Simulation of the Distributed Rainfall-Runoff Process

Huang Yuk Feng and Lee Teang Shui

*Department of Biological and Agricultural Engineering Faculty of Engineering, Universiti Putra Malaysia,
43400 UPM Serdang, Selangor*

ABSTRACT

A deterministic model to simulate rainfall runoff from pervious and impervious surfaces is presented. The surface runoff model is based on an established one-dimensional, variable width, kinematics wave approximation to the Saint Venant equations and Manning equation, to mathematically route overland and channel flow, using the finite element method. The Galerkin's residual finite element formulation utilizing linear and quadratic one-dimensional Lagrangian elements is presented for the spatial delimitation of the nonlinear kinematics runoff equations. The system of nonlinear equations was solved using successive substitutions employing Thomas algorithm and Gaussian elimination. The whole formulation was set up using the MapBasic and MapInfo Geographical Information System. A laboratory rainfall runoff physical model was set up to test the numerical model. Parameters considered include, surface roughness, plane slope, constant or changing rainfall intensities. Linear element simulation was found to give results as accurate as the quadratic element simulation. Increasing the number of elements to simulate runoff from a homogenous surface did not give any added advantage. Whilst the Courant Criterion gives maximum time step increment for computation, it is however recommended that as small a time increment be used to eliminate any oscillatory instability. Time increment for channel flow routing was found to be always smaller when compared to lateral overland flow. Thus, the chosen time step increment for channel flow routing must be a common factor of that of lateral overland flow in order to satisfy the linear interpolation of overland outflow hydrograph as input into the channel. For laboratory scale catchments, smaller upstream plane and larger downstream plane roughness, 0.033 for bare soil surface upstream and 0.300 for grass surface downstream, respectively, can result in small oscillatory disturbances at the rising limb. Such discrepancy does not occur when upstream roughness is larger then downstream roughness. Differences in elemental interface slope can be catered for rather well in the model. A hypothetical watershed and imaginary tropical rainstorm was also studied to verify the stability of the model in larger runoff catchments. Channels, which are initially dry or with existing flows can be simulated incorporating additional rainfall. Large catchments with large physical elemental roughness and slope differences can be well simulated, without oscillations that are evident in laboratory scale tests.

Keywords : *Rainfall runoff, finite element, kinematics-wave modeling, overland flow, geographical information system*

INTRODUCTION

A hydrologic system model is an approximation of the actual system, in which its inputs and outputs are measurable hydrologic variables and are linked by a set of equations. The flow of water through the soil plane and stream channels of a watershed, however, is a distributed process, since the flow rate, velocity, and depths usually show temporal and spatial variation throughout the watershed. Therefore, by using a distributed hydraulic model, flow rates can be computed as a function of space and time. Most of the hydraulic models require a large number of input data and might produce a large set of output data. A complex, large-area, multi-basin drainage study requires significant effort in terms of data organization, development of models, and presentation of results. To overcome these problems and difficulties, a GIS system can be used to organize, store, and display spatial (maps) and non-spatial (characteristic) data for the study.

In the actual flow process, the velocity of flow in a river varies along the river, across it and differs from the water surface to the riverbed. However, the first two spatial variations can be ignored. The flow process is assumed varying in only one space dimension that is along the flow channel or in the direction of flow. The Saint Venant partial differential equations for continuity and momentum respectively, are the governing equations for one-dimensional, unsteady flow in an open channel. Based on these equations, the simplest distributed or hydraulic routing model is the kinematics wave model, which assumes that the friction and gravity forces balance each other, and the flow condition is steady and water surface profiles are uniform. Direct numerical methods for solving partial differential equations can either follow the finite difference approach, or the finite element method.

Judah used the Galerkin's residual method in the formulation of a flood routing model and obtained satisfactory results [10]. The Galerkin's residual method of the finite element method was also used by Al-Mashidani and Taylor [1] solve the non-dimensional form of the shallow water equations for surface runoff. Cooley and Moin [3] also applied Galerkin's residual method to a finite element solution of open channel flow and obtained good results. Taylor et al. developed a numerical finite element for the analysis of watershed direct runoff problem [12]. White had demonstrated the application of the FEM in watershed analysis [14]. Jayawardena and White presented an analytical basis for the formulation of a distributed catchments model within the flexible framework of the finite element method (FEM) [9]. In this model, the solutions in the space and time domain, is carried out by using the finite element and the finite-difference method, respectively. A finite element storm hydrograph model (FESHM) has been developed as a distributed parameter model to simulate flow on ungauged watersheds [11].

Blandford and Meadows presented a Galerkin finite element formulation, utilizing linear, quadratic, and cubic one-dimensional Lagrangian elements, for the spatial delimitation of the nonlinear kinematics runoff equations [2]. A method to estimate a suitable computation time-step size based on the Courant condition is also presented. However, using a time increment approximately equal to the presented Courant time increment may not produce accurate results with both explicit and implicit schemes. Viessman et al. stated that for the explicit time integration scheme, the best results are obtained with a time increment of 20% off of that defined by the Courant condition [13]. Giammarco et al. (1995) developed a conservative finite elements approach to overland flow, known as the control volume finite element (CVFE) method [5]. This CVFE method is said to be extremely useful and flexible not only for overland flow studies but also for flood plain modeling.

The objective of this study is to develop a deterministic GIS based finite element model to simulate the rainfall runoff process. A laboratory scale model with various surface conditions and uniform rainfall simulation will be used to verify the model's computation stability, accuracy and differences between the linear and quadratic element based models used. The model is then to be checked for its stability when applied to fictitious real world catchments, albeit only in its computation stability, so that it can be used later for real life simulations.

