

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Prior to stating my research, it is important to have a deep understanding on the existing pages of microstrip array antenna. The main sources of information for the dissertation are books, journal, theses and dissertations and the internet. There are three major areas of reading in the literature review. Which are antenna design, methods for improving performance of microstrip patch antenna and related simulation software. These chapters are divided into two major sections which is research paper literature review and basic antenna theory.

2.2 Research Paper Literature Review

In order to start the project, the first step is to study the research papers that have been performed previous by other researchers. The paper that has related to this title are chosen and studied. With the help from this literature review, it gives more clear understanding to perform this project

2.2.1 Design of a Multi-layer Transmit / Receive Dual-Frequency Microstrip Patch Antenna Array [8]

In this article, Bhartia, P. and Bahl, I. has describes the design of a multi-layer, dual linearly polarized microstrip patch array antenna with resonant frequencies at 28.9 and 29.4 GHz respectively. The array contains 256 elements (16 x 16). The patches are printed on the top surface of a two-layer back-to-back Duroid 5880 substrate with a ground layer in between. The dielectric constant of the Duroid is 2.2, the thickness of each layer is 0.025 cm, and the thickness of the adhesive is 0.0038 cm. Figure 2.0 shows a 2 x 2 sub-array. The vertical polarization is excited through a corporate feed structure on the top surface of the top layer as shown by the solid lines in Figure 2.0. The horizontal polarization is similarly excited by a corporate feed (dashed lines in Figure 2.0) printed on the bottom surface of the bottom layer. This corporate feed electromagnetically excites the slots provided in the common ground plane and they in turn couple their energy to the radiating patches. A differential 180° phase shifter is provided for alternate elements of each column on the bottom layer and each row on the top layer as shown in Figure 2.0 so that all the elements of the array are excited in phase. The element dimensions are 0.314 x 0.325 cm and the spacing between elements is 0.76 cm. The slot dimensions are 0.0225 x 0.1875 cm.

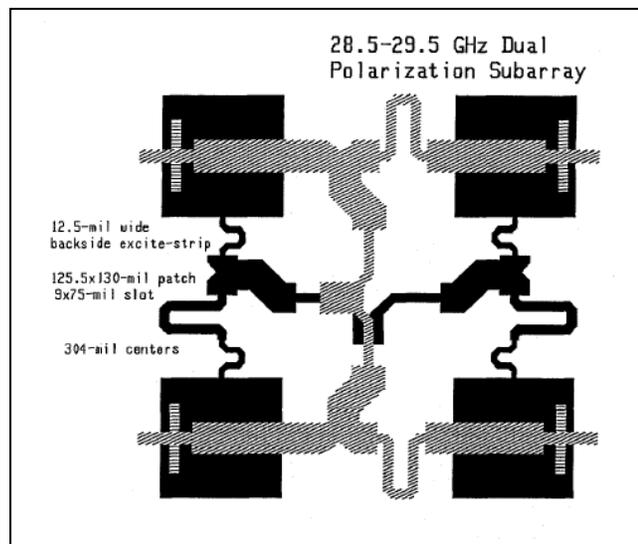


Figure 2.0: 2 X 2 Dual-Polarized Sub-arrays.

2.2.2 New Considerations in the Design of Microstrip Antennas [9]

In this paper it describes about printed microstrip antennas and arrays that have limitations in terms of bandwidth and efficiency, all imposed by the very presence of the dielectric substrate. Microstrip arrays printed on a very thin film and separated from the ground-plane by foam were successfully designed; however, the fabrication difficulties associated with the use of foam considerably increases the fabrication cost. In this paper, a new concept is presented. Rather than using superstrata geometry, the “printed circuit” is etched out of metal and supported in “strategic points” by (metallic or nonmetallic) posts. The main motivation for this work was to obtain large microstrip arrays, which exhibit a higher efficiency than conventional ones, and can be fabricated using inexpensive large quantity production techniques. However, this technology was also used to develop many new types of microstrip antennas. Microstrip elements and arrays based on this technology were designed and fabricated at L, S, and C bands.

