AN ANALYTICAL STUDY OF FLUID TANK SYSTEM FOR REDUCING THE ROLLING OF A SHIP

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ABSTRACT

There have long been attempts to reduce the rolling of ships, prompted by a desire to increase comfort of travel, improve weapon control measures, extend the operation of aircraft, and so on. In many passenger and naval ships anti-rolling tanks are fitted in order to reduce rolling. Anti-rolling tank systems operate on the principle that a fluid, usually water, moves from one tank to another and generates a moment on the ship that counteracts the rolling motion. The principal aim of the present paper is to explain the characteristics of oscillation of fluid in anti-rolling tanks. Using the principle of conservation of energy, a detailed theoretical analysis is performed for the oscillation of water in two identical circular tanks connected by a narrow pipe. A new expression is derived for the frequency of oscillation of water as a function of the diameter of the circular tanks, the depth of water in the tanks, and the length and diameter of the connecting pipe. Subsequently, the expression for the frequency of oscillation of water is modified for the case of two identical rectangular tanks. Then it is analytically demonstrated how the aforesaid tank system can be used to neutralize the rolling motion of a ship. Based on the aforesaid theoretical analysis, a computer program is developed in FORTRAN 95 *and executed on the* DELL OPTIPLEX 780 *computer. The computational results are plotted in order to show how the frequency of oscillation of water is influenced by the diameter of the tanks, the depth of water in the tanks, geometry of the tanks and the length-to-diameter ratio of the connecting pipe. The present theoretical analysis is expected to be useful in the design of anti-rolling tanks which will effectively reduce the rolling motions of a ship.*

Keywords: *Anti-Rolling Tank, Connecting Pipe, Depth of Water, Diameter of Tanks, Frequency of Oscillation, Identical Tanks, U-Tank*

1.0 INTRODUCTION

A ship moving on the surface of a sea is almost always in oscillatory motion. There are six fundamental forms of ship motions. These motions are heaving, swaying, surging, rolling, pitching and yawing. All six motions are oscillatory in nature. The first three are linear motions, heaving being the vertical motion of the ship, whilst swaying and surging are respectively the athwartship motion and the fore-and-aft motion. Rolling is a rotation about a longitudinal axis, while pitching is a rotation about a transverse axis and yawing is a rotation about a vertical axis.

The motions of ships and the control of these motions have been the focal point of extensive research over the years. A ship in a seaway undergoes complex motions that may reduce the operational range of the ship and can be uncomfortable or even dangerous for the crew. In certain circumstances, the captain may be forced to alter the course or slow the ship to reduce large motions and thus reduce the combat readiness of naval vessels, cause the loss of containers on cargo vessels, and reduce the operational parameters of various ships. In extreme cases, the ship may capsize.

If a ship could possibly roll among waves unresisted, that is, without being subject to any resistance from wind or water and having her natural period of rolling synchronizing with that of the waves, the effect would be that the continued impulses given by the synchronizing waves would eventually capsize her, whatever might be her stability.

The great value of bilge keels in increasing resistance and reducing rolling has frequently been proved by experiments with models and experience with ships in service. Bilge keels produce good results upon large ships but the effect is equally apparent upon smaller craft with a short period.

Various other methods have been adopted in the past to reduce rolling. In some passenger and naval ships anti-rolling tanks were fitted and in a few a type of gyroscope has been fitted. Whatever method is tried, whether by external fittings such as bilge keels, fins, etc., or internal appliances, the principal object is to absorb the energy imparted to the ship by the wave motion, and the degree of violence of the rolling depends on the rate at which the resisting medium can keep pace in absorbing the energy which the waves are imparting to the ship.

There have been many types of internal tanks designed to produce counter-rolling moments, and they have been tried with varied degrees of success. No attempt will be made here to describe all these various systems. Rather, it would be more pertinent to single out one of the most successful of this type of anti-rolling device which was originally developed by Frahm in 1911. The famous Frahm tank consists basically of a U-shaped tank system transversely arranged from side to side (*Figure 1*). Actually, the system is composed of two reservoirs (one on the port side and the other one on the starboard side) connected by a horizontal duct. The horizontal duct is of a smaller sectional area than the reservoirs. The fluctuating column of water within the reservoirs comprises an oscillatory system, possessing a definite natural period of oscillation. For the effective operation of the