METHODOLOGY

Several assumptions were made:

(i) evaporation and evapotranspiration are assumed to be zero for the purpose of this study in order to reduce the complexity of the model. This is an event driven model and this assumption can be valid for the duration of rainfall runoff process;

(ii) excess rainfall is the only inflow onto the overland;

(iii) the net inflow into the channel is contributed from the direct rainfall onto the channel as well as from lateral overland flow; and

(iv) assuming that the kinematics overland and channel flows have only a forward characteristic with no backwater effects. The Saint-Venant (1871) equations of continuity and momentum form the basis for the solution. The kinematics wave based model neglects the local accelerations, convective acceleration, and pressure terms in the momentum equation, and thus assumes that the friction and gravity forces balance each other, that is, $S_o = S_f$. and is approximated using the Manning equation.

FINITE ELEMENT FORMULATION

The finite element method is especially adaptable to the problem of evaluating the impact of land-use changes on flood flows since a watershed and channel can be divided into a finite number of sub-areas or elements. The hydrologic properties of one or all of the elements can then be altered to simulate the effect upon the hydrologic response of the entire watershed system. The results from the overland flow are considered as input for the subsequent channel flow computation, ignoring direct rainfall into the channel. The same finite element formulation can be applied for the both stages.

The derivation of the finite element equation involves the development of algebraic equations from a governing set of differential equations. Galerkin's residual method was used to derive the individual element equations because it has been demonstrated to be a good formulation procedure for surface flow problems. For the finite element grid consisting of more than one element, it must be arranged in a form, which embodies the total number of elements. The direct stiffness method is used to obtain the assembled matrices. The algebraic equations must be solved as a set of simultaneous equations to obtain the primary unknowns, area of flow A, at the nodes. Here, the Thomas algorithm and the Gaussian elimination are used to obtain the solution. The solutions of the system equations are next used to calculate the secondary unknowns, discharge Q, at the nodes.

DETERMINING STEP TIME INCREMENT

Selection of a proper time increment to be used for flow routing process in the model is essential for an efficient and accurate solution. A large value of time increment may produce an inaccurate result or an instability problem. On the other hand, a time increment that is too small requires larger number of computations. In the finite element model, a time step is chosen to satisfy the Courant condition. Courant condition time increment is the time taken by the kinematics wave to travel from node to node (element length). The equation to estimate the maximum Courant condition time increment (Blandford and Meadows, 1990), applicable in the model is given by:

$$\Delta t_c = \frac{\Delta x}{\left(m.\alpha^{1/m}\right)}\left(I..i_{\max}\right)^{\frac{1}{m}-1} \tag{1}$$

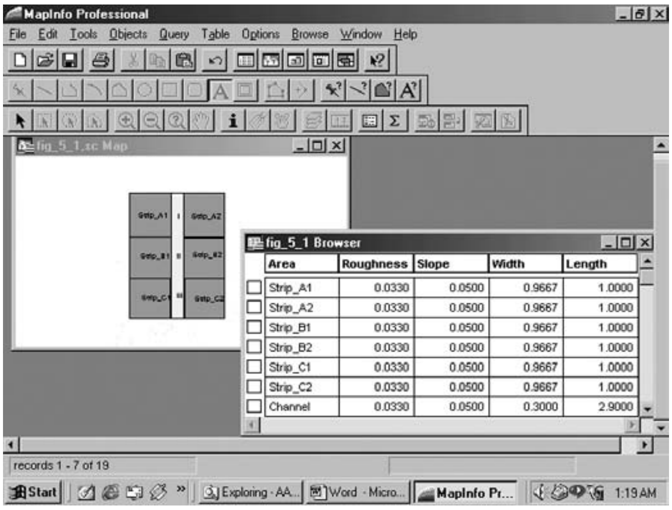
In cases where more than one value of n and S is applied, the smallest n and biggest S should be selected. The calculated time increment approximates the maximum time increment that should be used in the model. In the case of channel flows, using this calculated time increment for flow routing, produced inaccurate results. Here, the value of maximum rainfall intensity should be replaced by the maximum lateral inflow value from the lateral strip of the channel. This is because, the lateral inflow into the channel is very high compared to the direct rainfall falling on to the channel. Thus, depending on the rainfall duration, a suitable value of the time increment, which must be at least equal to or less than the Courant condition time increment, should be chosen for both the overland and channel flows routing.

TEMPORAL EXCESS RAINFALL DELIMITATION

Typically, excess rainfall event data are reported and displayed at regular time interval, say, every 1 min, 5 min, or at breakpoint intervals. Such a discontinuous loading function would cause convergence problems. To eliminate the abrupt discontinuities in the excess rainfall, a linear transition over two time steps is used. The transition scheme adopted here is such that it conserves the excess rainfall volume. This transition strategy will result in less oscillatory results in runoff simulation.

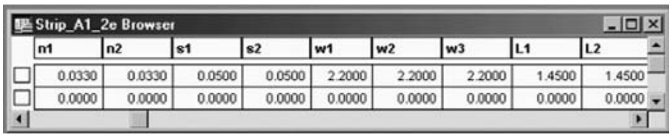
MODEL

A model based upon the mathematical equations delineated was programmed using the MapBasic Language and run concurrently with the desktop MapInfo Geographical Information System. The simple rectangular laboratory set up is used as an example to illustrate some of the geographical information system functions in running the model and are as shown in Figures 1 - 5.