2.2.3 Microstrip Patch Antenna Receiving Array Operating in the Ku Band [10]

In the interest of the consumer, the cost of manufacturing and the dimensions of each array will be kept to a minimum. Therefore, all the microstrip antenna elements will be stripline fed enabling each element to share a common ground plane. This will allow all the antenna elements to be fabricated on a single dielectric sheet. The dielectric substrate, separating the ground plane and the antenna patches, has a dielectric constant (ϵ_r) of 2.2 and a thickness of 0.051 cm (20 mil). Single rectangular patches will first be fabricated to determine the proper dimensions for a patch operating in the Ku band (1.7 GHz - 12.7 GHz). Once the dimensions of a single patch were determined an 8x8 and 16x16 planar arrays were fabricated using duplicates of the aforementioned patch. Each array was first tested without a parasitic patch layer. The E-plane and H-plane patterns were measured using the Far-Field Antenna Testing Range at NASA Lewis. Using an HP8510 Network Analyzer, the S_{11} parameter versus frequency was measured and the percent bandwidth for

each array was determined from these measurements (where $S_{11} < 10$ dB). Once these tests were made, parasitic patches were introduced for each array and investigations were made on the effects they have on antenna gain. An estimate of either array's antenna gain can be made by comparing to the gain of a standard gain horn.

2.2.4 Microstrip Antenna with Parasitic Elements [11]

In this paper, the main purpose of this invention to provide an improved antenna which has a beam with a main lobe tilted approximately twenty degrees from a normal to a missile, such while retaining the low profile, low volume attributes. Another object of this invention is to provide an improved antenna which is readily adaptable to flush-mounting on a missile. These and other objects of this invention are attained generally by providing a microstrip patch antenna with parasitic elements flush-mounted to the side of the missile to produce an antenna beam with main lobe directed approximately twenty degrees off the normal to the missile, such antenna here including a driven patch antenna, a reflector element and two director elements, with the reflector and director elements being parasitic elements in combination with appropriate connector elements.

2.2.5 Guidelines for Design of Electromagnetically Coupled Microstrip Patch Antennas on Two-Layer Substrates [12]

In this paper, the graphical guidelines for design of electromagnetically coupled square and circular microstrip antennas are given. Substrates composed of two different dielectric layers are considered. The analysis is extended to electrically thick substrates. Given the required resonant frequency and the bandwidth, material parameters are selected. Patch dimension and the optimal position of the feedline are obtained thereafter from the provided graphs. The design data were computed by applying the method of moments in

the spectral domain to solve the integral equation for the currents on the patch and portion of the microstrip feed line. The integral equation was formulated using the appropriate dyadic Green function for the grounded multilayered slab.

2.2.6 A High-Gain Microstrip Patch Array Antenna Using a Superstrate Layer [13]

In this article it explain about the comparison of the gain of a single patch with a superstrate withthat of a single patch without a superstrate and reports the experimental results for a 2×8 array antenna with a superstrate and a 4×8 array antenna without a superstrate. A dielectric superstrate layer above a microstrip patch antenna has remarkable effects on its gain and resonant characteristics. This paper experimentally investigates the effect of a superstrate layer for high gain on microstrip patch antennas. The gain of antennas with and without a superstrate is measured and found that the gain of a single patch with a superstrate was enhanced by about 4 dBi over the one without a superstrate at 12 GHz. The impedance bandwidths of a single patch with and without a superstrate for VSWR < 2 were above 11%. The designed 2×8 array antenna using a superstrate had a high gain of over 22.5 dB and a wide impedance bandwidth of over 17%.

2.2.7 Planar Square and Diamond Microstrip Patch Array Antennas for Dual-Polarization Operation [14]

In this literature review, it presented two compact low-cost planar microstrip array antennas devised to achieve dual-polarization capability. The arrays are composed of five square patches with a central aperture-coupled element feeding the other patches through a straight forward network of resonant microstrip lines. Dual-edge and dual-corner feeding arrangements for the external patches are considered as alternatives. By appropriately driving the two antennas at their input ports, dual-linear as well as circular polarization of the

radiated field can be accomplished. Preliminary numerical investigations of the proposed antenna configurations are reported and the relevant performances are comparatively assessed.

2.3 Basic Antenna Theory

In this section, basic antenna theory is discussed, followed by an understanding on the basics of microstrip line and microstrip antenna. It is important to understand this basic knowledge before proceeding to a more in-depth understanding of how microstrip antennas operate.

Antenna is the interface between transmitter lines and space. The basic concept to understand the antennas is that they are passive devices where they require no supply voltage to operate. Besides that, they do not alter nor process RF signals especially to amplify the energy of RF signals. In the other words, an antenna only converts an electromagnetic signal to an electrical signal at a receiver or electrical signal to an electromagnetic signal at a transmitter.

