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The Effect of Two-Species Competition on a Lotka-Volterra Fishery Model in the Presence of Toxicity

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ABSTRACT

Competition is a crucial ecological interaction between organisms. Despite the belief that competition benefits more vigorous species since lesser species tend to die out owing to lack of resources, there are times when even stronger species populations collapse. In this research, two species of the fish population that are subject to compete for the same resources are presented and discussed. This research focuses on analyzing the influence of the competition coefficient between these two species that are exposed to toxic substances. To accomplish this, first and foremost, the competition coefficient is chosen as a bifurcation parameter. Then, several bifurcation graphs, phase planes, and time series are presented using mathematical computing software such as Maple, Matlab and XPPAUT. This research indicates that different competition coefficient rates can affect the dynamic behavior of both species. By using one-parameter bifurcation analysis, it is analyzed that there is an existence of a transcritical bifurcation point. Findings revealed that when the competition parameter passes the transcritical bifurcation point, the stability of the two species shifted from unstable saddle to asymptotically stable steady state.

Keywords: Bifurcation Analysis, Competition Interaction, Lotka-Volterra Model, Transcritical Bifurcation.

1 INTRODUCTION

Interaction between species refers to beneficial and harmful relationships between species that promote or restrict mutual population expansion and evolution. Competition, predation, parasitism, commensalism, and mutualism are all examples of interaction between species. Competition occurs when two or more species of the same or different species compete for resources. When food supplies are in short quantity relative to demand, much of the competition occurs. Nevertheless, organisms may compete for refuge, light, and substrate. Competition may be classified into numerous forms based on a variety of characteristics. Some of the competitive engagement falls into more than one of these categories. Based on taxonomic connection, competition is classified as interspecific and intraspecific. Essentially, interspecific competition occurs when two individuals from different species compete for the same resource. As a result, weaker species populations decline while more substantial species populations thrive. Interspecific competition is a crucial regulator of biological communities and a natural selection mechanism. Intraspecific competition happens when individuals of the same species strive for limited natural resources.

Nearly a century ago, Lotka and Volterra established a firm, if massively oversimplified, the theoretical basis for competition. Ecological theory has benefited tremendously from their competitive equations, which are an outstanding illustration of how mathematics can be used to represent important ecological phenomena. For several years, this phenomenon has always been an area of concern to many researchers [1-4]. According to Vasilyeva and Lutscher [5], interspecies rivalry serves as a powerful tool for defining species variety and shaping society. Gavina et al. [6] has done extensive research on the multi-species coexistence in competitive systems with high crowding efficiency. According to the findings of the research, some species want to coexist. Haque and Sarwardi [7] investigated the effect of toxicity on the harvested fisheries model of two competing fish species where both species released harmful toxic compounds. The Lotka-Volterra prey-predator model has been used, in which the model system's behavior is localized around any constant state. According to their results, the intensity of the toxins released by both species may change the phenomenological nature of the proposed system. Furthermore, the progressive rise in toxicity generated by both species has detrimental impacts on the environment and may ultimately lead to the extinction of both species. According to Swain and Chatterjee [8], estimating the competitive factors is a fundamental competition component, relying on the Lotka-Volterra population dynamical system. An intuitive method for estimating a competitive coefficient has been presented in these articles [9-12], which considers factors including resource availability, the proportional value of a given resource to a particular species, and resource energy consumption.

Many researchers have developed mathematical systems to investigate population or species interactions, such as prey-predator interactions. The toxicity substance and harvesting effort of the fisheries model or population, for example, were employed by several of the researchers in their studies [13-16]. Although numerous studies have been done, the research on the competition coefficients with toxicity substance employing bifurcation analysis remains limited. Consequently, to address the gap in the literature, this paper will discuss the bifurcation analysis of the coefficient of competition in situations when both species in a population are competing for the same resources. The aim of this research was to investigate the impact of varying the coefficient of competition on the dynamic behavior of both populations. The structure of this paper is represented as follows: Section 2 highlights the competition Lotka-Volterra model with the presence of toxicity substance. Following that, the presence of steady states, stability, and bifurcation analysis is presented. Section 3 reports on simulation performance, while Section 4 concludes with a final discussion and analysis of the current study's results in ecological terms.