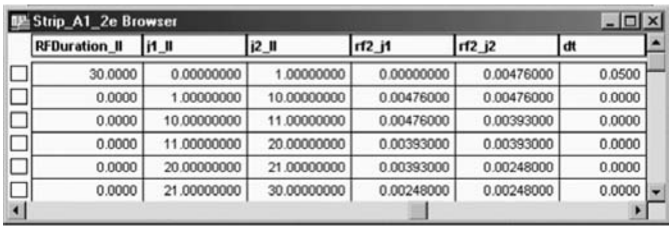
	Area	Roughness	Slope	Width	Length
☐	Strip_A1	0.0330	0.0500	0.9667	1.0000
☐	Strip_A2	0.0330	0.0500	0.9667	1.0000
☐	Strip_B1	0.0330	0.0500	0.9667	1.0000
☐	Strip_B2	0.0330	0.0500	0.9667	1.0000
☐	Strip_C1	0.0330	0.0500	0.9667	1.0000
☐	Strip_C2	0.0330	0.0500	0.9667	1.0000
☐	Channel	0.0330	0.0500	0.3000	2.9000

Figure 1: Example of the digitized map of the Laboratory Setting with corresponding MapInfo data table



	n1	n2	s1	s2	w1	w2	w3	L1	L2
☐	0.0330	0.0330	0.0500	0.0500	2.2000	2.2000	2.2000	1.4500	1.4500
☐	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Figure 2: MapInfo Data Input Table for a 2-element overland flow strip

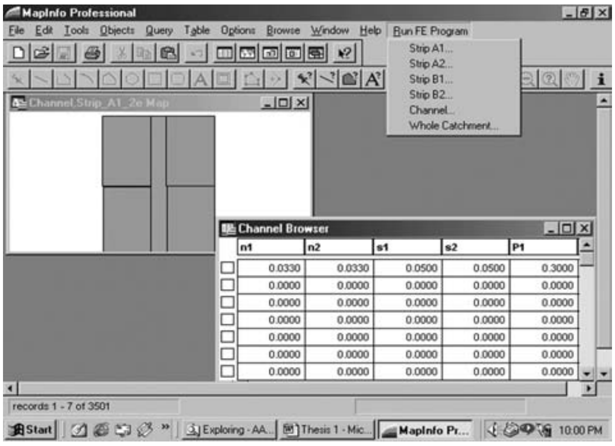


	RFDuration_II	j1_II	j2_II	rf2_j1	rf2_j2	dt
☐	30.0000	0.00000000	1.00000000	0.00000000	0.00476000	0.0500
☐	0.0000	1.00000000	10.00000000	0.00476000	0.00476000	0.0000
☐	0.0000	10.00000000	11.00000000	0.00476000	0.00393000	0.0000
☐	0.0000	11.00000000	20.00000000	0.00393000	0.00393000	0.0000
☐	0.0000	20.00000000	21.00000000	0.00393000	0.00248000	0.0000
☐	0.0000	21.00000000	30.00000000	0.00248000	0.00248000	0.0000

Figure 3: Excess rainfall data and time increment MapInfo Input Table

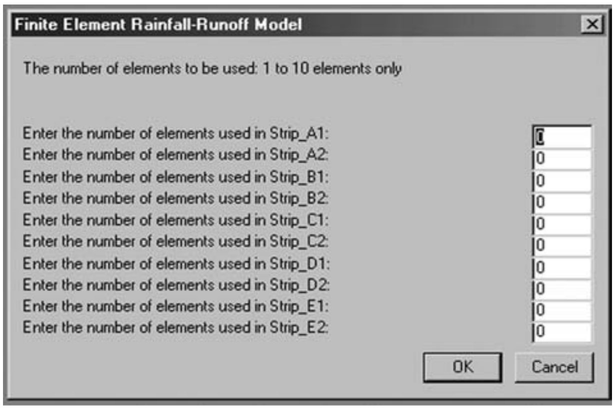
LABORATORY TESTS

The laboratory apparatus consist of a rainfall simulator, runoff basin, runoff collection drain (discharge measurement), and infiltration-percolation collecting tray. The rainfall simulator was made such that the raindrops are formed by a large number of sprinklers spaced at 30 cm apart, and inserted into seven equal lengths of 32 mm diameter PVC pipes. The pipes were placed parallel to each other at a spacing of 30 cm. The sprinklers were set in a 30 cm × 30 cm (one-square-foot) rectangular grid with one sprinkler at each corner. The simulator was suspended at 45 cm above the runoff basin. Water is supplied through a gate valve controlled water pump, which pumped water uniformly and continuously from a constant-head tank. According to Yoon et al. and Hall et al. [16; 7], droplet size and droplet velocity were reported to have a negligible effect to the flow mechanics of the plane compared to the rainfall intensity.



	n1	n2	s1	s2	P1
☐	0.0330	0.0330	0.0500	0.0500	0.3000
☐	0.0000	0.0000	0.0000	0.0000	0.0000
☐	0.0000	0.0000	0.0000	0.0000	0.0000
☐	0.0000	0.0000	0.0000	0.0000	0.0000
☐	0.0000	0.0000	0.0000	0.0000	0.0000
☐	0.0000	0.0000	0.0000	0.0000	0.0000
☐	0.0000	0.0000	0.0000	0.0000	0.0000

Figure 4: MapInfo Menu for MaBasic program execution for a 2-element channel



The number of elements to be used: 1 to 10 elements only

Enter the number of elements used in Strip_A1: 0
Enter the number of elements used in Strip_A2: 0
Enter the number of elements used in Strip_B1: 0
Enter the number of elements used in Strip_B2: 0
Enter the number of elements used in Strip_C1: 0
Enter the number of elements used in Strip_C2: 0
Enter the number of elements used in Strip_D1: 0
Enter the number of elements used in Strip_D2: 0
Enter the number of elements used in Strip_E1: 0
Enter the number of elements used in Strip_E2: 0

Figure 5: MapInfo Dialog Query Box for number of elements to be used

The dimensions of the basin are 2.9 meters in length and 2.2 meters in width. The soil used consisted of mining sand and clayey loam soil, each of equal volume, with the clayey loam soil on top. The total thickness of the plane was 30 cm and the surface was lightly compacted. The runoff surface plane of the basin when required was formed by changing to the appropriate type of materials; namely, bare clayey loam soil, Taiwanese Grass, a combination of bare clayey loam soil surface and grassed surface, and plywood. In addition, different values of plane slope were also set in the runoff basin for the overland flow and overland with channel flow cases. For overland with channel flow, the runoff basin was divided symmetrically into two equivalent sections along the longitudinal axis of basin, with the channel placed right in the middle of the basin, as shown in Plate 1.

