Coefficient of restitution of sports balls: A normal drop test

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Coefficient of restitution of sports balls: A normal drop test

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Abstract. Dynamic behaviour of bodies during impact is investigated through impact experiment, the simplest being a normal drop test. Normally, a drop test impact experiment involves measurement of kinematic data; this includes measurement of incident and rebound velocity in order to calculate a coefficient of restitution (COR). A high speed video camera is employed for measuring the kinematic data where speed is calculated from displacement of the bodies. Alternatively, sensors can be employed to measure speeds, especially for a normal impact where there is no spin of the bodies. This paper compares experimental coefficients of restitution (COR) for various sports balls, namely golf, table tennis, hockey and cricket. The energy loss in term of measured COR and effects of target plate are discussed in relation to the material and construction of these sports balls.

Keyword: coefficient of restitution, sports balls, normal drop test, normal impact, energy loss

1. Introduction

Predicting the dynamics behaviours of sports balls on court surfaces during ball impacts can be extremely complicated. Dynamic behaviour of sports balls with interacted surfaces during ball impacts plays a major role in many popular sports, particularly, tennis, golf, and cricket and can have great effect on the style of play adopted by players, such as the changes seen in the game of tennis when moving from hard to clay surface, normally classified as fast to slow court respectively. In such sports as mentioned, ball –surface interaction can also affect the outcome of a match as well.

Recently, the dynamics behaviours of spherical sports balls during impacts have been investigated from theoretical, numerical and experimental viewpoints incorporated with some approximations [1-12]. According to these approaches of studies within the specific purpose noted, theoretical model or similarly mentioned as mathematical model used to be the main component of the analyses attempted. By considering to engineering practices and disciplines respectively, theoretical model is initiated the studies while either numerical or experimental studies as well are necessarily important to prove or verify the theoretical analyses. Thus, it is crucially important to determine the dynamics behaviours of sports balls experimentally for better understanding on impact mechanism.

This paper presents experimental measurements of COR for various sports balls, namely, golf, table tennis, hockey and cricket. The theoretical calculation for COR is presented next. The experiments and measurements using a high speed camera are explained thoroughly. Measurements of position are calibrated and validated to ensure accuracy of the results. Variations of COR between various sports balls at different targets are discussed in details.

2. Coefficient of restitution (COR)

According to most of dynamics literature, coefficient of restitution specifically for one dimensional collision is defined to be the ratio of the velocity of separation (post impact velocity) to the velocity of approach (pre impact velocity). Coefficient of restitution is a common and popular parameter used in many collision models particularly for collision without friction because of its simplicity and usually applied as constant values between zeros to one [12-14]. It is usually denoted by *e* and also recognized as a referred parameter for energy loss due to the motion in normal direction. To comply with work-energy principle, condition $0 < e < 1$ is implemented with the end conditions corresponding to perfectly plastic ($e = 0$) and perfectly elastic impact ($e = 1$) [12, 13]. Theoretically in the latter case, no energy loss is occurred and vice versa. Furthermore, plastic deformation usually occurs in most collisions which contributed to energy dissipation, as well. Consequently, COR is neither zero nor one in actual cases [12, 13]. The coefficient of restitution depends on many elements, such as the geometry of the bodies in contact, the approach velocity, the material properties, the duration of contact and, possibly, friction [13].

In Newton's model the coefficient of restitution is defined as the ratio of final to initial velocity. This model is based on a kinematic point of view and only the initial and final values for the relative normal velocity are taken into account. Meaningly, Newton presumed that the coefficient of restitution is a material property [10-15]. In Stronge's model; based on the internal energy dissipation hypothesis, the coefficient of restitution is defined as the square root of the ratio of energy released during restitution to the energy absorbed during compression [13].