If they are 100% efficient, they radiate no more power than is delivered to their input terminal. This is because all the energy of the signal is absolutely absorb. Antenna can be defined as a conductor. Here the system of conductors used for radiating electromagnetic energy into space or for collecting electromagnetic energy from space.

2.4 Antenna Properties

The basic properties that are used to describe the performance of the antenna include impedance, Voltage Standing Wave Ratio (VSWR), bandwidth, radiation patterns, 3 dB beamwidth, gain and finally polarization.

2.4.1 Impedance

The input impedance of the antenna must identically match the characteristic impedance of the transmission line in order to archive maximum energy transfer between a coaxial transmission line and an antenna. If the input impedance of the antenna does not match with the characteristic impedance of the transmission line, a reflected wave will be generated at the antenna terminal and travel back towards the energy source. This reflection of energy results in a reduction in the overall system efficiency. This loss in efficiency will occur if the antenna is used to transmit or receive energy.

2.4.2 VSWR

VSWR (Voltage Standing Wave Ratio) is the ratio between the maximum voltage and the minimum voltage along the transmission line. The VSWR is given by the equation shown below.

$$\text{VSWR} = \frac{1 + \Gamma_L}{1 - \Gamma_L} \quad (2.1)$$

$$\text{Where} \quad \Gamma_L = \frac{Z_{\text{in}} - Z_{\text{out}}}{Z_{\text{in}} + Z_{\text{out}}} \quad (2.2)$$

The VSWR indicate that how closely or efficiently an antenna’s terminal input impedance is matched to the characteristic impedance of the transmission line. The larger the number of VSWR, the greater the mismatch between the antenna and the transmission line.

In many systems, the antenna is required to operate with a VSWR better than 1.5:1 with 50 Ω impedance. Therefore an antenna VSWR should be closely to the 50 Ω of the antenna impedance. The system are perfectly match if the VSWR equals to 1:1 where there is no power reflected and all the energy are absorbed at their input terminal.

2.4.3 Bandwidth

Bandwidth is refers to the range of frequency that the antenna will radiate effectively where the antenna meets a certain set of specification performance criteria. When antenna power drops to ½ (3 dB), the upper and lower extremities of these frequency have been reached and the antenna no longer performs satisfactorily. An antenna that operates over a wide frequency range and still maintain satisfactory performance must have compensating circuits switched into the system to maintain impedance matching. The relationship between VSWR and the bandwidth is shown in Figure 2.1.

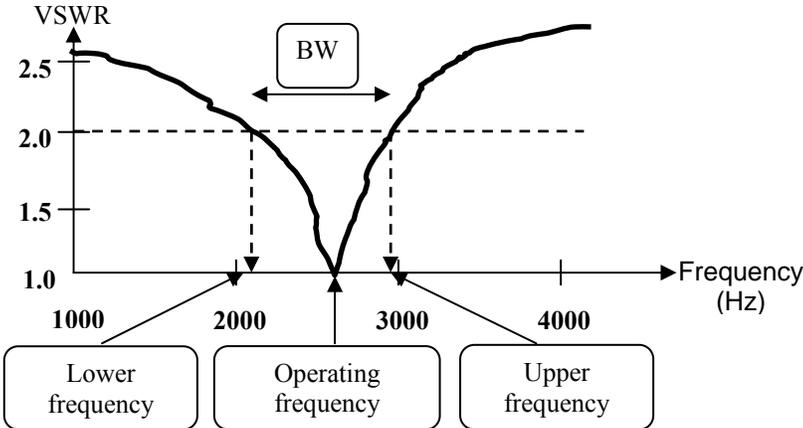


Figure 2.1: Frequency Response for the Antenna

2.4.4 Radiation Pattern

An antenna's radiation pattern is the characteristics that most affect system coverage and performance. The radiation pattern of antenna simply describes how an antenna focuses or directs the energy it radiates or receives. All antennas do not radiate more total energy than is delivered to their input connector. Antenna radiation pattern are typically presented in the form of a polar plot for a 360° angular pattern in one of two sweep planes and it is presented on a relative power dB scale as shown in Figure 2.2.