RANGE OF EXPERIMENTAL CONDITIONS

In runoff hydrograph measurements for overland flow planes, two different slopes, 5% or/and 10%, were set for each of the surface condition. The surface conditions are bare clayey loam soil (Plate 2), Taiwanese grass (Plate 3), bare clayey loam soil and Taiwanese grass interface (Plate 4), and plywood (dimension: 1.125m × 2.400m). In addition, a clayey loam soil overland flow

Plate 1: Runoff basin for overland with channel flow simulation

Plate 2: Runoff basin with bare clayey loam soil surface

Plate 3: Runoff basin with Taiwanese grass surface

Plate 4: Runoff basin with bare clayey loam soil (upstream) and Taiwanese grass (downstream).

plane with a combination of two different slopes (5% and 10%) was used. For the overland with channel flow experiments, the only surface condition used was clay soil, and the side slopes were set to either 5% or 10% slope. The longitudinal flow channel of the plane was also built up using clay soil and fixed at 5% slope for all experiments (Plate 1).

The duration of all constant rainfall events was 20 minutes, except for the plywood surface, which was set to 5 minutes. The duration for all the experiments with increasing or decreasing rainfall intensity was set at 10 minutes per rainfall intensity, making a total of 30 minutes for an experiment run over three different intensity. Similarly, 5 minutes duration (a total of 15 minutes for the experiment) were set for the experiments on plywood surface.

RESULTS AND DISCUSSIONS EFFECTIVE RAINFALL

In the laboratory set up, the only infiltration rate that can be determined is the maximum infiltration rate, which occurred when the measured infiltration rate become constant. This is the time after which all the soil in the model is well wetted and absorbs no more water. The excess rainfall that is calculated during this period can be considered as the maximum excess rainfall for that rainfall event. Since the laboratory model is very small, it can be assumed that the excess rainfall rate will be constant when the measured runoff-discharge volume first becomes constant, as long as the rainfall intensity is constant. Figure 6 shows the discharge hydrograph of the laboratory test which was set at 5% bare soil slope for overland flow with 4.76×10^{-3} m^2/min excess rainfall, 2.2m width, 2 equal 1.45m length elements; using linear element.

An excess rainfall hyetograph and discharge hydrograph for a changing rainfall intensity event is illustrated in Figure 7 (Laboratory Test: 5% bare soil slope for overland flow with 4.76 ×

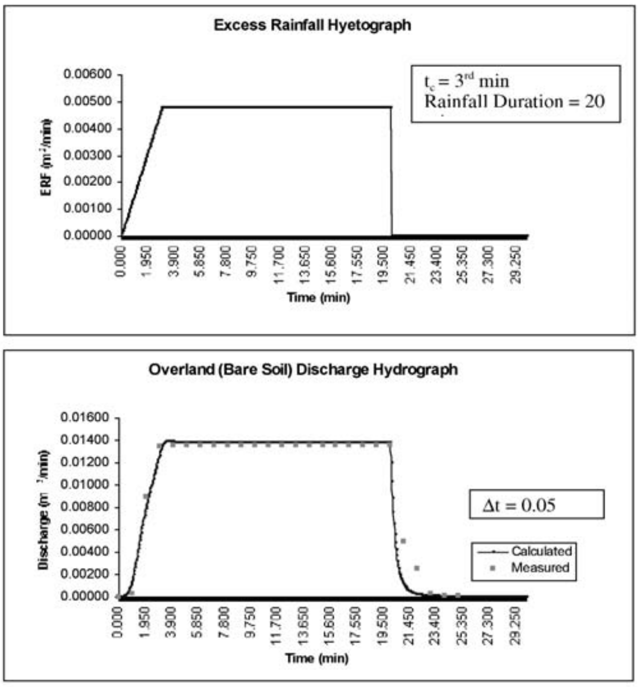


Figure 6 : Excess Rainfall Hyetograph and corresponding Discharge Hydrograph for 5% slope bare soil (n = 0.033) overland flow. Rainfall intensity 2.67 x 10^{-3} m/min

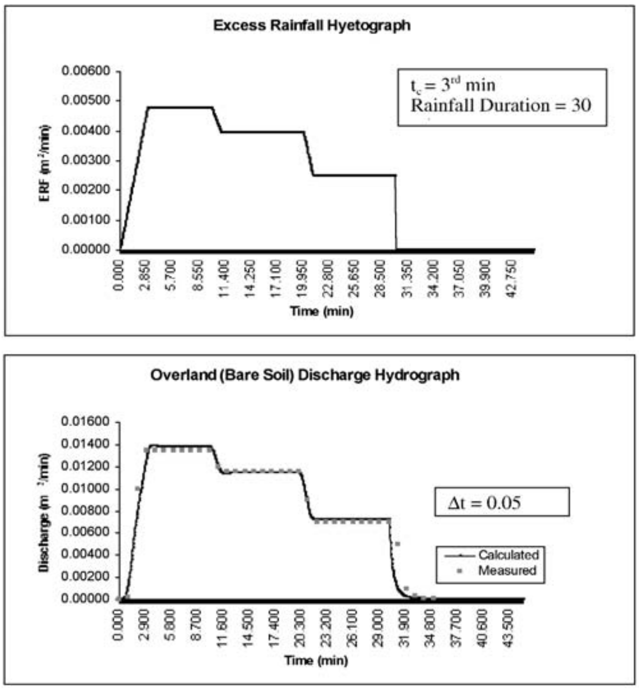


Figure 7: Excess Rainfall Hyetograph and corresponding Discharge Hydrograph for 5% slope bare soil (n = 0.033) overland flow. Consecutive rainfall intensities 2.67 x 10-3, 2.25 x 10-3, 1.44 x 10-3 m/min

10^{-3}, 3.93×10^{-3}, and 2.48×10^{-3} m^2/min excess rainfall, 2.2m width, 2 equal 1.45m length elements; using linear element simulation model

TIME INCREMENT SELECTION

It was observed that the value of time increment depends on the length of element used, the surface roughness, slope, maximum rainfall intensity, and the length of the whole system. Hence, time increment must be chosen so that the Courant condition for that particular case is always satisfied to avoid kinematics shocks that produce instability. In addition, the selected time increment must be a common factor of the rainfall duration also (required for the purpose of interpolation in excess rainfall between two adjacent time period). If a smaller time increment were to be used, it will give a more stable and accurate result.