In terms of the validation of the basic theories of restitution, experimental verification is applied. As discussed by Gilardi & Sharf (2002), the final (post-impact) velocity was calculated by using initial impact velocity and the measured displacements and purposely used to compute the coefficient of restitution for a central direct impact of two spheres. The qualitative relation for both parameters is shown in figure 1, shows that the coefficient of restitution tends to level out with increase in impact velocity. Moreover, the experimental data were used to find the dependencies between different quantities, such as coefficient of restitution and the initial impact velocity, as well as to check the limits of the known impact theories. The results obtained for the coefficient of restitution clearly demonstrated the dependence of this parameter on the geometry and material of the impacting bodies, as well as the initial impact geometry and velocity. More specifically, the coefficient of restitution decreases with the increase of the initial impact velocity, and for most materials, it is significantly smaller than unity, even at very low impact speeds.

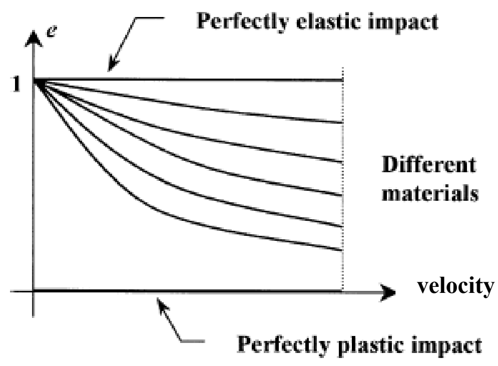

Figure 1. Coefficient of restitution as a function of approach velocity [13].

Instead of the theoretical overview regarding COR discussed above, simplification due to some approximations in calculation of COR seems to be more practically used in wide cases. For instance, the simplest experimental study on the effect of COR for impact between a spherical ball with planar surface is using the method of normal drop test [10-12]. In this case, we consider that a ball drops over a surface from an initial or inbound height of h_1 and it rebounds to a height of h_2. Moreover, as referred to previous literatures in which collision between two bodies have been investigated; air drag can be neglected during drop of bodies from small height [10-12].

According to the impact condition given, the COR is defined as:

$$e = \frac{v_2}{v_1} \tag{1}$$

where v_1 and v_2 are the inbound and rebound speed respectively [4, 6]. Thus, if the ball drops vertically with speed of v_1 impacting to a resting planar surface in normal impact configuration, it rebounds with speed of v_2. Using the energy principles, the inbound and rebound speed relate to the drop height and can be defined as follows;

$$v_1 = (2gh_1)^{1/2} \tag{2}$$

$$v_2 = (2gh_2)^{1/2} \tag{3}$$

While the change in potential energy, ΔE, is given as:

$$\Delta E = mg\,(h_2 - h_1) \tag{4}$$

Using the Equations (1), (2) and (3), the COR can be related to height as:

$$e = \frac{v_2}{v_1} = \left(\frac{h_2}{h_1}\right)^{\frac{1}{2}} \tag{5}$$

3. Normal Drop Test of Sports Balls

Basically, the experimental setup consists of a drop tower with a ball releasing system, a steel table with changeable target surface that can be inclined from 0^o to 45^o; as an optional feature for oblique impact experiment in the future.

Furthermore, the drop tower is equipped with adjustable height mechanism up to 3 m drop height and also equipped with ball releasing system. The vacuum nozzle is attached at the top of drop tower. The sports ball is held to the nozzle by vacuum created using vacuum pump. To release the ball, the negative pressure in the hose is removed by switching off the vacuum pump. The table is made of steel and designed to be equipped with a mechanical inclination system and structurally made for changeable target surface which is embedded on top of the table. In order to minimize the effect of vibration, the impact plate compartment's is necessarily to be lined with a layer of rubber and possibly to be clamped to reduce vibration. Steel and wood plates have been chosen to represent the hard and soft impact surfaces respectively.

