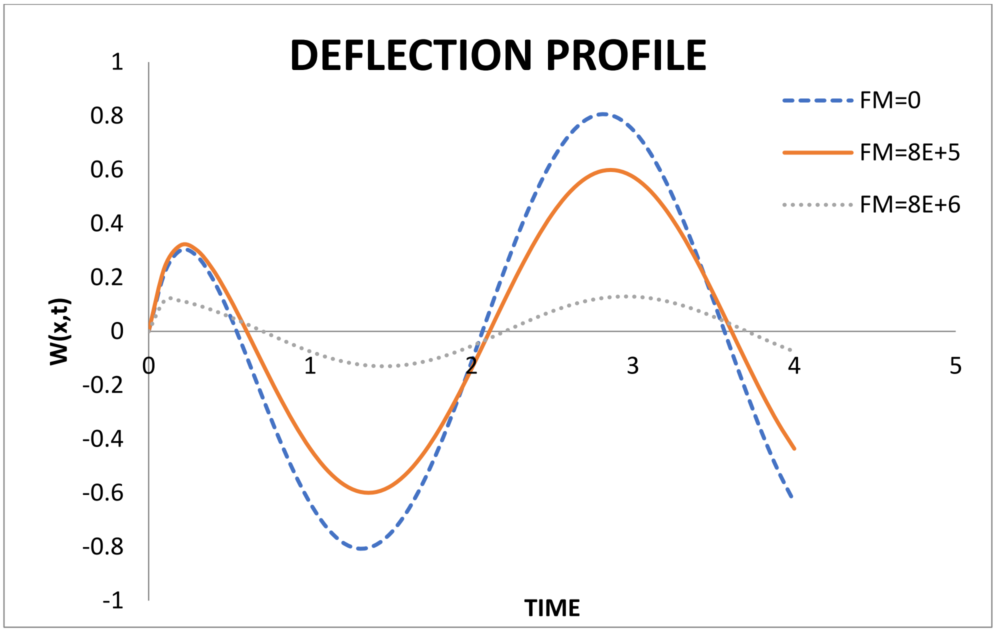

Figure 3: Displacement response of a simply supported beam resting on an elastic foundation and subjected to action of constant loads for various values of foundation modulus K and fixed value of axial force $N = 200{,}000$.

6 CONCLUSION

As discussed herein, the effect of variable axial force on the vibration of a thin beam subjected to both constant and variable loads has so far been investigated. These were assumed to have travelled with same loads under constant velocity.

We have so far looked into the fourth order partial differential equation governing the motion of the beam. The analytical solution of this PDE` is obtained by the use of a robust technique called the Galerkin's method and integral transformation method (the Laplace transform) in conjunction with convolution theorem.

This study has shown that as the magnitude of the axial force Increases, the deflection of the vibrating loaded beam decreases, and the higher the value of foundation modulus the lower the deflection profile of the beam for fixed value of axial force.

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