Figures 8, 9 and 10 show the results of using different time increment values for a laboratory test (2 elements, linear element simulation), where $\Delta t > \Delta t_C$, $\Delta t = \Delta t_C$, and $\Delta t < \Delta t_C$ respectively.

In this case, the time increment that should be used according to Courant condition is about 0.54 minute. However, this value is not a common factor of the rainfall duration (20 minutes). The biggest common factor that is less than 0.54 is 0.50, which is about 7.4% off the Courant condition time increment. Thus, the recommended values of time increment that can be used in simulation include 0.50, 0.40, 0.20, 0.10, 0.05, or other values, as long as it is a common factor of the rainfall duration and less than 0.54.

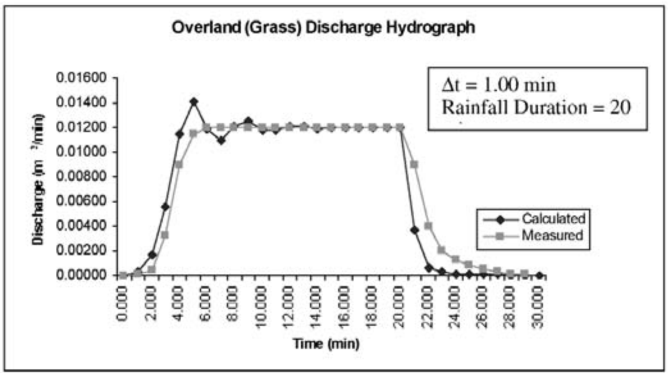


Figure 8: Discharge Hydrograph for 10% slope grass (n = 0.300) overland flow. Rainfall intensity 2.25 × 10^{-3} m/min, $\Delta t > \Delta t_c$

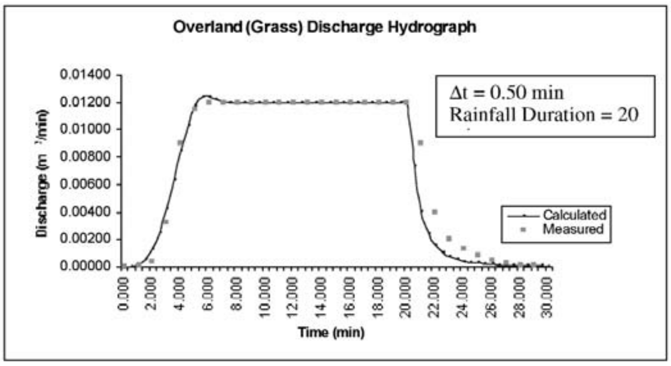


Figure 9: Discharge Hydrograph for 10% slope grass (n = 0.300) overland flow. Rainfall intensity 2.25 × 10^{-3} m/min, $\Delta t > \Delta t_c$

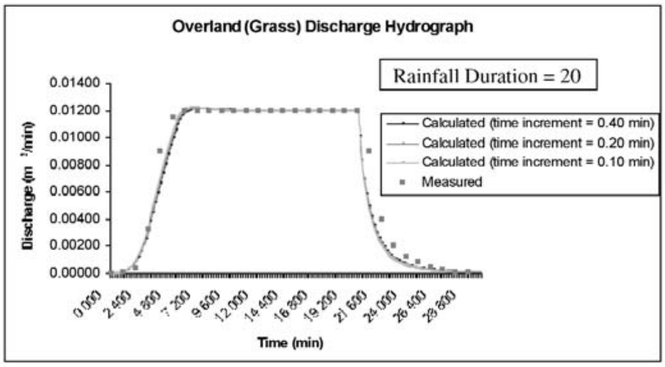


Figure 10: Discharge Hydrograph for 10% slope grass (n = 0.300) overland flow. Rainfall intensity 2.25 x 10^{-3} m/min, $\Delta t > \Delta t_c$

However, in channel flows routing process, the time increment is calculated according to the peak overland flow runoff discharge volume into the channel from lateral strips. It is recommended that the time increment used in channel flow routing be as small as possible. This is because the maximum rainfall intensity in the equation to determine time increment is now replaced by the maximum lateral inflow from the overland strips from both sides. A large volume of lateral inflow will result in a very small Courant condition time increment value.

The dissipative mechanism can be quickly dampened by the judicious choice of the time increment. Although it is said that the maximum time of computational time increment is the Courant Criterion, it can be concluded that the time increment selected should be as small as possible. It is not true that satisfying the Courant criterion will result in solutions that are inherently stable. With increased iterations, flow behavior is more precisely simulated than that of using a single time leap for the wave to travel to the element node. This linear time discrete is thus an important consideration in the solution of the algorithm.

SELECTION OF SURFACE ROUGHNESS

The Manning roughness coefficients for the bare soil and grass were selected using values recommended by Engman [4] for overland flow. The recommended values for bare clay-loam (eroded) ranges from 0.012 – 0.033, and 0.170 – 0.300 for dense grass. In all the laboratory tests, the value of roughness coefficient chosen for bare soil is 0.033, and 0.300 for dense grass (Taiwanese grass). The Manning roughness coefficient for the plywood surface is estimated as 0.015 using the value recommended by Schwab al. [15], which ranges from 0.010 to 0.015 for planed wood. The highest value of roughness coefficient from the ranges is always selected for all cases. This is because the laboratory model is very small relatively, and will need a very small time increment value to produce results without undue divergence, especially when the rainfall intensity used is high, or the number of element used is large.