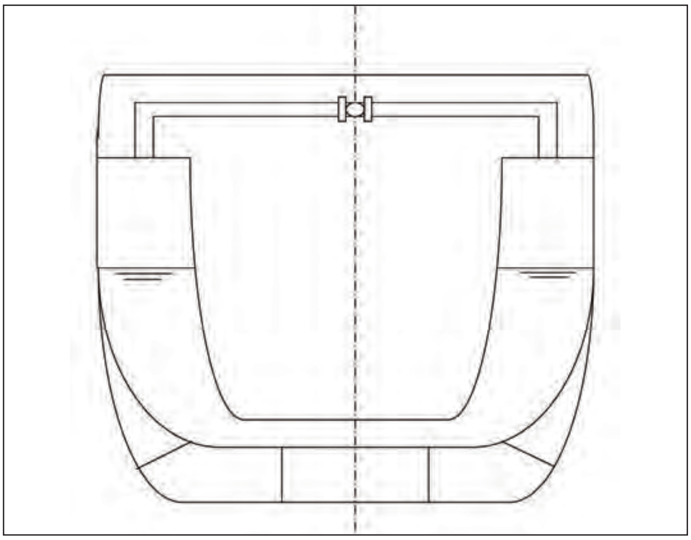

Figure 1: Schematic diagram of Frahm's tank

reservoirs, it is essential that the period of oscillation of the water column within the reservoirs be equal to the period of rolling of the vessel in still water. This is attained by the proper choice of dimensions for the section of the connecting water canal. As the vessel experiences a resonant, regular sea, there arises the phenomenon of dual resonance between the action of the waves and the rocking of the vessel, and between the motions of the vessel and the oscillations of the water in the reservoirs. The oscillations of the vessel lag, in phase, the oscillations of the exciting force by $\pi/2$, and the oscillations of the water within the reservoirs lag the oscillations of the vessel by $\pi/2$. As a result, the lag between the phase of the oscillations of the water and the phase of the exciting moment comprises π, and the stabilizing moment created by the oscillation of the water bears a sign opposite to that of the exciting moment of the wave action (Blagoveshchensky, 1962).

The reservoirs are connected across the top by an air line whose dimensions are also critical in affecting the period of the water in the system. This air line is equipped with valves which, upon adjustment, can control the amount of water transferred between reservoirs and the phase relation between the roll of the ship and the transfer of the water. The phase lag is adjusted so that when the ship is rolling to port the water in the tank is rolling to starboard. This type of tank contains no mechanical apparatus at all, although the water passes through structural constrictions as it passes from side to side, and is efficient only when the quantity of water in motion has been carefully calculated (Walton and Baxter, 1970).

In recent years, there has been a significant interest in U-tanks in relation to their potential for reducing the likelihood of ship parametric roll resonance. Holden *et al*. (2011) presented two novel nonlinear models of U-shaped anti-roll tanks for ships, and their linearization. The models were derived using Lagrangian mechanics. This formulation not only simplifies the modeling process, but also allows one to obtain models that satisfy energy-related physical properties. The proposed nonlinear models and their linearization were validated using model-scale experimental data.

There are many more researchers who have made important contributions towards the improvement of anti-rolling tanks. Some of them are Watanabe (1930), Vasta *et al*. (1961), Okada and Takagi (1965), Watanabe *et al*. (1966), Webster (1967), Takaishi (1971), Barr and Aukudinov (1977), Cox and Lloyd (1977), Webster *et al*. (1988), Honkanen (1990), Yamaguchi and Shinkai (1992), Lee and Vassalos (1996), Hsueh and Lee (1997), Treakle (1998), Gawad *et al*. (2001), Tanizawa *et al*. (2003), Moaleji and Greig (2007), Umeda *et al*. (2008), Marzouk and Nayfeh (2009), Winden (2009), Holden *et al*. (2011), etc.

Many researchers have studied the design of anti-rolling tanks. However, most of them concentrated on studying the performance of anti-roll tanks in damping the rolling motion of the ship. Little attention has been paid to the fluid motion inside the tank itself. Another important issue is the tank tuning. Proper tuning of the anti-rolling tank, to match the ship's natural frequency, is very important in reducing the rolling motion. This paper concentrates on the most familiar type, which is the U-tube passive tank as a mechanical absorber of rolling motion. A detailed mathematical analysis of the oscillation of water in two identical tanks connected by a narrow duct is presented. An expression is derived for the frequency of oscillation of water as a function of the diameter of the tanks, the depth of water in the tanks, and the length and diameter of the connecting duct. The present theoretical model is expected to be useful in the design of anti-rolling tanks which will effectively reduce rolling motions of a ship.

2.0 THEORETICAL ANALYSIS

Consider two identical circular cylindrical tanks of cross-sectional area A. The tanks are connected at their bottom by a pipe of length L and cross-sectional area a, as shown in *Figure 2*. The tanks are filled with water, and depth of water in each tank is h.

Suppose that oscillatory motion occurs in the tanks due to some external disturbance. In such a case, the water level will fall in one tank, with a corresponding rise in the other. The head of water, causing flow in the pipe, will be the difference between the two water levels.

Let the water level in Tank 1 fall down by y. Then the water level in Tank 2 will rise up by y. Let x be the horizontal displacement of water in the pipe. The volume of water that has passed from Tank 1 is Ay. From constancy of volume of water, the volume of water can be written as Equation (1)

