Theoretical and Computer Analysis of High Rise Structures with Coupled Walls and Shear Walls

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Introduction

In tall buildings, shear walls are used as the elements to resist lateral loads, such as wind loads or loads due to earthquakes. They are usually located at the sides of the building. A coupled shear wall is a shear wall that has one or multiple rows of openings. Encased in the openings are doors or windows. The combination of coupled walls and shear walls is termed as 'equivalent coupled shear walls' in this article. Typically, an assembly of equivalent coupled shear walls and moment resisting frames constitutes of a reinforced concrete tall building.

Given the importance of equivalent coupled shear walls in the construction of tall buildings, it is no surprise that its behaviour is studied exhaustively by engineers and scientists. A variety of experimental, theoretical and numerical techniques were employed in the past in order to understand the response of coupled shear walls under realistic loads. This article introduces an analytical model called continuum method that can be used in the preliminary analysis of equivalent coupled shear walls.

Some popular methods used in the analysis of coupled shear walls

1. Finite Element method

This is the most versatile and powerful method in structural analysis. A coupled shear wall is often modeled as a number of finite elements. Plane stress elements are used for walls, frame elements for the coupling beams and plate elements for the floor slabs [1]. The benefits of this method are that it is applicable to any geometrical arrangement of the openings and any shear wall shape. The drawback is that for high-rise building, a large number of elements have to be generated and this requires efficient meshing techniques. Besides that, the processing of individual elements and assembling them cost a significant amount of computer time. This method is unsuitable for hand or spreadsheet calculations.

2. Continuum Method

This is an approximate method that analyses the shear wall as a whole. This method enables one to derive the physical quantities of interest in analytical form, and thus its solution is readily attainable by hands. We shall see more of this in the next section.

The modeling of coupled shear wall using continuum method

Figure 1 shows a view of a coupled shear wall with openings. The idea of the continuum method is to replace the discrete connecting beams with a continuous lamina that has an equivalent bending and shear properties. With this, two differential equations can be derived [3]:

$$\frac{d^2T}{dx^2} - \alpha^2 T = \beta M \quad (1)$$

$$EI\frac{d^2y}{dx^2} = M - TL \quad (2)$$

with the boundary conditions:

$$y(H) = 0 \quad (3)$$

$$\frac{dy(H)}{dx} = 0 \quad (4)$$

$$T(0) = 0 \quad (5)$$

$$\frac{dT(H)}{dx} = 0 \quad (6)$$

and

$$\alpha^2 = \frac{12 I_{beam} L^2 (1+\lambda)}{(I_1 + I_2) h b^3} \quad (7)$$

$$\beta = \frac{\alpha^2}{L(1+\lambda)} \quad (8)$$

λ is defined by

$$\lambda = \frac{IA}{L^2 A_1 A_2} \quad (9)$$

For other nomenclatures, please refer to the Appendix 1.

The equivalent coupled walls and shear walls

In reality, coupled shear wall rarely exists alone. Building normally consists of several coupled walls and shear walls. To apply the above formulation to an equivalent coupled shear walls system, one replaces I, A, λ, α and β in the above equations with I_t, A_t, λ_t, α_t and β_t, that are defined by [4]:

$$I_t = I + I_w \quad (10)$$

$$\lambda_t = \lambda \frac{I_t}{I} \quad (11)$$

$$\alpha_t^2 = \alpha^2 \frac{\lambda(1+\lambda_t)}{\lambda_t(1+\lambda)} \quad (12)$$

$$\beta_t = \beta \frac{I}{I_t} \quad (13)$$

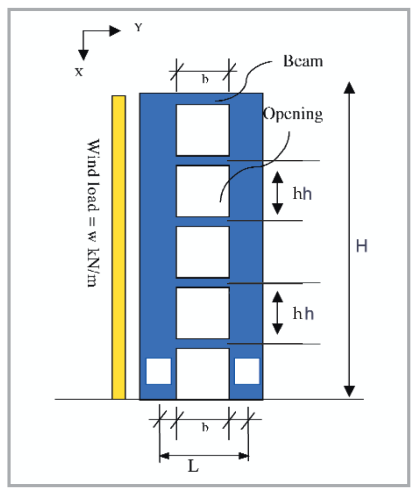

Figure 1: A side view of a coupled shear wall with multiple openings

Axial load, displacement in coupled walls and moment in beams

The solution for axial load and displacement can be determined from the above differential equations (1) and (2) subjected to the boundary conditions (3)–(6). For uniformly distributed load (UDL) and triangularly distributed load (TDL), the solutions are given in Appendix 2. Shear forces and moment in beam are also given.