NUMBER OF ELEMENTS AND ELEMENT LENGTH

According to the Courant condition time increment equation, the time increment value used in a system mainly depends on the element length of the system. The shorter the element length, the smaller is the time increment needed for the model simulation. The criteria of element length selection and the number of element used in a system mainly depend on the topography condition of the simulation area, such as surface roughness and slope. Areas with the same roughness coefficient and slope should be selected as an element, instead of dividing it into two or more elements. Two simulations had been carried out to prove that the number of elements used to assess a homogenous surface did not affect the simulated results: (1) using different number of elements for a homogeneous overland flow routing system; (2) using different number of elements for a homogeneous overland flow with channel flow routing system. In both cases, the results were not significantly different in terms of absolute values. On the other hand, in situations where the use of shorter element length and bigger number of element cannot be avoided, for example, in a natural catchments due to the various types of surface physical properties, selection of a smaller time increment value is still needed and cannot be avoided.

LINEAR VERSUS QUADRATIC INTERPOLATION FUNCTION MODELS

From simulations performed it was found that simulated results obtained through using either the linear or quadratic function models were similar. However, it is noted that quadratic element simulation model need a smaller time increment value than that predicted by the Courant condition time increment because it has a bigger matrix iteration for the same number of element compared to the linear element simulation model.

ACCURACY, STABILITY, AND CONVERGENCE

Various sets of test (not shown here) with different physical and rainfall conditions have been carried out for this purpose. From all the plots, it can be concluded that almost all the simulated result matched quite closely with the measured results. However, some of the peak discharge and the volume of runoff for certain events have been either over-predicted or under-predicted although the deviations are not large. The inconsistency can be due to the assumption of overall homogeneity of infiltration, roughness, rainfall intensity, slope, and uniform pumping pressure in all the laboratory experiments. But in reality, this is virtually impossible to achieve all at once for each individual experiment.

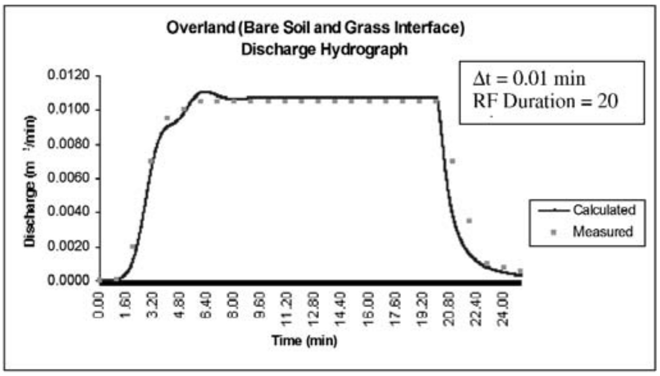


Figure 11: Discharge Hydrograph for 5% slope bare soil (upstream, n = 0.033) and grass (downstream, n = 0.300) interface overland flow. Rainfall intensity 2.25 x 10^{-3} m/min

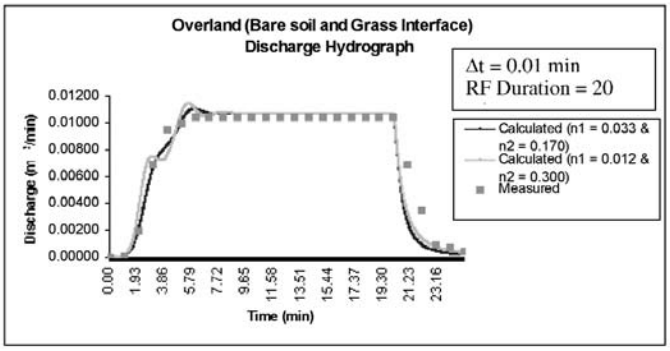


Figure 12: Discharge Hydrograph for 5% slope bare soil (upstream, n = 0.033) and grass (downstream, n = 0.300) interface overland flow. Rainfall intensity 2.25 x 10^{-3} m/min, two sets of different roughness

Almost in all the events, the rising curves have been underestimated by model. This discrepancy may be caused by the assumption of linear transition of the simulation of the excess rainfall, from zero to the maximum constant rate. In the recession curve of all the hydrographs, the simulated results always show faster recession and underestimation due the residual effect of remaining water in the pipes as explained earlier.

A laboratory test using bare soil surface with two different slopes, 5% (upstream side) and 10% (downstream side), divided into two equal length elements, was also set to check the ability of finite element method in simulating runoff discharge for an area with different slopes. Generally, the results simulated by the model for both constant rainfall intensity, and changing rainfall intensity events are close to the measured results. Further evaluation included a laboratory test using a bare soil (upstream) and grass (downstream) interface as runoff surface, divided equally into two elements (each with 1.45m of length), to check the ability of finite element in simulating runoff discharge from an area with different roughness coefficients. The simulated discharge hydrograph compared to the measured discharge hydrograph is shown in Figure 11. The rising curve in the simulated hydrograph has shown small oscillations. The roughness coefficient values used in the model is 0.033 for bare soil surface, and 0.300 for grass surface, a factor of 10 difference. This may be the main factor that affects the rising curve simulated by the model. This is where the kinematics-wave theory may fail due to dam effect at the interface.

When two closest values of roughness coefficient for bare soil and grass are used, 0.033 for bare soil and 0.170 for grass, and compared to the case where two as-far-apart-as-possible roughness coefficient values for these two surfaces is used, 0.012 and 0.300 for bare soil and grass surfaces respectively, the simulated rising curves have shown that the former case would produced less oscillatory result, as shown in Figure 12. Thus, when the two values of roughness coefficient used in simulation differs by a large margin, there will be more instability. The spurious oscillatory behavior can be suppressed when the difference between adjacent values of roughness (and perhaps slope) is made very much smaller. On the contrary, if the upstream elemental roughness has a bigger value compared to the downstream end, the resulting simulated hydrograph would be normal. However, this is only a conclusion made from the laboratory condition, where the catchments model is very small. If the same situation is applied in bigger catchments, this oscillatory may not occur. The scale factor of the physical model may be contributory impedance in model simulation.