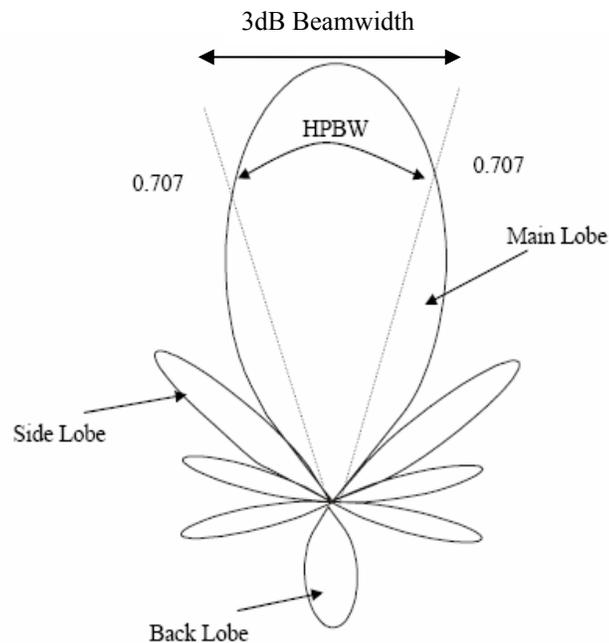


Figure 2.2: Radiation Pattern and 3dB Beamwidth

From Figure 2.2, the field strength drops to 0.707 (measure in $\mu(V/m)$) of the maximum voltage at the center of the lobe. These points are known as the half power points or Half Power Beamwidth (*HPBW*). It is defined as “in a plane containing the direction of the maximum of a beam, the angle between two directions in which the radiation intensity is one-half of the beam” [1].

The other radiation pattern properties of significance are the antenna's side lobes, back lobes and front to back ratio (f/b). In practice, it is impossible to eliminate antenna side lobes and back lobes completely. Antenna side and back lobes affect antenna and system performance in several ways. First, energy delivered to or received by side and back lobes is from a direction other than the intended region of coverage and is therefore wasted. At a transmitter, energy delivered to side and back lobes may be directed towards other receive systems causing interference. Then at a receiver, energy from other transmit sites may be received through the side and back lobes causing interference within the system.

Main lobe is the radiation lobe containing the direction of the maximum radiation. For side lobe, it is a radiation lobe in any direction other than the intended lobe direction. It is usually adjacent to the main lobe and occupies the hemisphere in the direction of the main beam. A radiation lobe which axis is at 180° with respect to the main beam. It usually refers to a small lobe that occupies the hemisphere in a direction opposite to a main lobe are the definition for back lobe.

2.4.5 Gain

In the definition of the antenna gain level, an isotropic antenna is typically used as a reference standard. An isotropic antenna is a theoretical antenna radiating energy equally in all directions of space. This antenna has a directivity of 0 dB since energy is distributed equally in all directions. The gain of an antenna must equal to its directivity if the antenna is 100% efficient. The gain of an antenna is therefore equal to the directivity less any losses in the antenna. Gain which is referred to an isotropic radiator is expressed as "dBi". Gain which is referred to a half wavelength dipole, which has an isotropic gain of 2.16 dBi. Gain which is defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction" [1].

2.4.6 Polarization

The polarization of an antenna in a given direction is defined as “the polarization of the wave radiated by the antenna”. Polarization in the direction of maximum gain is taken when the direction is not stated. In details, polarization refers to the direction in space of the E-Field (electric vector) portion of the electromagnetic wave being radiated by the transmission system. Low frequency antenna is usually vertically polarized because of ground effect (reflected waves) and physical construction methods while high frequency antennas are generally horizontally polarized. All radiated waves are generally defined as elliptically polarized which have two components that lie in the same plane [1].

The two most known and common cases of elliptical polarization are circular in which the two E-Field components are equal in magnitude and oriented at 90° to each other (90° out of phases) and linear in which the wave has a single E-Field component. These two E-Field components may be of different strength and are oriented at different angles.

2.5 Microstrip Patch Antenna

A microstrip antenna in its basic form consists of a metallic patch on a ground substrate, which is an extension of a microstrip transmission line. Advantages of microstrip antenna include low profile, conformable to planar and non-planar surfaces, inexpensive and simple to manufacture, and they are versatile in terms of resonant frequency, polarization, pattern and impedance. There are disadvantages of the microstrip antenna as well. They include low efficiency, high Q which results in a narrow frequency bandwidth, low power, poor scan performance and spurious feed radiation.