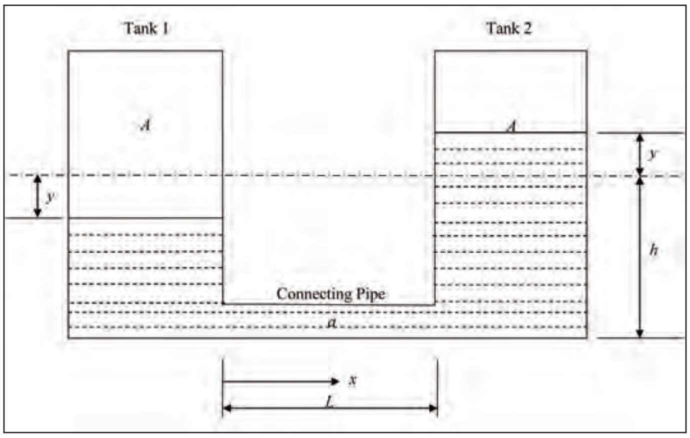

Figure 2: Natural oscillations of water in two identical tanks

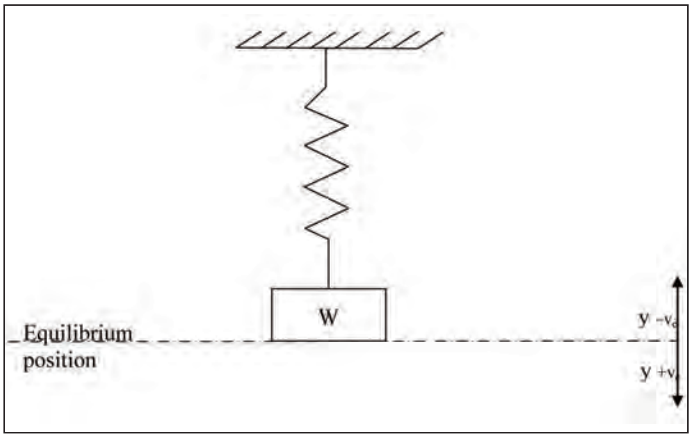

Figure 3: Simple spring-mass system

$$Ay = ax \tag{1}$$

$$\therefore x = \frac{Ay}{a} \tag{2}$$

Using the principle of conservation of energy, natural frequencies of oscillating systems can be conveniently determined. The simplest case of a free, natural oscillation is that of a weight suspended on a spring, as shown in *Figure 3*. When the weight is disturbed from its position of equilibrium, a restoring force is applied by the change in length of the spring and free oscillations are sustained by the elastic force in the spring alone. For the oscillating system, the energy is partly kinetic and partly potential. The kinetic energy (T) is associated with the velocity of the mass and the potential energy (U) is due to the strain energy stored in the spring. Thus,

$$T + U = \text{Total energy} = \text{constant} \tag{3}$$

Therefore, the rate of change of energy is zero.

$$\frac{d}{dt}(T+U)=0 \tag{4}$$

The kinetic energy of the mass is given by

$$T=\frac{1}{2}mv^2=\frac{1}{2}m\dot{y}^2 \tag{5}$$

The potential energy of the system is due to strain energy stored in the spring and is given by

$$U=\int_0^y ky\,dy=\frac{1}{2}ky^2 \tag{6}$$

where k is the spring constant.

The expression for the potential energy of the water oscillating in the tanks is as shown in Equation (7). In case of oscillation, the water falls in one tank, with a corresponding rise in the other. The restoring force (F) is due to the difference between the two water levels. Thus, it can be written as:

$$F = 2Ay\rho g \tag{7}$$

where ρ is the density of water.

In this problem there is no specific "spring", but still the force of gravity tends to restore the water level to an equilibrium position. Thus we have a "gravity spring", of which the spring constant by definition is the force per unit deflection.

$$F = ky$$

$$\therefore k=\frac{F}{y}=\frac{2Ay\rho g}{y}=2A\rho g \tag{8}$$

Equation (6) and Eqn (8) can be combined and rewritten as:

$$U=\frac{1}{2}(2A\rho g)y^2$$

$$\therefore U = A\rho g y^2 \tag{9}$$

Differentiating Eqn (9) with respect to time t,

$$\frac{dU}{dt}=2A\rho gy\frac{dy}{dt}=2A\rho gy\dot{y} \tag{10}$$

The kinetic energy of the oscillating mass of water is given by

$$T=\frac{1}{2}Ah\rho\left(\frac{dy}{dt}\right)^2+\frac{1}{2}Ah\rho\left(\frac{dy}{dt}\right)^2+\frac{1}{2}aL\rho\left(\frac{dx}{dt}\right)^2$$

$$\text{or,}\quad T=Ah\rho\left(\frac{dy}{dt}\right)^2+\frac{1}{2}aL\rho\left(\frac{dx}{dt}\right)^2 \tag{11}$$

Equation (2) and Eqn (11) can be combined and rewritten as:

$$T=Ah\rho\left(\frac{dy}{dt}\right)^2+\frac{1}{2}aL\rho\left(\frac{A}{a}\right)^2\left(\frac{dy}{dt}\right)^2$$

$$\text{or,}\quad T=Ah\rho\dot{y}^2+\frac{1}{2}aL\rho\left(\frac{A}{a}\right)^2\dot{y}^2$$

$$\text{or,}\quad T=\frac{1}{2}\rho\left[2Ah+aL\left(\frac{A}{a}\right)^2\right]\dot{y}^2 \tag{12}$$

Differentiating both sides of Eqn (12) with respect to time t,

$$\frac{dT}{dt}=\frac{1}{2}\rho\left[2Ah+aL(A/a)^2\right]2\dot{y}\ddot{y}$$

$$\text{or,}\quad \frac{dT}{dt}=\rho\left[2Ah+aL(A/a)^2\right]\dot{y}\ddot{y} \tag{13}$$

From Eqn (4), Eqn (10) and Eqn (13),

$$\rho\left[2Ah + aL(A/a)^2\right]\dot{y}\ddot{y} + 2A\rho g y\dot{y} = 0$$

Dividing the above expression by $\rho\dot{y}$ throughout,

$$\left[2Ah + aL(A/a)^2\right]\ddot{y} + 2Agy = 0 \tag{14}$$

$$\text{or, } \ddot{y} + \left[\frac{2Ag}{2Ah + aL(A/a)^2}\right]y = 0$$

$$\text{or, } \ddot{y} + p^2 y = 0 \tag{15}$$

where p is the natural frequency of oscillation in radian per second. Thus,

$$p^2 = \frac{2Ag}{2Ah + aL(A/a)^2}$$

$$\text{or, } p^2 = \frac{2g}{2h + L(A/a)} \tag{16}$$

Let D be the diameter of each tank and d the diameter of the connecting pipe. Therefore,

$$A = \frac{\pi D^2}{4}$$

$$a = \frac{\pi d^2}{4}$$

$$\therefore \frac{A}{a} = \frac{\pi D^2}{4} \times \frac{4}{\pi d^2} = \left(\frac{D}{d}\right)^2 \tag{17}$$