Example:
Two 20-storey building models are constructed. One with the floor view as shown in figure 3. Another is similar to the first one, except that it has 4 openings (dimension 2mX0.3m) around the 2 coupled walls. Figure 2 shows a coupled wall view with a door of the dimension 2X3.15m. A lateral load of 120kN/m is applied along the height of the structure. The result for continuum method is checked against the finite element method (FEM). Some important parameters are calculated as below:

$E = 26kN/mm^2$

Coupled wall characteristics:

$A_1 = 0.3*3 = 0.9m^2$

$A_2 = 0.3*2 = 0.6m^2$

$I_1 = 0.3*3^3/12 = 0.675m^4$

$I_2 = 0.3*2^3/12 = 0.2m^4$

$I = (0.675 + 0.2)*2 = 1.75m^4$

$I_{beam} = 0.3*0.6^3/12 = 0.0054m^4$

$L = (2+3)/2+2 = 4.5m$

Shear wall and lift core moment inertia:

$I_{sw} = 0.3*5^3/12 = 3.125m^4$

$I_{lc} = (3.3^4 - 2.7^4)/12 = 5.454m^4$

Equivalent coupled shear wall:

$$\lambda = \frac{1.75*(0.9 + 0.6)*2}{4.5^2*2*0.9*2*0.6} = 0.12$$

$$\alpha^2 = \frac{12*2*0.0054*4.5^2*1.12}{1.75*3.75*2^3} = 0.056$$

$$\beta = 0.056/4.5/1.12 = 0.011$$

$$I_t = 1.75+2*3.125+5.454 = 13.454m^4$$

$$\lambda_t = \lambda*13.454/1.75 = 0.923$$

$$\alpha_t^2 = \frac{\alpha^2*0.12*(1.923)}{0.923*(1.12)} = 0.0125$$

$$\beta_t = 0.011*1.75/13.454 = 0.00145$$

Using the above parameters, the deflection and the axial load of the coupled wall are generated as shown in Figure 4. It is clear that the agreement between continuum method and the finite element method is excellent. Figure 6 plots the shear force and bending moment reaction in the beam with respect to the storey of the building.

Maximum shear force in beam occurs at the 5th floor, approximately ¼ of the height of the building. The continuum method predicts that the moment should be the same regardless of the configuration of the diaphragm. However, according to FEM method, openings around the coupled wall could change the bending moment in beam considerably. The existence of a diaphragm between 2 walls causes the moment in the beam to be redistributed to the diaphragm. But openings around the coupled wall reduce this effect. It is clear that continuum method cannot be used to predict the bending moment in beam, since it does not take the diaphragm effect into account.

Figure 5 shows how the moment in the wall changes with respect to the number of storey. It can be seen that FEM method consistently predicted higher moment compared to the theoretical values. The difference is about 15% and it is postulated that this discrepancy is due to the diaphragm effect. Further investigation may be warranted.

Conclusion

The methods used for the analysis of equivalent coupled shear wall are introduced. The continuum method is shown to be capable of calculating the displacement of the walls. (A simple program for this can be obtained at no