2 MATERIAL AND METHODS

In general, the Lotka-Volterra model, often known as the interspecific competition model, is used to describe interspecific competition. The model is represented by

$$\frac{dX}{dt} = r_1 X\left(\frac{K-X}{K}\right), \qquad \frac{dY}{dt} = r_2 Y\left(\frac{L-Y}{L}\right). \tag{1}$$

The variable X and Y denotes the population sizes of the two species. The parameters r_1 and r_2 represent the growth rates of each species X and Y, respectively. The parameters K and L reflect how many species may be supported by a given amount of habitat. As a result of competition, we must add a new variable to the system (1), which now extends to another one:

$$\frac{dX}{dt} = r_1\mathrm{X}\left(1 - \frac{X}{K}\right) - \alpha XY, \qquad \frac{dY}{dt} = r_2 Y\left(1 - \frac{Y}{L}\right) - \beta XY. \tag{2}$$

The competitive coefficients for species Y and X, respectively, are determined by the parameters α and β as the competitive cofactors for X and Y. In the exclusion of the other species, α and β, they compete for using exterior sources such as food, allowing the species to expand as development demands.

The following model illustrates the impact of toxicity on the interactions of competing species. Both species produce toxic compounds that are detrimental to the population's reproductive system. Thus, the system (2) has been expanded to include toxic substances, represented as follows:

$$\frac{dX}{dt} = r_1\mathrm{X}\left(1 - \frac{X}{K}\right) - \alpha XY - \gamma_1 X^3 Y, \qquad \frac{dY}{dt} = r_2 Y\left(1 - \frac{Y}{L}\right) - \beta XY - \gamma_2 XY^2. \tag{3}$$

The toxicity coefficient is based on the parameters $\gamma_1$and γ_2. The term $\gamma_1 X^3$ refers to a form of functional response of the Y-species to the density of the X-species, which occurs through the production of toxic substances by the Y-species to prevent the X-species from using shared resources. The term $\gamma_2 Y^2$ has a similar meaning. Since $\frac{d}{dx}(\gamma_1 x^3) = 3\gamma_1 x^2 > 0$ and $\frac{d^2}{dx^2}(\gamma_1 x^3) = 6\gamma_1 x > 0$, the production of the toxic substance accelerates as the density of competing species increases. All parameters are presumptively positive.

In the competitive fisheries model, the populations of two species will compete, and the consequence of the competition would either be a win or loss. Population interaction is essential for the survival of a population. In this research, we investigated the interaction in the competition coefficient of two species with toxicity and densities of $X(t)$ and $Y(t)$, respectively. Then, MAPLE software is employed to perform the stability analysis on competition coefficient systems (3).

System (3) is assumed to be equal to zero in order to determine the steady states or critical points. By using the Maple software, the Jacobian matrix for system (3) is formed, which is represented as:

$$J_{XY} = \begin{bmatrix} r_1 - \left(\frac{2r_1}{K}\right) - \alpha Y - 3\gamma_1 X^2 Y & -\alpha X - \gamma_1 X^3 \\ -\beta Y - \gamma_2 Y^2 & r_2 - \left(\frac{2r_2}{L}\right) - \beta X - 2\gamma_2 XY \end{bmatrix}.$$

Therefore, the eigenvalues of each steady state are then determined in the above Jacobian matrix by substituting the value of steady states. When performing a stability analysis, the system is considered asymptotically stable if its eigenvalues are less than zero or negative. Otherwise, if all eigenvalues of the steady state are greater than zero or positive, the system is unstable. Finally, if the eigenvalues are in opposite sign, it is considered as unstable saddle steady state.

To conduct the bifurcation analysis, we applied the numerical software called XPPAUT to capture the transcritical bifurcation point on the bifurcation diagram. The parameters used in the model of a competitive fishery (3) are listed in Table 1.

Table 1 : The parameter used in competitive fishery system (3).