From Eqn (16) and Eqn (17), it can be written as:

$$p^2 = \left[\frac{2g}{2h + L(D/d)^2}\right] \tag{18}$$

$$\text{or, } p = \left[\frac{2g}{2h + L(D/d)^2}\right]^{1/2} \text{ rad/sec} \tag{19}$$

We know that

$$p = 2\pi N$$

where N is the frequency of oscillation in cycles per second.

$$\therefore\ N = \frac{1}{2\pi}\left[\frac{2g}{2h + L(D/d)^2}\right]^{1/2} \text{ cycles/sec} \tag{20}$$

Consider a very special case when the diameter of the connecting pipe is equal to the diameter of the twin tank, that is, $D = d$. Then Eqn (20) reduces to

$$N = \frac{1}{2\pi}\left[\frac{2g}{2h + L}\right]^{1/2} \text{ cycles/sec} \tag{21}$$

Then consider two rectangular tanks in place of the aforesaid circular cylindrical tanks. The length and breadth of each tank is D and the depth of water is h. The two tanks are connected by a pipe of diameter d. Therefore, it can be written as:

$$A = D \times D = D^2 \text{ and}$$

$$a = \frac{\pi d^2}{4}$$

$$\therefore \frac{A}{a} = \frac{D^2}{\pi d^2/4} = \frac{4}{\pi}\left(\frac{D}{d}\right)^2 \tag{22}$$

Combining Eqn (16) and Eqn (22), it can be shown that:

$$p^2 = \frac{2g}{2h + \frac{4L}{\pi}\left(\frac{D}{d}\right)^2}$$

$$\therefore p = \left[\frac{2g}{2h + \frac{4L}{\pi}\left(\frac{D}{d}\right)^2}\right]^{1/2} \text{ rad/sec}$$

$$N = \frac{1}{2\pi}\left[\frac{2g}{2h + \frac{4L}{\pi}\left(\frac{D}{d}\right)^2}\right]^{1/2} \text{ cycles/sec} \tag{23}$$

Consider a U-tank of uniform cross-sectional area A, as shown in *Figure 4*. The tank is filled with water. Let the total length of the water column be L', and the density of water be ρ. If the water oscillates back and forth, the mass in motion is $\rho AL'$. The force of gravity tends to restore the water level to an equilibrium position. Suppose that the water level in one arm of

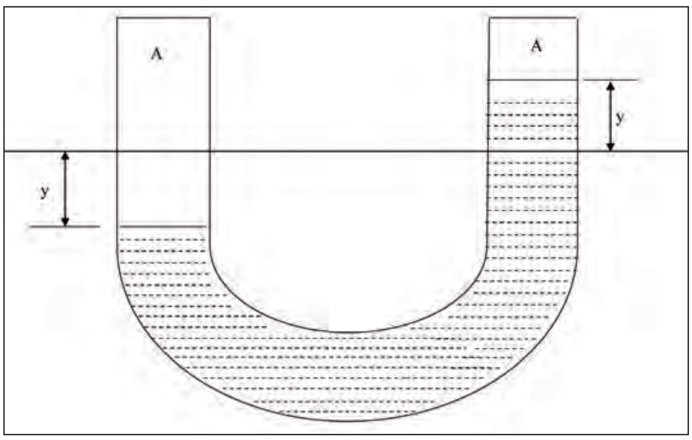

Figure 4: Natural oscillations of water in a U-tank

the U-tank is raised by y, then it will fall in the other arm by y. This gives an unbalanced weight of $2y$ water column. Thus the inertia force (Fi) and the restoring force (F) can be expressed as follows:

$$F_i = AL'\rho\ddot{y}$$

$$F = 2Ay\rho g$$

According to Newton's law of motion,

$$AL'\rho\ddot{y} = -2Ay\rho g$$

$$\text{or, } AL'\rho\ddot{y} + 2Ay\rho g = 0$$

$$\text{or, } \ddot{y} + \left(\frac{2g}{L'}\right)y = 0 \tag{24}$$

Equation (24) is of the well-known form:

$$\ddot{y} + p^2 y = 0$$

where $$p^2 = \frac{2g}{L'} \tag{25}$$

Thus the frequency of oscillation can be expressed as

$$p = \left(\frac{2g}{L'}\right)^{1/2} \text{rad/sec}$$
and (26)

$$N = \frac{1}{2\pi}\left(\frac{2g}{L'}\right)^{1/2} \text{cycles/sec} \tag{27}$$

It may be noted that Eqn (21) gives an expression for the frequency of oscillation of water in two identical circular tanks when the diameter of each tank is equal to that of the connecting pipe. Then Eqn (27) provides an expression for the frequency of oscillation of water in a simple U-tank. A careful observation reveals that there is no fundamental difference between these two expressions, although they are independently derived. This confirms the validity of the present theoretical analysis.

Let us now discuss how the aforesaid tank system can be used to neutralize the rolling motion of a ship. Suppose these two tanks are installed in a ship, one on the starboard and the other on the port side and are connected by a pipe. In case of rolling, the water will roll from side to side with a period similar to that of the ship but exactly opposite in phase, that is, when the ship is rolling to port the water in the tank will be rolling to starboard. In *Figure 5* the main spring-mass system (k_1, m_1) is the analogue of the ship and the small spring-mass system (k_2, m_2) is the analogue of the anti-rolling tanks. A sinusoidal wave exciting force $P \sin \omega t$ acts on the main mass m_1.