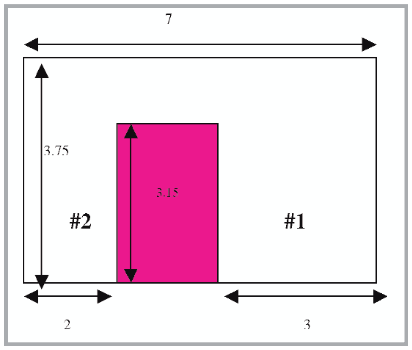

Figure 2: A view of a coupled wall in the structure. The door is in purple

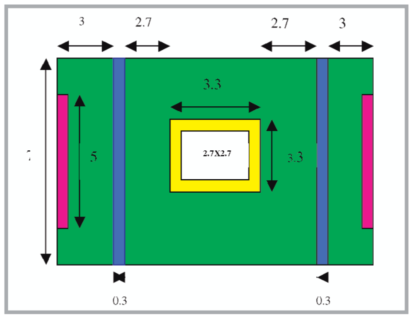

Figure 3: A view of a floor in the structure. Shear walls are labeled as purple, coupled shear walls are in blue, diaphragm in green and lift core in yellow. Void is left as white

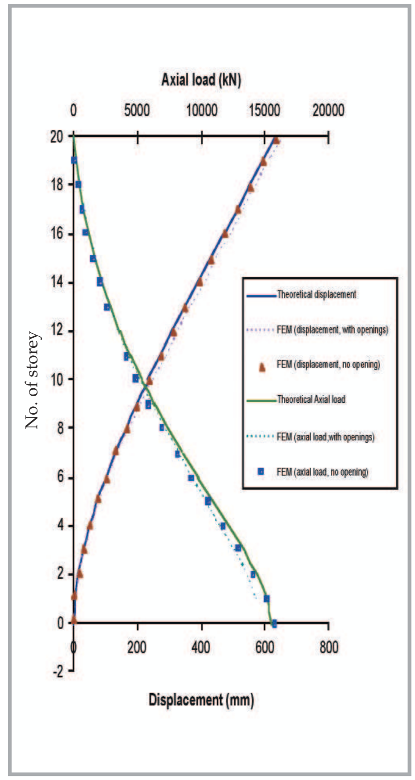

Figure 4: Displacement of the coupled wall vs. storey

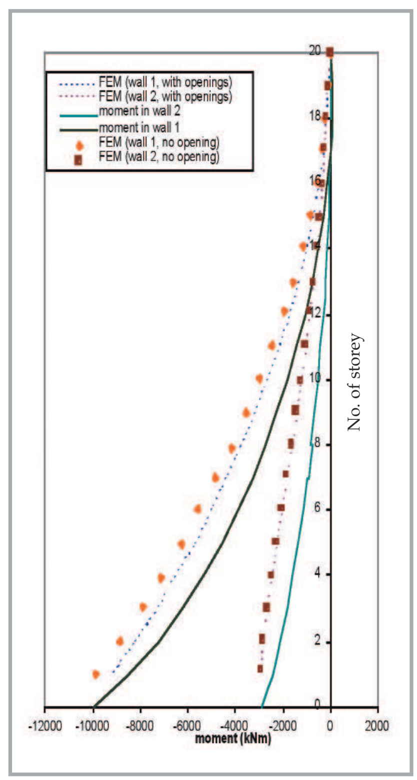

Figure 5: moment in the coupled walls vs. storey

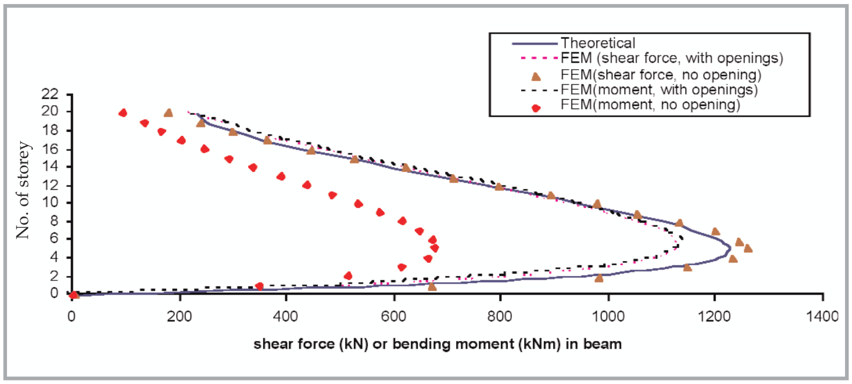

Figure 6: Shear force and bending moment in beam vs. storey

charge from the author, email soonhui@esteemsoft.com). An example is presented to demonstrate the accuracy of continuum method compared to the more exact, but time-consuming finite element method. It is found that continuum method does not include diaphragm effect, and this omission may render the calculation of the moment in beam and in the wall less accurate. ■