Parameter	Definition	Value
α	Competition coefficient of species Y	0.015
β	Competition coefficient of species X	0.01
r_1	Growth rate on X	3.7
r_2	Growth rate of Y	1.2
γ_1	Toxicant rate of X	0.0004
γ_2	Toxicant rate on Y	0.0003
K	Carrying capacity of species X	250
L	Carrying capacity of species Y	100

3 RESULTS AND DISCUSSION

3.1 Stability Analysis

In this section, the stability analysis of the population system's competition coefficient is investigated in further detail. Table 2 summarizes the results of the stability analysis.

Table 2: The stability analysis results.

Steady states	Eigenvalues	Stability results
$E_1 = (0,0)$	$\lambda_1 = 3.7, \lambda_2 = 1.2$	Unstable
$E_2 = (0,100)$	$\lambda_1 = 2.2, \lambda_2 = -1.2$	Unstable saddle
$E_3 = (250,0)$	$\lambda_1 = -1.3, \lambda_2 = -3.7$	Asymptotically stable
$E_4 = (118.352,0.347)$	$\lambda_1 = 0.3712, \lambda_2 = -6.0257$	Unstable saddle
$E_5 = (8.973,75.571)$	$\lambda_1 = -0.8578, \lambda_2 = -5.2525$	Asymptotically stable

As indicated in Table 2, there are five steady states with their corresponding eigenvalues. For $E_1(0,0)$, both eigenvalues have positive real roots, resulting in an unstable pattern. Whereas for the steady states $E_2(0,100)$ and $E_4(118.352, 0.347)$, the eigenvalues indicate an opposite sign. This leads to an unstable saddle pattern. For the steady states $E_3(250,0)$ and $E_5(8.973, 75.571)$, the eigenvalues have negative real roots. As a result, the asymptotically stable pattern is obtained.

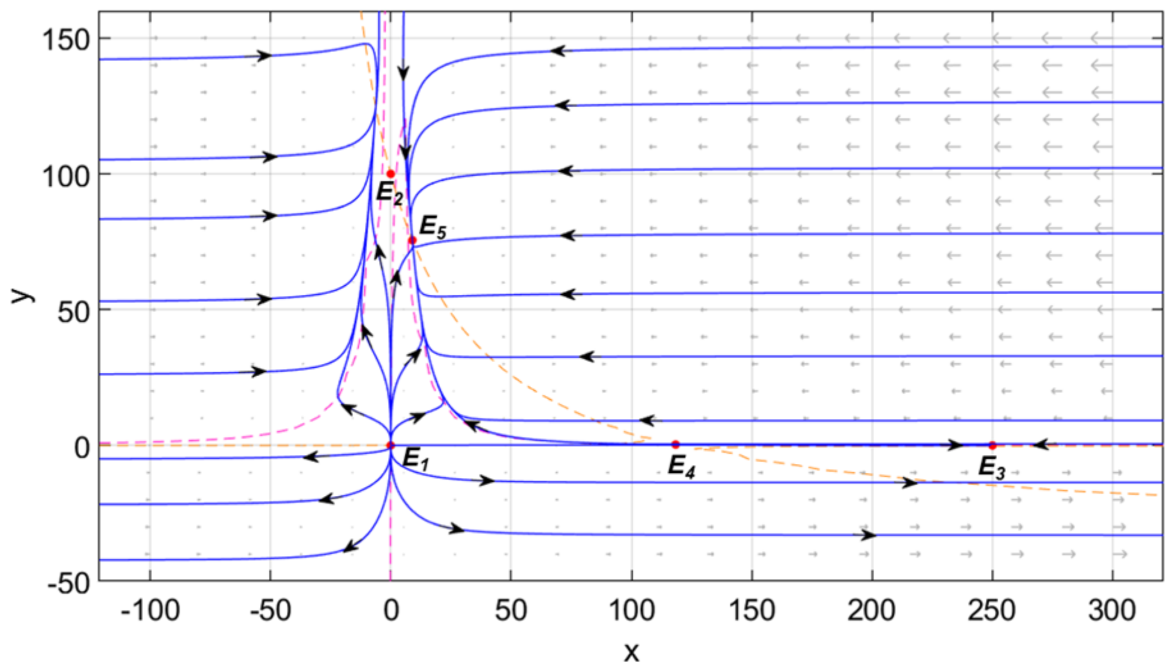


Figure 1: The phase portrait of fish species X and Y with parameter values presented in Table 1.