A high speed camera complete with accessories and an image processing system is used to capture the impact. The Fastec Inline high speed camera is equipped with 1 kW video light and 17mm high speed lens, F0.95. It is a monochrome type high speed camera with speed between 60 to 1000 frames per second(fps). Moreover, necessarily for images analyses is an image processing system. It consists of Fastec InLine Monitoring System (FIMS) software for video recording purpose, PAC Player software for playback and editing a frame subset of a video event and MIDAS Player software to convert the video to images files. At the final stage, selected images were analysed using Adobe Illustrator software to obtain the data on kinematics measurement. A schematic set-up of all the equipment used in the experiment is illustrated in figure 2 and 3.

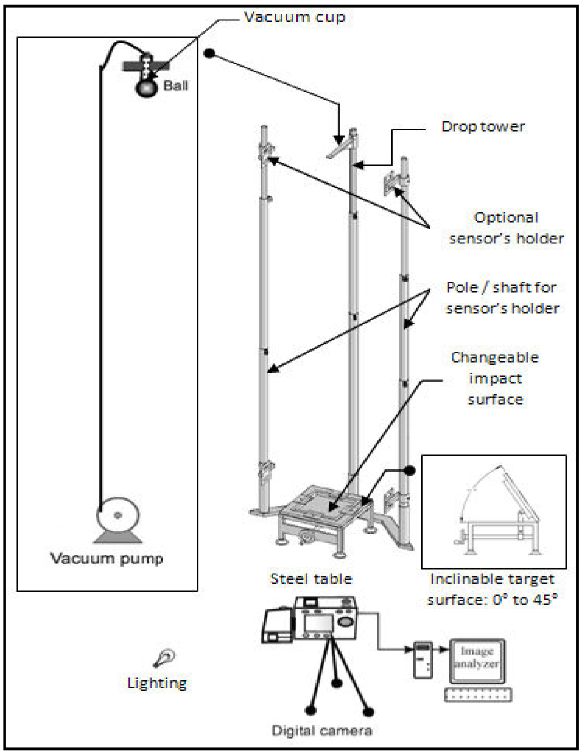

Figure 2. Schematic diagram of experimental set-up(not to scale)

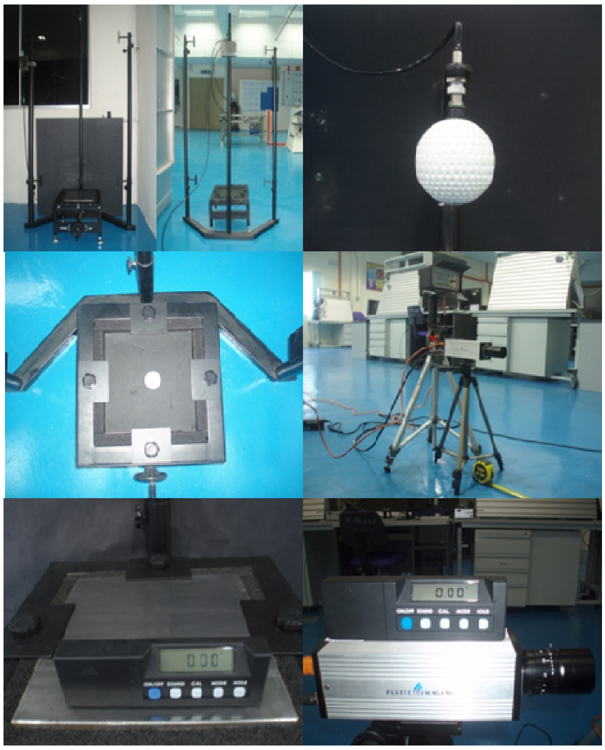

Figure 3. Apparatus used in the drop test

3.1 Technical set-up and procedures

The normal drop test of sports balls were done using test apparatus as shown in figure 2 and 3 above. Several of sports balls, namely, golf, table tennis, hockey and cricket were used in this study. The properties of the sports balls are given in table 1. The initial heights of drop for each ball were set from 1.3 m to 1.7 m with increment of 0.1 m between each drop, specifically. The hard and soft impact target of steel and wood plates with 260mm (L) x 250mm (W) x 25mm (H) in dimensions were used interchangeably during the experiments. The impact target surfaces and high speed camera used are measured to be horizontally, 0º, as required to be parallel in plane during capturing the images by using the measurement tool which is a digital inclinometer. The sports balls are dropped vertically without rotation. Purposely, the impact configuration is actually collinear and its impact property due to impact occurrence is described by the only parameter which is coefficient of restitution.