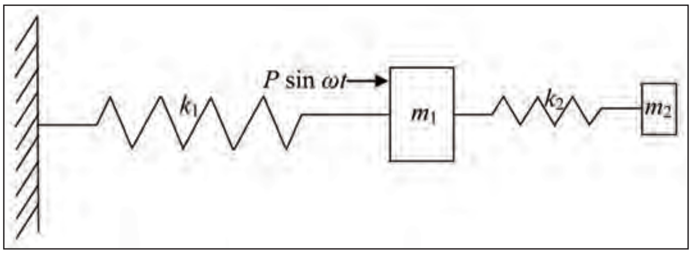

Figure 5: Equivalent spring-mass system of a ship under rolling-motion

The equation of motion can be written as

$$m_1 \frac{d^2 x_1}{dt^2} = k_2(x_2 - x_1) - k_1 x_1 + P \sin \omega t \tag{28}$$

$$m_2 \frac{d^2 x_2}{dt^2} = -k_2(x_2 - x_1) \tag{29}$$

The Cardinal Principle of forced vibration of a linear system is that a harmonic force acting on a linear system produces a harmonic motion of the same frequency but with a phase shift. Since the system under consideration is undamped, the phase angles can be taken as constant and equal to zero. Hence, the solution of Eqn (28) and Eqn (29) can be written in the form

$$x_1 = a_1 \sin \omega t \tag{30}$$

$$x_2 = a_2 \sin \omega t \tag{31}$$

Using Eqn (30) and Eqn (31) in Eqn (28) and Eqn (29), it can be shown that:

$$a_1(-m_1\omega^2 + k_1 + k_2) - a_2 k_2 = P \tag{32}$$

$$-a_1 k_2 + a_2(-m_2\omega^2 + k_2) = 0 \tag{33}$$

Defining

$$\left.\begin{aligned} \frac{k_1}{m_1} &= p_1^2 \\ \frac{k_2}{m_2} &= p_2^2 \\ \frac{P}{k_1} &= x_s \\ \frac{m_2}{m_1} &= \mu \end{aligned}\right\} \tag{34}$$

and rewriting Eqn (32) and Eqn (33) as

$$a_1\left(1 + \frac{k_2}{k_1} - \frac{\omega^2}{p_1^2}\right) - a_2\left(\frac{k_2}{k_1}\right) = x_s \tag{35}$$

$$a_1 - a_2\left(1-\frac{\omega^2}{p_2^2}\right)=0 \tag{36}$$

Solving Eqn (35) and Eqn (36),

$$\frac{a_1}{x_s}=\frac{\left(1-\frac{\omega^2}{p_2^2}\right)}{\left[\left(1-\frac{\omega^2}{p_2^2}\right)\left(1+\frac{k_2}{k_1}-\frac{\omega^2}{p_1^2}\right)-\frac{k_2}{k_1}\right]} \tag{37}$$

$$\frac{a_2}{x_s}=\frac{1}{\left[\left(1-\frac{\omega^2}{p_2^2}\right)\left(1+\frac{k_2}{k_1}-\frac{\omega^2}{p_1^2}\right)-\frac{k_2}{k_1}\right]} \tag{38}$$

Assuming the reduction factor

$$\Delta=\left[\left(1-\frac{\omega^2}{p_2^2}\right)\left(1+\frac{k_2}{k_1}-\frac{\omega^2}{p_1^2}\right)-\frac{k_2}{k_1}\right] \tag{39}$$

In Eqn (37) and Eqn (38), the magnification factors of the main mass ($\underline{a}_1$) and the anti-rolling tank ($\underline{a}_2$) can be written as:

$$\underline{a}_1=\frac{a_1}{x_s}=\frac{\left(1-\frac{\omega^2}{p_2^2}\right)}{\Delta} \tag{40}$$

$$\underline{a}_2=\frac{a_2}{x_s}=\frac{1}{\Delta} \tag{41}$$

Equation (40) gives us the condition for reducing the rolling motion of the ship to zero. If p_2 is made equal to ω (that is, the frequency of the anti-rolling tank is made equal to the frequency of the wave exciting force), then a_1 becomes zero. Thus the rolling motion of the ship does not exist any more.

If p_2 is made equal to ω then Eqn (34) and Eqn (39) lead to:

$$\Delta=-\frac{k_2}{k_1}=-\frac{p_2^2}{p_1^2}\frac{m_2}{m_1}=-\frac{\omega^2}{p_1^2}\mu \tag{42}$$

Combining Eqn (41) and Eqn (42):

$$\underline{a}_2=-\frac{p_1^2}{\omega^2\mu} \tag{43}$$

3.0 RESULTS AND DISCUSSION

A detailed mathematical analysis of the oscillations of water in two identical tanks connected by a narrow pipe is presented in the preceding section. Based on the aforesaid theoretical analysis, a computer program is developed in FORTRAN 95 and executed on the DELL OPTIPLEX 780 computer. It may be noted that all computations are performed initially for a pair of identical circular tanks and then for a pair of identical rectangular tanks whose length and breadth are equal to the diameter of the circular tanks. The computational results are then plotted in order to show the characteristics of oscillation of water in circular as well as rectangular tanks.