The phase portrait of X and Y species with a competition coefficient of $\alpha = 0.015$ are plotted using Matlab software, as shown in Figure 1. The steady state $E_1(0,0)$ exhibits an always unstable pattern, with trajectories leading from $E_1(0,0)$ to the stable steady states $E_3(250,0)$ and $E_5(8.973, 75.571)$. While the steady states $E_3(250,0)$ and $E_5(8.973, 75.571)$ show a continuously stable pattern. The steady state $E_4(118.352, 0.347)$ demonstrates an eternally unstable saddle pattern, with trajectories shifting and moving away from the steady state. Nevertheless, if we increase the competition coefficient for $E_2(0,100)$, there are switches in system stability, which will be discussed in the following section.

3.2 Numerical Bifurcation Analysis

With the help of XPPAUT numerical tools, the parameter variation technique is performed for numerical bifurcation analysis. The model system (3) was evaluated, and bifurcation diagrams were plotted by using numerical software, XPPAUT. For simplicity, the parameters were set to $\beta = 0.01, r_1 = 3.7, r_2 = 1.2, \gamma_1 = 0.0004, \gamma_2 = 0.003, K = 250, L = 100$ and the competition parameter α is varied from 0.036 to 0.038. Figures 2(a) and 2(b) indicate the switches steady state, E_2. The horizontal solid red and black lines represent the stable and unstable constant steady state, respectively. The dashed green vertical line indicates the transcritical bifurcation point at $\alpha = 0.037$.