Table 1. Properties of sports balls

Type of sports ball	Mass (g)	Diameter (mm)
Golf Ball	46	42.80
Table Tennis Ball	2	39.25
Hockey Ball	162	71.00
Cricket Ball	170	71.15

Additionally, the Fastec Inline high speed camera is set to 250fps, 640 x 480 pixels; trigger position is set to END and with the duration of recording is up 17.5 seconds. Furthermore, the interfacing of this high speed camera is by Fastec InLine Monitoring System (FIMS) software with connection to computer using common computer interface cable. Network connection with the host computer should be set-up in order to operate the camera and Fastec InLine Monitoring System (FIMS) software. Briefly, InLine cameras must be connected to a host computer to operate. Two application software's are used with each InLine camera which is Fastec InLine Monitoring System (FIMS) for camera control and PAC Player for image saving and playback. FIMS will playback image clips from the on-board camera memory and download RAW image files to the host computer. Images downloaded and saved in RAW format can be played, cut and clipped and converted to the AVI file format with PAC Player. This was continued by image processing system which is involved the process of converting video file; AVI file to image file; bitmap file or jpeg file, by using MIDAS player software. At the final stage, selected images were analyzed using Adobe Illustrator software to obtain the data on kinematics measurement which is displacement of the bodies.

3.2 Experimental measurements

Regarding the experimental set-up and procedures discussed above, the measurements of displacements for each sports ball noted were calculated thoroughly. For instance, figure 4 shows the method for measuring displacement; rebound height, h_2, of a hockey ball dropped from initial height, h_1, of 1.3 m is calculated using Adobe Illustrator software. To ensure reliability and minimal errors of data, each ball tested was dropped 3 times for all set-up planned. Consequently, the displacements data's were tabulated and calculated results of v_1, v_2 and COR were incorporated in the same table; table 2, in accordance with the equations discussed earlier. The speeds, v_1 and v_2, are obtained from curve fitting on the displacement data for 5 frames each, before and after the impact respectively.

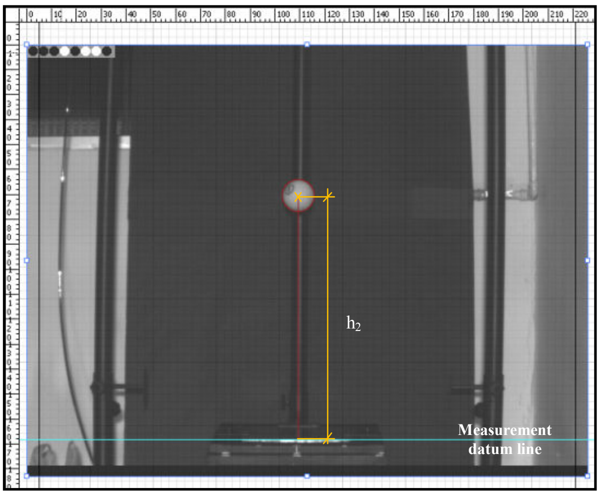

Figure 4. Captured image after the impact. The image is calibrated against known dimensions to obtain displacement of the ball

Table 2. Example of measured displacement and calculated COR for drop from 1.3 m

No. of drop	h_1 (m)	h_2 (m)	COR	v_1 (m/s)	v_2 (m/s)	COR
1	1.30	0.51	0.627	5.050	3.168	0.627
2	1.30	0.57	0.665	5.050	3.358	0.665
3	1.30	0.55	0.648	5.050	3.270	0.648
		Average	**0.647**			**0.647**

4. Results and discussions

Drop test were conducted to study variations of COR between different sports balls and the effect of target surfaces on the COR values. Table 3 and 4 below show COR of the sports balls at different target surfaces and drop heights.