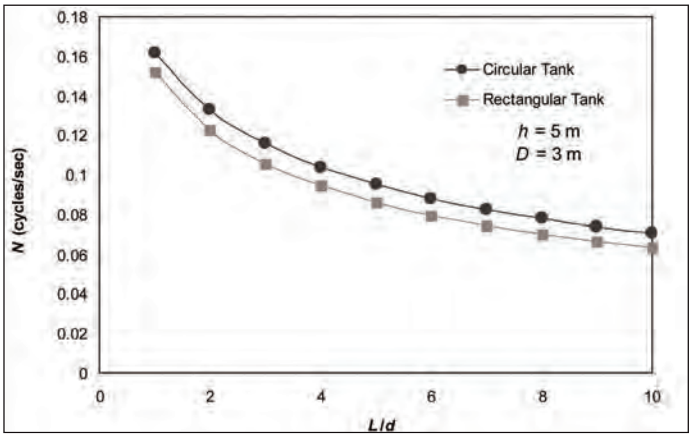

Figure 6: Variation of the frequency of oscillation (N) with the length-to-diameter ratio (L/d) of the connecting pipe when the depth of water (h) is 5 m and the diameter of each tank (D) is 3 m

Figure 6 shows the variation of the frequency of oscillation of water (*N*) with the length-to-diameter ratio (*L/d*) of the connecting pipe. The computations are performed assuming that the diameter of each tank (*D*) is 3 m and the depth of water (*h*) is 5 m. It is observed that the frequency of oscillation of water attains a high value (0.16 cycles/second) when the length-to-diameter ratio of the connecting pipe is equal to unity, and then gradually decreases as the length-to-diameter ratio increases. The frequency of oscillation is 0.08 cycles/second when *L/d* is 10. The frequencies of oscillation of water in the rectangular tanks are always found to be lower than those in the circular tanks. In the case of rectangular tanks, *N* is equal to 0.15 cycle/second when *L/d* is equal to 1 and *N* is equal to 0.07 cycle/second when *L/d* is equal to 10. It may be noted that Attwood *et al*. (1953) have provided a very useful table containing the rolling periods of different types of ships namely, battleship, fleet aircraft carrier, cruiser, destroyer and liner. Typical periods of rolling are found to vary from 7.5 seconds to 24 seconds. In other words, the frequency of rolling varies from 0.042 to 0.133 cycle/second. Thus the frequencies of oscillation

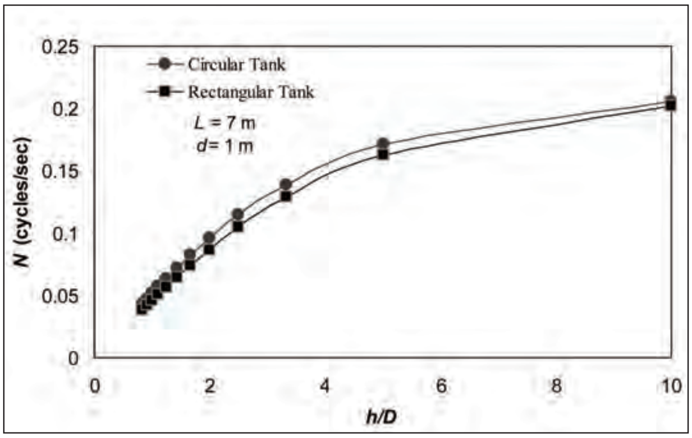

Figure 7: Variation of the frequency of oscillation (N) with the non-dimensional depth of water (h/D) in the tanks when the length of the connecting pipe (L) is 7 m and its diameter (d) is 1 m

of water obtained from the present theoretical analysis are found to be in complete agreement with the range of frequencies of rolling reported by Attwood *et al*.

Figure 7 shows the variation of the frequency of oscillation of water (N) with the non-dimensional depth of water (h/D) in the tanks. The computations are performed assuming that the length (L) and diameter (d) of the connecting pipe are 7 m and 1 m respectively. It is observed that the frequency of oscillation of water increases almost linearly with the increase of non-dimensional depth of water in the tanks. This observation on the variation of the frequency of oscillation of water holds good for both the circular and the rectangular tanks. However, the frequencies of oscillation of water in the circular tanks are always found to be higher than those in the rectangular tanks.

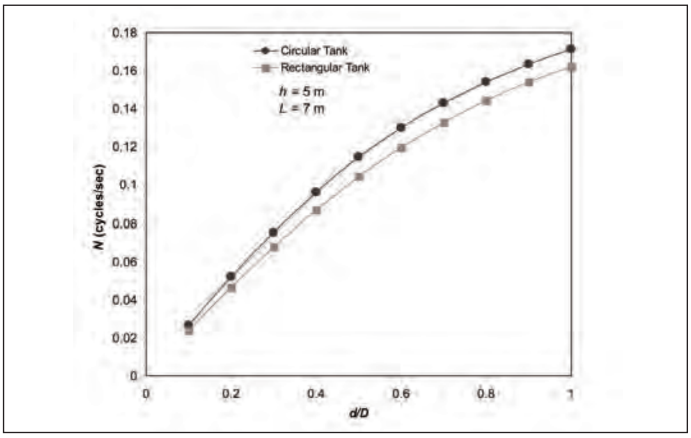

Figure 8: Variation of the frequency of oscillation (N) with the non-dimensional diameter (d/D) of the connecting pipe when the depth of water (h) is 5 m and length of the pipe (L) is 7 m

Figure 8 shows the variation of the frequency of oscillation of water (N) with the non-dimensional diameter (d/D) of the connecting pipe. The computations are performed assuming that the depth of water (h) is 5 m and the length of the connecting pipe (L) is 7 m. It is observed that the frequency of oscillation of water rapidly increases with the increase of non-dimensional diameter and attains the maximum value when d/D = 1. It is interesting to note that the maximum value of frequency of oscillation of water is obtained when the diameter of the connecting pipe is equal to that of the twin tank. This statement is equally true for both the circular and the rectangular tanks. However, the frequencies of oscillation of water in the circular tanks are always higher than those in the rectangular tanks.