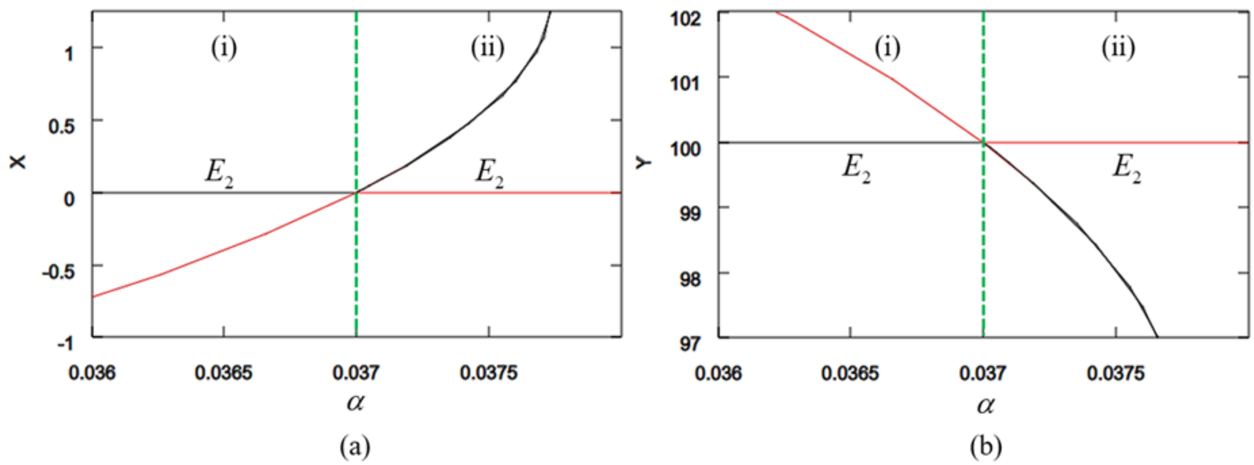


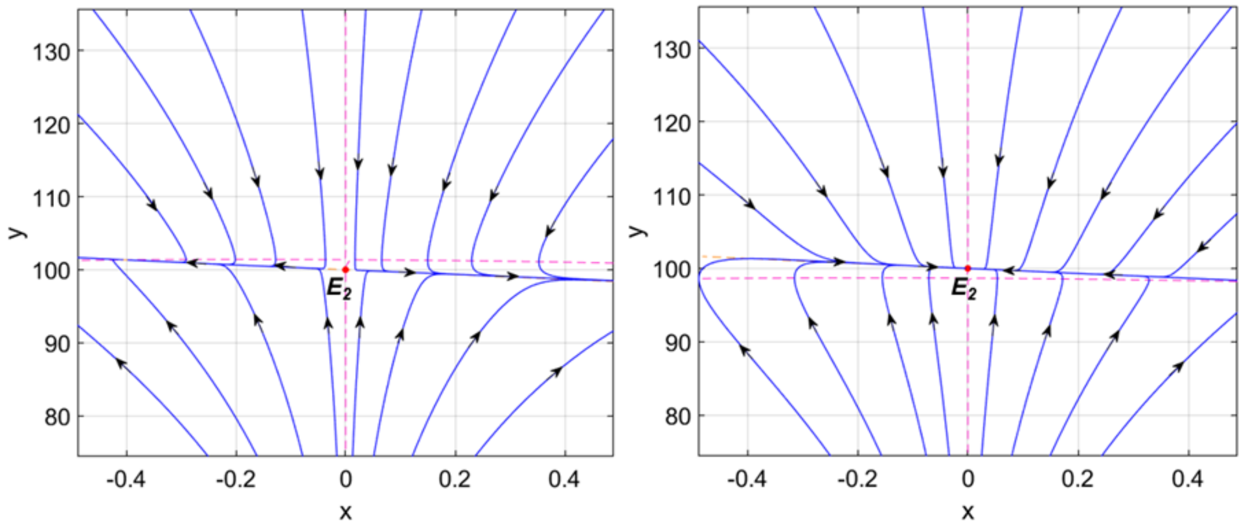
Figure 2: Bifurcation plot of competition fishery system (3) with respect to competition parameter α.

The transcritical bifurcation point demonstrates how the system stability switches for the population species. Using the numerical bifurcation analysis, we observed that the coefficient of competition changes its behaviour in various ways. When the competition parameter α is set to 0.0365, the steady state $E_2(0, 100)$ indicates an unstable saddle node. However, as the competition parameter $\alpha = 0.0375$ crosses the transcritical bifurcation point, the steady state $E_2(0, 100)$ switched from an unstable saddle to an asymptotically stable. The summary of stability and bifurcation analysis results is shown in Table 3.

Table 3: The stability and bifurcation analysis result with respect to competition parameters α

Bifurcation parameters	**Critical points**	**Eigenvalues**	**Stability results**
$\alpha = 0.0365$	$E_2 = (0{,}100)$	$\lambda_1 = 0.05 , \lambda_2 = -1.2$	Unstable saddle node
$\alpha = 0.037$		Transcritical bifurcation point	
$\alpha = 0.0375$	$E_2 = (0{,}100)$	$\lambda_1 = -0.05 , \lambda_2 = -1.2$	Asymptotically stable node

The phase portrait in Figure 3(a) depicts the first region (i) in Figures 2(a) and 2(b) before crossing the transcritical bifurcation point. In these regions, the steady state E_2 exhibits an unstable saddle pattern. Meanwhile, the phase portrait in Figure 3(b) depicts the second region (ii) in Figures 2(a) and 2(b) after crossing the transcritical bifurcation point. We can see the shifting in system stability pattern in these regions, where E_2 shifted to an asymptotically stable pattern.



(a) (b)

Figure 3: The phase portrait of species X and Y with bifurcation parameter (a) α = 0.0365 and (b) α = 0.0375.

3.3 Dynamical Behavior of Species Interaction as Competition Parameter Increases

In this section, four different competition parameter values are selected to observe the dynamics of the population as the competition parameter increases. The steady state $E_5(x*, y*)$ has been selected since it is always stable in the parameter range of interest. Hence, Table 4 summarizes the findings.

Table 4: The critical points for the chosen values of competition parameter α

Competition parameter values	$E_5\ (x*, y*)$
$\alpha = 0.0362$	$(3.355, 89.681)$
$\alpha = 0.0367$	$(3.004, 90.686)$
$\alpha = 0.0373$	$(2.439, 92.337)$
$\alpha = 0.0377$	$(1.718, 94.509)$

Table 4 illustrated that as the competition parameter rises, species X declines while species Y increases. Because they occupy the same niche, both species will face competition. As a result, one species will win while the other will lose. This scenario explains why the number of species X decreases as the competition parameter increases, owing to the increased competition for survival. As a result, species Y has survived, while species X will become locally extinct.