2.5.1 Introduction to Microstrip Patch Antenna

A Microstrip Patch Antenna consists of a very thin patch that placed a small fraction of a wavelength above a conducting ground plane. The patch and the ground plane are separated by a dielectric. The patch conductor is normally copper and can assume any shape but for this project half rectangular patch is used and this simplifies the analysis and performance prediction. The patches are usually photo etched on the dielectric substrate and the substrate is usually non-magnetic. The relative permittivity of the substrate is an important parameter to consider. It is because relative permittivity will enhances the fringing fields that account for radiation. This type of antenna is characterized by its length L , width w , and thickness h , as shown in Figure 2.3.

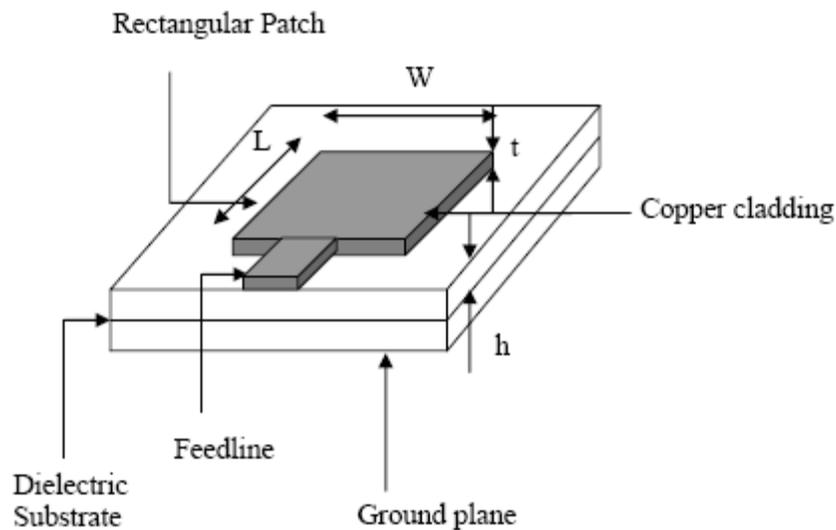


Figure 2.3: Rectangular Microstrip Patch Antenna.

2.5.2 Operation of Microstrip Patch Antenna

Consider a basic rectangular patch microstrip antenna first as shown in Figure 2.3; the different shapes of microstrip antennas will be discussed later. Assuming a voltage input at the feedline, when operating in the transmitting mode, current is excited on the feedline

to the patch and a vertical electric field between the patch and the ground plane [14]. So therefore the patch element resonates at certain wavelength and this result in radiation.

2.5.3 Shape of Microstrip Antenna

Microstrip antennas are often referred as patch antennas because the radiating element is normally a patch. This radiating patch comes can be in different shapes such as square, rectangular, circular, triangular and many others. However, the two commonly used shapes; the rectangular and circular patch, are describe below.

A simple rectangular patch antenna is shown in Figure 2.3. This is the one of the basic design of the microstrip antenna and for this project a rectangular patch antenna will be design.

Another shape of a microstrip antenna is the circular patch antenna and is shown in Figure 2.4.

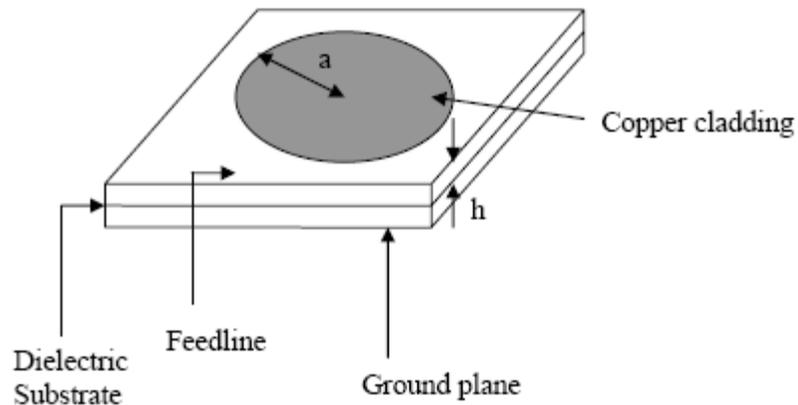


Figure 2.4: Circular Patch Antenna

The summarizations of basic operation for microstrip patch antenna's parameters are discussed below.

The antenna substrate dielectric constant is given as ϵ_r .

The ϵ_r is primarily affects the bandwidth and radiation efficiency of the antenna. The lower the permittivity will give a wider impedance bandwidth and reduce the surface wave excitation.