Table 3. COR of sports balls for impact with **steel** target surface

Type of sports ball	Drop height, h_1 (m)				
	1.3	1.4	1.5	1.6	1.7
Golf Ball	0.897	0.896	0.894	0.892	0.889
Table Tennis Ball	0.809	0.800	0.791	0.783	0.779
Hockey Ball	0.647	0.658	0.653	0.650	0.628
Cricket Ball	0.615	0.610	0.609	0.606	0.606

Table 4. COR of sports balls for impact with **wood** target surface

Type of sports ball	Drop height, h_1 (m)				
	1.3	1.4	1.5	1.6	1.7
Table Tennis Ball	0.786	0.769	0.746	0.748	0.743
Golf Ball	0.681	0.677	0.676	0.675	0.666
Hockey Ball	0.398	0.391	0.381	0.380	0.378
Cricket Ball	0.356	0.351	0.351	0.343	0.350

In addition, COR in both tables are plotted on several graphs in order further analyse the COR regarding the energy loss and effects of target plates in relation to the material and construction of those sports balls. Figures 5 and 6 show COR vs. drop height for steel and wood target surfaces respectively.

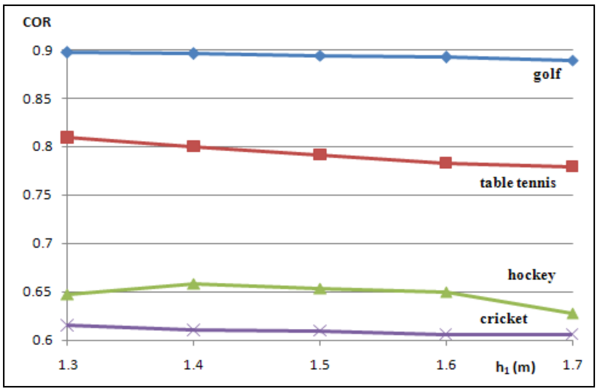

Figure 5. COR vs. drop height of sports balls with steel as impact target surface

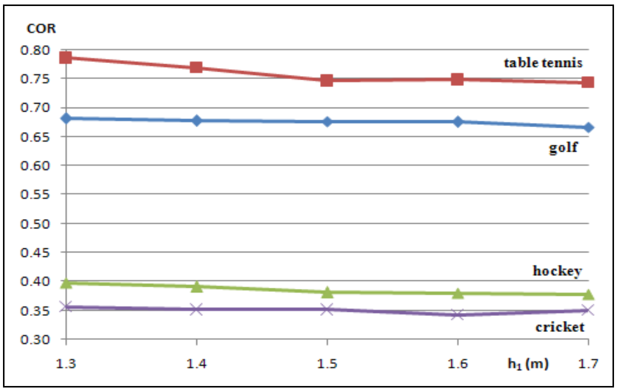

Figure 6. COR vs. drop height of sports balls with wood as impact target surface

The COR values reduce as the drop heights increase i.e. as the velocity increase. For the steel target surface, golf ball has the highest COR value at every drop height (Figure 5). This follows by table tennis, hockey and cricket balls. Golf ball is normally made from different layer of materials. Elastomer is one of its major constituents; thus it is bound to be elastic which can be seen by having the highest COR compared to the other sports balls. On the other hand, cricket ball is normally made from a core of cork, and is layered with tightly wound string and covered by leather. These materials are known to be dissipiative in nature and thus resulting the lowest COR values in all experiments.

However, when the target surface is changed to a softer target, that is wood, table tennis now has the highest COR (Figure 6). The COR values of every balls drop significantly when the target was changed from a hard to a soft surface as shown in figure 7. In details, COR of a table tennis ball drops by 4.3% in average when the target surface is changed. This follows by the golf ball with 24.46%, hockey ball by 40.41% and cricket ball by 42.51%. It seems that the percentage drop in COR is governed by the mass of the ball. Table tennis is the lightest and cricket is the heaviest among the sports balls tested; their percentage drop in COR is the smallest and largest respectively. The measured COR is also related to energy losses during the impact. When the target is changed to the soft surface, more energy is loss during the impact since the target is more deformable compared to the hard surface.