Figure 9 shows the variation of the frequency of oscillation of water (N) with the non-dimensional length of the connecting pipe (L/D) when the depth of water (h) is 5 m and the diameter of the connecting pipe (d) is 1 m. It is observed that the frequency of oscillation of water decreases with increase in non-dimensional length of the connecting pipe. The variation is, however, not linear. The frequency of oscillation decreases rapidly up to L/D = 6 and after that the rate of reduction is quite low. The frequency curves for both the rectangular tanks and the circular tanks exhibit similar trend. However, it must be noted that the frequency of oscillation of water in circular tanks is always greater than that in rectangular tanks.

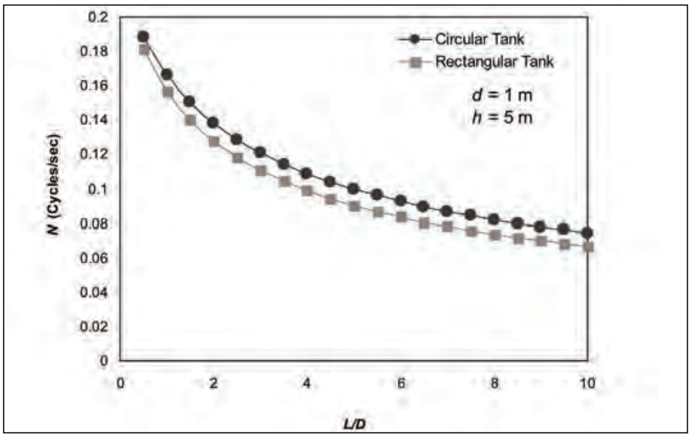

Figure 9: Variation of the frequency of oscillation (N) with the non-dimensional length of the connecting pipe (L/D) when the depth of water (h) is 5 m and the diameter of the connecting pipe (d) is 1 m

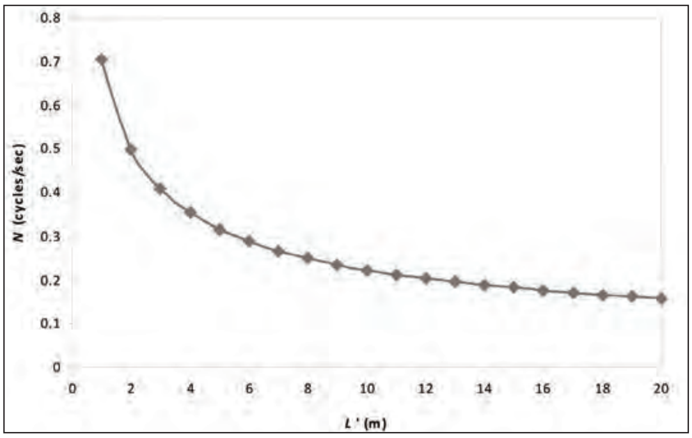

Figure 10: Variation of the frequency of oscillation (N) with the total length of water column (L') in the U-tank

Figure 10 shows the variation of frequency of oscillation of water (N) with the total length of the water column (L') in the U-tank. It is observed that the frequency of oscillation of water decreases at a sharp rate with the increase of the total length of the water column up to L' = 15 m. However, the frequency of oscillation does not remarkably change when the length of the water column exceeds 15 m.

Equation (40) provides the condition for the complete elimination of rolling motion of a ship. If the frequency of the anti-rolling tank (p_2) is made equal to the frequency of the wave force (ω), then a_1 becomes zero. Thus the rolling motion of the ship does not exist any more.

Figure 11 is plotted to show the relationship between the magnification factor of the anti-rolling tank ($\underline{a}_2$) and the mass ratio (μ) when the frequency of oscillation of the ship is equal to the frequency of the wave (exciting) force and the frequency of the anti-rolling tank. Actually, when the frequency of the wave (exciting) force equals the frequency of the ship, then resonance occurs. This is the worst case scenario and the main subject of interest. This figure shows the relationship between the magnification factor of the anti-rolling tank ($\underline{a}_2$) and the mass ratio (μ). From this figure it is evident that as μ increase, i.e., the mass of anti-rolling tank increases, $\underline{a}_2$ decreases. But a large mass overboard will reduce the stability. So, one must be careful in choosing an appropriate mass of anti-rolling tank.