The antenna substrate thickness is given as h .

The substrate thickness affects bandwidth and coupling level. A thicker substrate results in wider bandwidth, but less coupling for a given aperture size.

L is the microstrip patches length.

The length of the patch radiator determines the resonant frequency of the antenna.

The microstrip patches width is given as w .

The width, w of the patch affects the resonant resistance of the antenna, with a wider patch giving a lower resistance.

Microstrip patch antenna has a several advantages such as thin profile, light weight, portable and conformability to a shaped surface. But the disadvantage is the narrow bandwidth. But today there are so many methods to improve the bandwidth of microstrip patch antennas such as tuning the resonant frequencies. There are four methods to tune the resonant frequencies which are using Varactor diodes, shorting pins, adjustable air gap and optically controlled pin diodes.

2.6 Factors Affecting Microstrip Design

In this subtopic, it will highlight the factors affecting microstrip design. Types of microstrip discontinuity and losses will be discussed here. Feeding and analysis methods will also be covered. Last but not the least, the criteria for the selection of substrate will also be touched on. There are five main factors that affect the microstrip design, microstrip discontinuity, fringing effect, feeding techniques, substrate selection and patch dimension.

2.6.1 Microstrip Discontinuity

When there is an abrupt change in the dimension of the strip conductor, a change in the electric and magnetic field distributions will occur. This is known as microstrip discontinuity. Microstrip discontinuities include open end, gaps, step in width, bends, T and cross junctions. In this section, open end, step in width and right angle bend will be discussed as there are more relevant to the design of the antennas.

2.6.1.1 Open End

Open end discontinuity is common in a number of circuits such as matching subs and parallel coupled filters. The equivalent circuit of open end is represented by an excess capacitance or by an extension in length, Figure 2.5.

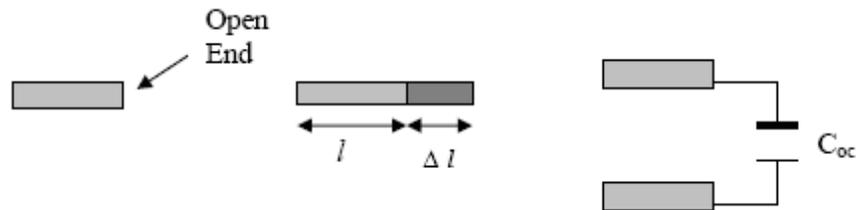


Figure 2.5: Open End Equivalent Circuit

2.6.1.2 Microstrip Line Step Discontinuity

Microstrip line step discontinuity occurs when two lines having different width are joined together. It is common in designing matching transformer, couplers and filters. The equivalent circuit consists of shunt capacitance and series inductance as shown in the Figure 2.6.

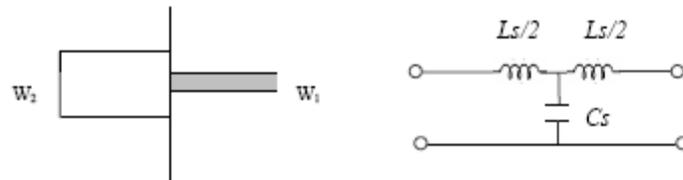


Figure 2.6: Open End Equivalent Circuit

2.6.1.3 Right Angle Bends

Bends such as right angle bends are normally incorporated into the circuit for introducing flexibility in the circuit layout. The equivalent circuit is made up of equal series inductance, L_b and a shunt capacitance, C_b and is shown in Figure 2.7. C_b has accuracy to within 5% for $2.5 \leq \epsilon_r \leq 4$ and $0.1 \leq w/h \leq 5$. The accuracy for L_b is about 3% for $0.5 \leq \epsilon_r \leq 2$. By using a method known as chamfering, discontinuity reactance can be reduce as shown in Figure 2.7.

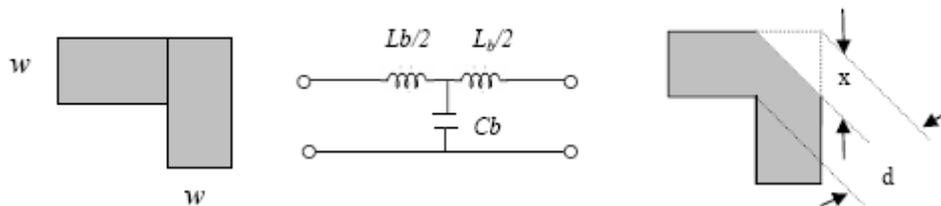


Figure 2.7: Equivalent Circuit for Bend

2.6.2 Fringing Field

Considering the rectangular patch antenna using microstrip line feed, the radiation from microstrip patch antenna can be found from the patch spaced of a wavelength above a ground plane as shown in Figure 2.8 and Figure 2.9.