References

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2. I. A. MacLeod, Lateral stiffness of shear walls with openings. In: *Proc. Symp. Tall Buildings*, University of Southampton, April 1966, pp.223-252. Pergamon Press, Oxford (1967).
3. R. Rosman, Approximate analysis of shear walls subject to lateral loads. *Jnl. Am. Conc.* Inst. 61, 717-733
4. B.S. Smith and D.P. Abergel, Approximate analysis of High-rise structures comprising coupled walls and shear walls. *Build and Environ.* 18, 91-96

Appendix 1:

- $A_{1,2}$ Cross sectional area of the walls
- A $A_1 + A_2$
- b Coupling beam length
- E Elastic modulus
- $I_{1,2}$ Moment of inertia in the coupled wall
- n_{cwall} Total number of coupled wall
- I $(I_1+I_2)*n_{cwall}$
- I_w The equivalent moment of inertia in the shear walls and the lift cores
- M Applied moment
- P The maximum intensity of the applied triangularly distributed load
- W The intensity of the Uniformly distributed load
- Q_{beam} Axial force in the coupling beam
- T Axial force in the wall
- M_{beam} Moment of the coupling beam

Appendix 2:

	Uniform Distributed load	Triangularly distributed load
T_0	βWH^4	$\beta PH^4/2$
r_0	$-1/(\alpha H)^4$	$-2/(\alpha H)^4$
r_1	0	$2/(\alpha H)^4$
r_2	$-1/(2(\alpha H)^2)$	$-1/(\alpha H)^2$
r_3	0	$1/(3(\alpha H)^2)$
W_{eff}	W	$P(1\text{-}x/(3H))$
r_4	$\left(r_0\sinh(\alpha H) - \frac{r_1+2r_2+3r_3}{\alpha H}\right) / \cosh(\alpha H)$	
$T(x)$	$T_0[r_0(1-\cosh(\alpha x)) + r_4\sinh(\alpha x) + r_1 x/H + r_2(x/H)^2 + r_3(x/H)^3]$	
Q_{beam}	$hT_0\{(aH)[r_4\cosh(\alpha Hz) - r_0\sinh(\alpha Hz)] + r_1+2r_2z+3r_3z^2\}/H$	
M_{beam}	$Q_c b/2$	
$M_{1,2}(x)$	$\frac{I_{1,2}(W_{eff}x^2/2 - T(x)L)}{I}$	

Table 1: Equations for axial force and moments in the walls and in beams

	Uniform Distributed load	Triangularly distributed load
y_0	$WH^4/(EI)$	$PH^4/(2EI)$
N_5	0	$-\lambda/[60(1+\lambda)]$
N_4	$\lambda/[24(1+\lambda)]$	$-5N_5$
N_3	0	$(1-12N_4)/[3(\alpha H)^2]$
N_2	$-(1-24N_4)/[2(\alpha H)^2]$	$-3N_3$
N_1	$-2N_2$	$(1+2N_2-12N_4)/(\alpha H)^2$
B_1	$-2N_2/(\alpha H)^2$	
B_2	$[\frac{N_1}{\alpha H} - B_1 \sinh(\alpha H)]/\cosh(\alpha H)$	
C_1	$-(N_1 + 2N_2 + 3N_3 + 4N_4 + 5N_5)$	
C_0	$-[(c_1 + N_2 + N_3 + N_4 + N_5 + B_1 \cosh(\alpha H) + B_2 \sinh(\alpha H)]$	
$y(x)$	$y_0[B_1 \cosh(\alpha x) + B_2 \sinh(\alpha x) + C_0 + C_1 x/H + N_2(x/H)^2 + N_3(x/H)^3 + N_4(x/H)^4 + N_5(x/H)^5]$	

Table 2: Equations for displacements in the wall