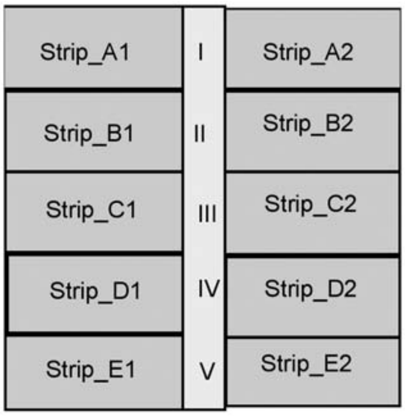


Figure 13: Schematics of a fictitious large natural catchments with 5 strips each side of overland flow and 5 element for channel flow

NATURAL CATCHMENTS

The parameters of a hypothetical larger natural catchments and imaginary rainstorms, was used to verify the stability of the model in simulation in large real catchments. A catchments area about 25km^2 (5 × 5 km) is considered for this purpose. The catchments is a square area with a channel flows in the middle of the catchments, as shown in Figure 13.

The overland components in the both sides of the channel are divided into five equal strips, each with 1 km width and 2.5 km long. The surface of the overland area (strips) has a 10% slope, and covered by a material with a coefficient of roughness, 0.20. Similarly, the channel has a 2% slope with 0.02 of roughness coefficient. Each strip is delimitated into five equal length elements (0.5 km each). Similarly, the channel is also delimitated into five equal 1 km length elements with 30 m width. A constant excess rainfall intensity, i = 50 mm/hr (8.333×10^{-4} m/min) with 1-hour rainfall duration is applied to the model. The result simulated from the catchments is shown in Figure 16.

The peak flow runoff discharge volume of 6,500.0 m^3/min (or, 108.33 m^3/sec) of the catchments simulated by the model is reasonable when compared to a catchments used as an example in DID [6], with the approximate size of area and rainfall intensity. Similarly, a constant rainfall intensity, i = 100mm/hr (1.667×10^{-3} m/min) is also applied in the model with the same catchments to check the stability of the model in simulating with different rainfall intensities (Figure 14).

In addition, an overland component (strip) in the catchments was tested with different rainfall durations of these rainfall intensities values and, was found that the model work as well. Also, with this overland component, different rainfall intensities amongst the elements in the system was applied to test the stability of the model in simulating a condition where the catchments system has different rainfall intensities, or some parts totally without rainfall.

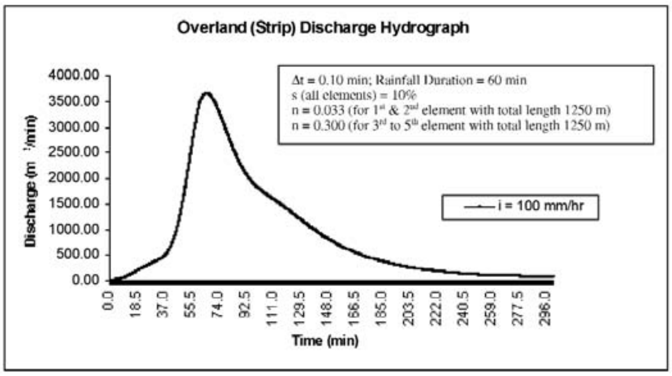


Figure 14: Comparison of fictitious large natural catchments runoff hydrographs for two rainfall intensities

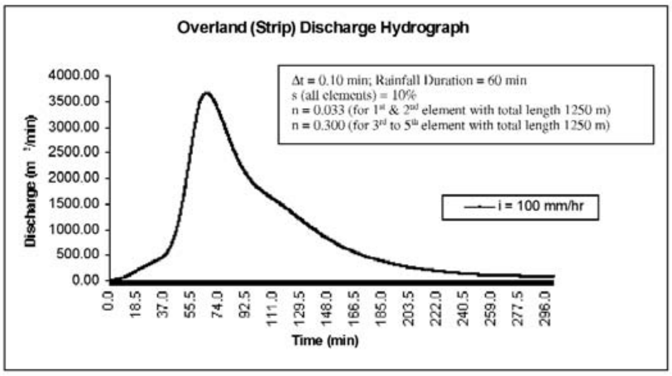


Figure 15: Oscillatory effects not evident in fictitious large natural overland component with different roughness coefficient values amongst the elements in the overland system

VARYING CATCHMENTS TOPOGRAPHY

In order to verify the capability of model to simulate rainfall-runoff events for different types of topography condition, the overland component from the catchments was also tested with different sets of physical conditions. As mentioned previously, when different values of roughness coefficient are used in the small-scale physical model for overland flow simulation, the results were oscillatory. However, when the same conditions were applied to this big overland component (divided into two equal 1,250 m

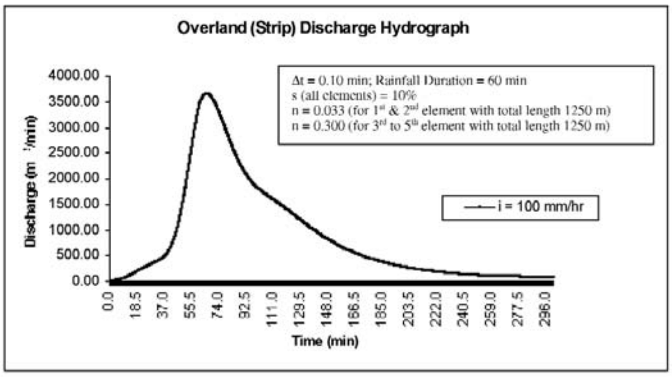


Figure 16: Oscillatory effects not evident in fictitious large natural overland component with different roughness coefficient values and slopes amongst the elements in the overland system

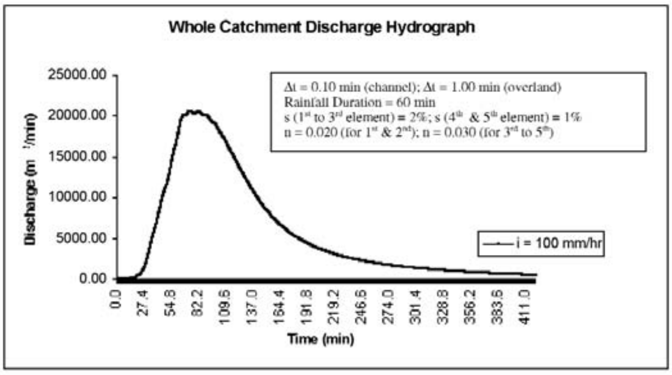


Figure 17: Oscillatory effects not evident in fictitious large natural catchments with different roughness coefficient values and slopes amongst the elements in the channel system

length with upstream roughness 0.033 and downstream roughness 0.300), the results are as illustrated in the following Figure 15.