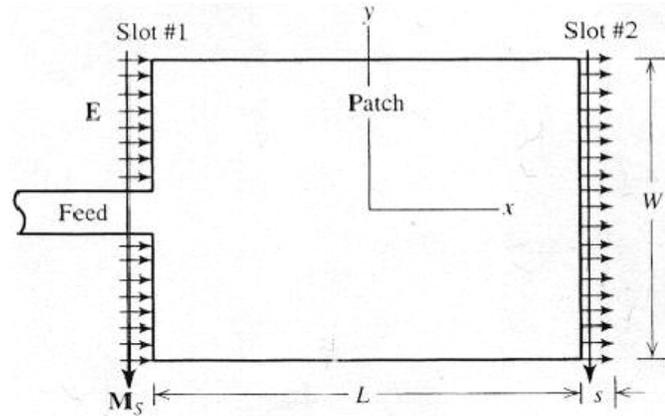


Figure 2.8: Top View of Patch and a Demonstration of the Electric Fringing Fields that is Responsible for Radiation

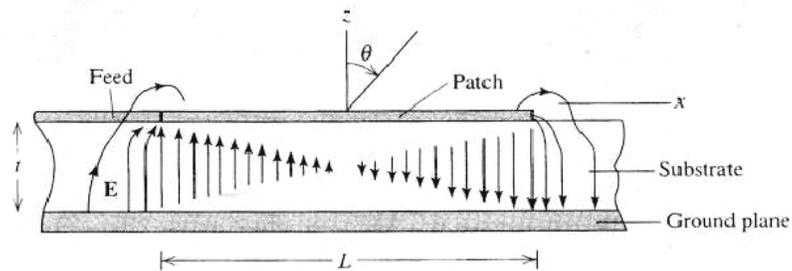


Figure 2.9: A Side View of Microstrip Patch Antenna and the Demonstrating the Electric Fringing Fields

When the length, L of the patch is about half wavelength long ($\lambda/2$), then the radiation generates from the fringing fields between the patch edge and the ground plane [8]. Here it is assumed that no variations of electric fields along the width and the thickness

of the antenna. As a summary, the fringing field is an effect of the electrons congregated at the surface of the conductor especially at the edge. So that, most of the radiated power is distributed from the edge of the patch.

2.6.3 Feeding Techniques.

The feed guides the electromagnetic energy from the source to the region under the patch. Some of this energy crosses the boundary of the patch and radiates into space. There are several methods to feed the radiated patch. There are coaxial feed, microstrip feed (feed line), aperture coupling feed and proximity coupling feed. With different feeding methods, different antenna properties of performance are constructed such as bandwidth, radiation pattern, polarization, gain and impedance. In practice, the coaxial or microstrip feed is the most common feeding method.

2.6.3.1 Coaxial Probe Feed

Coaxial line feed [1] is a feeding method in which that the inner conductor of the coaxial is attached to the radiation patch of the antenna while the outer conductor is connected to the ground plane. The advantages of coaxial probe feed are, it easy to fabricate and match and it also has low spurious radiation. However by using this technique, it will produce narrow bandwidth. It is also more difficult to model especially for thick substrates ($h > 0.02 \lambda_o$).

2.6.3.2 Aperture Coupling

Aperture coupling [1] consists of two different substrates separated by a ground plane. On the bottom side of the lower substrate there is a microstrip feed line whose energy is coupled to the patch through a slot on the ground plane separating the two substrates. This arrangement allows independent optimization of the feed mechanism and the radiating element. Normally the top substrate used is a thick low dielectric constant substrate while for the bottom substrate; it is a high dielectric substrate. The ground plane, which is in the middle, isolates the feed from the radiation element and minimizes interference of spurious radiation for pattern formation and polarization purity.