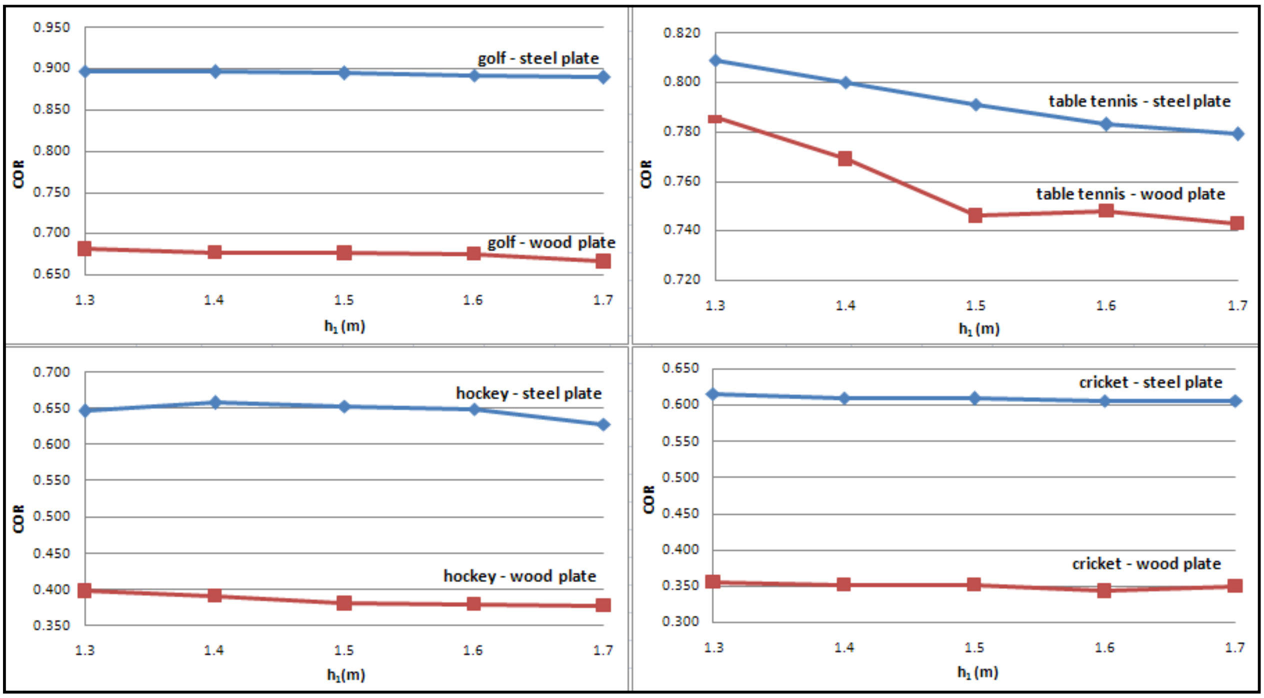

Figure 7. Comparison of COR for sports balls for impact with hard (steel) and soft (wood) impact target surfaces

5. Conclusion

Normal drop test were conducted to study variations of COR between different sports balls. Moreover, comparison on the different impact surfaces, COR for various sports balls, namely golf, table tennis, hockey and cricket due to hard and soft target surfaces were carried out from the results obtained. The energy loss in term of measured COR and effects of target plate are discussed in relation to the material and construction of these sports balls. The main conclusions from the experimental study are summarised below.

- The COR values reduce as the drop height is increased, i.e. as the impact speed increases.
- The COR values varies between different sports balls due to the material and construction of these sports balls. The measured COR is related to energy losses during the impact.
- As the impact surface changes from a hard to a soft target, the energy losses during impact increases (indicated by decreasing COR)

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