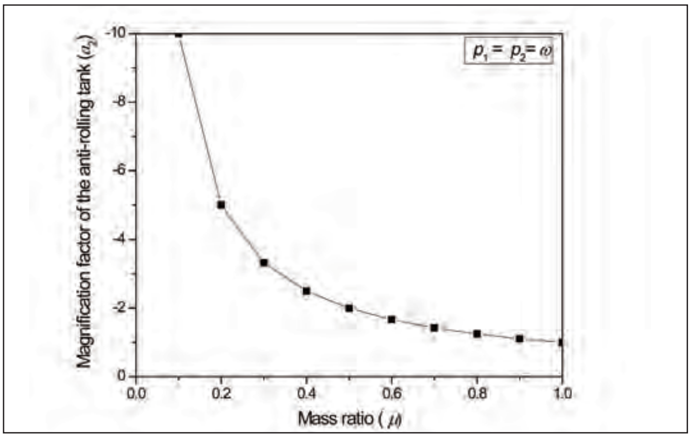

Figure 11: Relationship between the magnification factor of the anti-rolling tank ($\underline{a}_2$) and the mass ratio (μ)

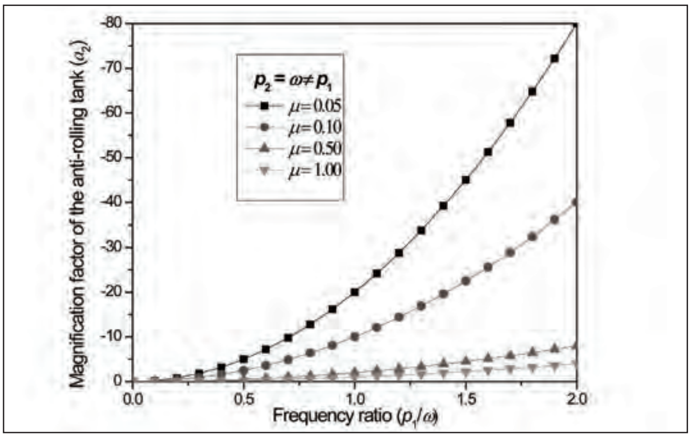

Figure 12: Relationship between the magnification factor of the anti-rolling tank ($\underline{a}_2$) and the frequency ratio (p_1/ω)

Figure 12 is plotted for the case when the frequency of the ship is not equal to the frequency of the exciting force and the frequency of the anti-rolling tank is tuned to that of the exciting force. It shows the relationship between the magnification factor of the anti-rolling tank ($\underline{a}_2$) and the frequency ratio (p_1/ω) for different values of the mass ratio (μ). For a given value of μ, $\underline{a}_2$ increases with the increase of p_1/ω. For a given system p_1 or ω can not be changed, so $\underline{a}_2$ can be controlled only by changing the mass ratio (μ).

NOMENCLATURE	
A	cross sectional area of the tank
a	cross sectional area of the connecting pipe
a_1	amplitude of oscillation of the ship
a_2	amplitude of oscillation of water in the U-tank
$\underline{a}_1$	magnification factor of the ship
$\underline{a}_2$	magnification factor of the U-tank
D	diameter of the circular tank
d	diameter of the pipe connecting the two tanks
F	restoring force
Fi	inertia force
g	acceleration due to gravity
h	depth of water in the tanks
k	spring constant
k_1	spring constant of the ship
k_2	spring constant of the U-tank system
L	length of the pipe connecting the two tanks
L'	total length of the water column in the U-tank
N	frequency of oscillation of water (cycles/sec)
m_1	mass of the ship
m_2	mass of the water in the U-tank
P	exciting force
p	frequency of oscillation of water (rad/sec)
p_1	frequency of oscillation of the ship
p_2	frequency of oscillation of water in the U-tank
T	kinetic energy
U	potential energy
x	coordinate measured along the horizontal axis
x_s	ratio of the exciting force to the spring constant of the ship
y	vertical displacement of the water level
Δ	reduction factor
μ	ratio of the mass of water in the U-tank to the mass of the ship (m_2/m_1)
ρ	mass density of water
ω	frequency of the exciting force

3.0 CONCLUSION

This paper presents an analytical study of the oscillation of water in two identical tanks connected by a narrow pipe. It also analyses how this fluid tank system can be used for reducing the rolling of a ship. The following conclusions can be drawn from the present work:

(a) Using the principle of conservation of energy, a detailed theoretical analysis is performed for the oscillation of water in two identical circular tanks connected by a narrow pipe. A new expression is derived for the frequency of oscillation of water as a function of the diameter of the circular tanks, the depth of water in the tanks, and the length and diameter of the connecting pipe. Subsequently, the expression for the frequency of oscillation water is modified for the case of two identical rectangular tanks.

(b) The frequency of oscillation of water in the circular tanks is always found to be higher than that in equivalent rectangular tanks. But the frequency curves exhibit similar trend for both circular and rectangular tanks.

(c) The frequency of oscillation of water decreases with increase in length-to-diameter ratio (L/d) of the connecting pipe.

(d) It is observed that the range of frequencies of oscillation of water is 0.08 cycle/second to 0.16 cycle/second for the circular tanks whereas the range is from 0.07 to 0.15 cycle/second for the rectangular tanks. The aforesaid range of frequencies is in complete agreement with the values of rolling frequency of ships presented by Attwood *et al*. (1953).

(e) The frequency of oscillation of water increases almost linearly with the increase in non-dimensional depth of water (h/D) in the tanks.

(f) The frequency of oscillation of water increases sharply with the increase of the non-dimensional diameter (d/D) of the connecting pipe. The twin tank system behaves like a U-tank when $d = D$.

(g) The frequency of oscillation of water decreases at a sharp rate with the increase of the total length water column (L') in the U-tank.
(h) If the frequency of the anti-rolling tank (p_2) is made equal to the frequency of the wave force (ω), the rolling motion of the ship is reduced to zero.
(i) The amplitude of oscillation of water in the anti-rolling tank a_2 can be controlled by changing the ratio of the mass of water in the tank to the mass of the ship (m_2/m_1).
(j) The present theoretical model is expected to be useful in the design of anti-rolling tanks which will effectively reduce rolling motions of a ship. ■

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