The configuration has advantages over its direct contact counterparts. Unlike the edge-fed patch antenna, independent optimization of the feed and antenna substrates can be achieved. Unlike the probe-fed configuration, no vertical interconnects are required, simplifying the fabrication processes and also adhering to the conformal nature of printed circuit technology. However, alignment issues can be important and also multilevel fabrication processes are typically required. A multilayered antenna can create other problems. The presence of small gaps between the layers of dielectric can significantly alter the input impedance nature of the antenna, especially at high frequencies, where these gaps appear larger electrically. Also, the material required to bond the layers could play a significant role in the performance of the antenna. If the bonding material is lossy and is located near, for example, the slot, the efficiency of the antenna is reduced.

2.6.3.3 Proximity Coupling

The second form of non-contact fed patches created to overcome the shortcomings of the direct contact fed patches is the proximity-coupled patch. The microstrip antenna consists of a grounded substrate where a microstrip feed line is located. Above this material is another dielectric laminate with a microstrip patch etched on its top surface. There is no

ground plane separating the two dielectric layers. The power from the feed network is coupled to the patch electromagnetically, as opposed to a direct contact. This is why this form of microstrip patch is sometimes referred to as an electromagnetically coupled patch antenna.

Proximity coupling has the largest bandwidth, has low spurious radiation and is easy to fabricate. However the fabrication is difficult. Length of feeding stub and the width-to-line ratio of the patch is used to control the match [1].

2.6.3.4 Microstrip Line Feed

Microstrip line feed is one of the easier method to fabricate as it is a just a conducting strip connecting to the patch and therefore can be consider an extension of the patch. It is simple to model and easy to match by controlling the inset position. However the disadvantage of this method is that as the substrate thickness increases, surface waves and spurious feed radiation increases which limit the bandwidth to typically 2-5 %. For this project, microstrip line feed will be used.

2.6.4 Substrate Selection

As the substrate is one of the most important materials in the design of microstrip, its selection must be treated with care. The properties of the substrates affect the dimensions and characteristics of the microstrip directly. The key parameters to take note when selecting a substrate are dielectric constant, loss tangent and substrate thickness.

Dielectric constant is the ratio of the amount of electrical energy stored in an insulator, when a static electric field is imposed across it, relative to vacuum. A high

dielectric constant will result in a smaller patch size which is good in most circuits though it will result in tighter fabrication tolerances. A high dielectric constant will also generally reduce the bandwidth. Loss tangent refers to the ratio of imaginary permittivity with real permittivity. A low loss tangent substrate will increase antenna efficiency and reduces microstrip losses. Thick substrate thickness will maximize bandwidth and efficiency but when it is too thick, surface-wave excitation will occur [15].

So therefore it is best to choose a substrate with the lowest possible dielectric constant if space permit, with a low loss tangent. Table 2.0 below lists some common substrate materials used in the design of microstrip antennas [5].

Table 2.0: Common Substrate Materials for Microstrip Antenna

Material	Dielectric Constant, ϵ_r	Loss Tangent, $\tan \delta$
Teflon (PTFE)	2.1	0.0005
Rexolite 1422	2.55	0.0007
Noryl	2.6	0.0011
FR4	4.7	0.0190
Alumina	9.8	0.0003

2.6.5 Patch Dimensions

The following calculations are based on the transmission line model of Derneryd [5]. The width w , of radiating edge is given by the equation

$$w = \frac{c[(\epsilon_r + 1)/2]^{-\frac{1}{2}}}{2fo} \quad (2.3)$$

The length L is slightly less than a half wavelength in the dielectric. The calculation of the precise value of the dimension L of the square patch is carried out by an iteration procedure. To obtain an initial value of L , equation (2.4) is used.

$$L = \frac{c}{2f_o \sqrt{\epsilon_r}} \quad (2.4)$$

In order to improved value of L , equation (2.5) is used.

$$L = \frac{c}{2f_o \sqrt{\epsilon_{eff}}} - 2\Delta L \quad (2.5)$$

Where ϵ_{eff} is stand for an effective relative permittivity for microstrip lines with $w/h \geq 1$ by means of equation (2.6).

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(\frac{1}{\sqrt{1 + 12h/w}} \right) \quad (2.6)$$

With the value of ϵ_{eff} , the fringe factor ΔL can be calculated using equation (2.7).

$$\Delta L = 0.412h \frac{(\epsilon_{eff} + 0.300)(w/h + 0.264)}{(\epsilon_{eff} - 0.258)(w/h + 0.800)} \quad (2.7)$$

With this value of ΔL and ϵ_{eff} , the value of L finally can be calculated using equation (2.5).