Next, the time series graphs are plotted in Maple in order to analyze the fundamental patterns and behaviours of the species population over time. Figure 4 depicts a time series plot of the species X and Y as competition parameter α rises. The dynamics of species X and Y are represented by the blue, red, green, and black curves as the competition parameters are $\alpha = 0.0362, \alpha = 0.0367, \alpha = 0.0373$

and $\alpha = 0.0377$. It is discovered that the population of species Y gradually grows at first until reaching a maximum and achieving stable steady state. This situation occurs since the impact of species Y on species X is more significant than the impact of species Y on species X.

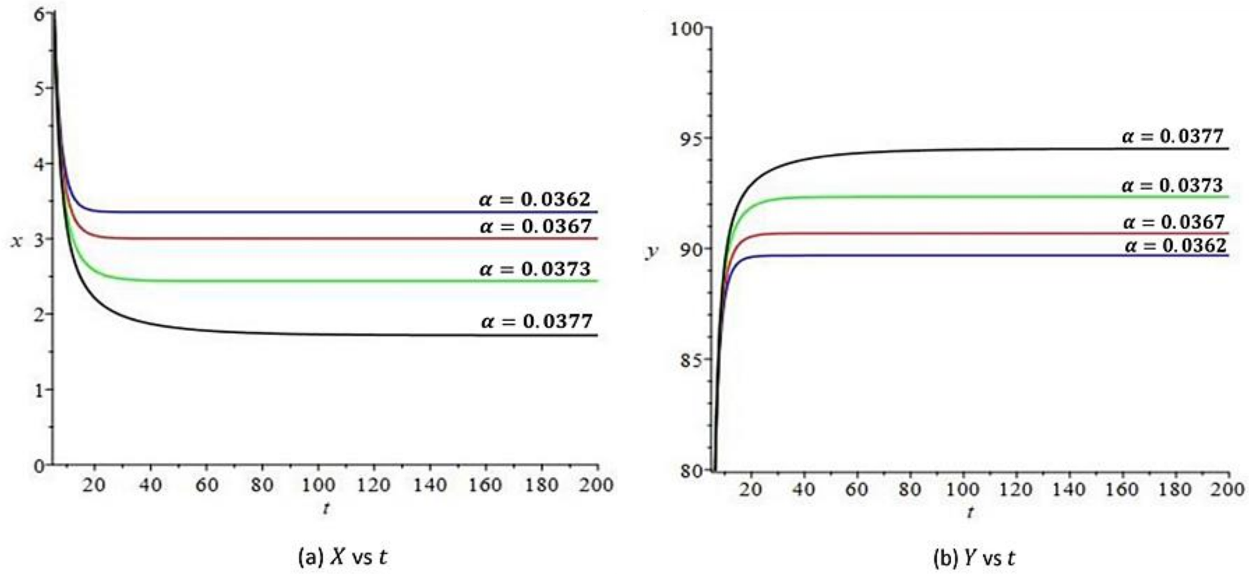


Figure 4: The time series plot of species X and Y with varying competition parameters α.

4 CONCLUSION

The competition coefficients involving two species in a population with toxicity have been considered and discussed in this study. The system's behavior was easily analyzed using a one-parameter bifurcation analysis based on this model. Our findings suggest that different rates of competition coefficient can influence the dynamical behavior of both species. The competition coefficient parameter α was treated as a bifurcation parameter since it is an essential parameter that influences the competition interaction model. As the bifurcation parameter exceeds the transcritical bifurcation point, the stability of steady states, E_2 switches. We also discovered that as the bifurcation parameter α increases, species X decreases while species Y increases. This phenomenon indicates that species X will become extinct locally while species Y will survive. Competition is not a static operation. Environmental disturbances, for example, can disrupt the ecosystem and eliminate the competitor's advantage. The model presented in this work is primarily concerned with what happens in population species and the stability of those populations when the competitive coefficient for each species is varied. Therefore, the bifurcation review of three toxic species' stability and dynamic activity is proposed for future studies.

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