The finite element method can work well in large-scale catchments, with different roughness coefficient values amongst the elements in the system. In addition, the same condition as used in the previous case, but with different values of plane slopes, was also simulated, and the results shown were reasonable, as in Figure 16.

It was also applied to the same whole catchments system (with channel) used previously, where the roughness coefficients and slopes used in the channel system are set with different values. The results shown in Figure 17 indicated that the model could be used to simulate a channel flow routing system with different physical conditions accurately.

CHANNELS WITH EXISTING FLOWS

All the theoretical cases discussed before were without existing flows in the channel. The model was also tested with the consideration of an existing flow in the channel. For this purpose, a volume of discharge, Q = 989 m^3/min (or, 16.48 m^3/sec), is assumed to exist uniformly in the channel. The hydrograph produced by the model for this is shown in Figure 18. This plot indicated that the existing flow would continue to discharge in combination with rainfall and lateral flows input into the channel. A similar condition was also applied to the system where a larger volume of existing flow is assumed, and where Q = 5,742 m^3/min (or, 95.70 m3/sec). The hydrograph is illustrated in Figure 19.

It can be noted that the sum total runoff discharge volume below the hydrograph (without existing flow) and the hydrograph (with existing flow only), is always equal to the runoff discharge volume of the catchments hydrograph (with existing flow, Q = 5,742 m^3/min). The outflow discharge hydrograph for the combination of existing flow with rainfall and lateral flows input, initially is contributed mainly by the existing flow in the channel. The rainfall and lateral inflows into the channel initially did not show any obvious contributions. It is illustrated that, after about twenty-one minutes, the rainfall and lateral inflows started to contribute to the channel runoff discharge. This situation would be continued until about the forty-ninth minute, whereby thereafter, the channel runoff discharge is almost only contributed by the rainfall and lateral inflows (assuming the whole volume of existing flow has been routed out). Thus the model could be used to simulate runoff discharge of a catchments system with an existing flow in the channel. For the case of a perpetual uniform existing flow, then the superimposition principle holds.

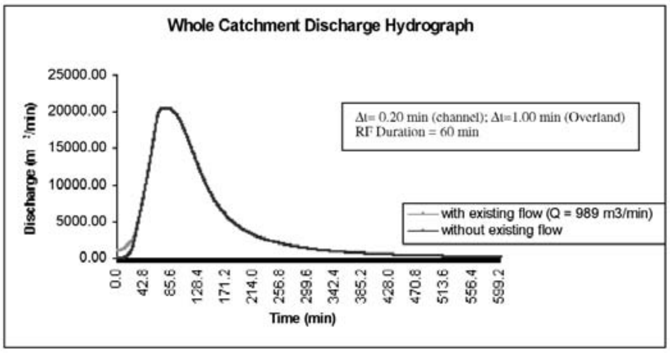


Figure 18: Simulation of fictitious large natural catchments discharge for channel without/with existing flows (lower volume)

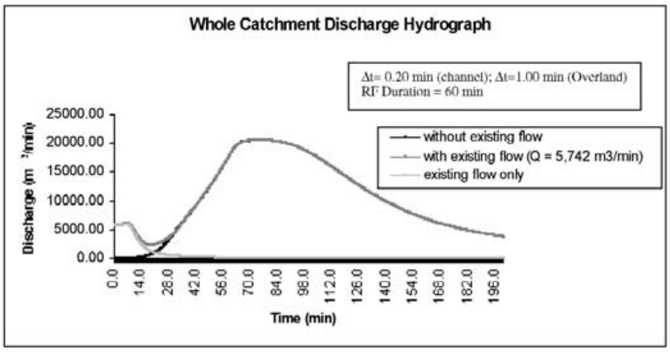


Figure 19: Simulation of fictitious large natural catchments discharge for channel without/with existing flows (higher volume)

CONCLUSIONS

The following conclusions were drawn from this study: (i) it is confirmed that the kinematics wave equation solved by the finite element standard Galerkin's residual method is able to simulate the runoff for overland plane and channel accurately; (ii) the spurious oscillatory behavior for the overland flow and channel flow can be suppressed by using a smaller time increment value (the smaller the better), governed by the Courant criterion; (iii) the temporal excess rainfall discrete scheme adopted has shown good results with less oscillatory disturbance at the point where discontinuous excess rainfall data are prescribed; (iv) spatial variations in geometry, hydrologic properties, and precipitation can be easily incorporated using geographical information systems; (v) number of elements used in runoff simulation did not significantly affect the simulated results. The consideration of number of element to be used in the model mainly depends on the topography, and/or climatic properties of the catchments. The linear and quadratic element simulation methods gave similar predictions of peak runoff volume, and the rising and receding curves pattern; (vi) simulation with differential elemental roughness whereby the upstream roughness is smaller than downstream roughness, have indicated inconsistent result in the upper end of the rising limb. However, when upstream roughness is larger than the downstream roughness, this discrepancy did not appear. Scale effects and/or storage detention at the interface by the rougher surface downstream, seems to be the reason for this phenomenon. However, all this does not appear in larger catchments and; (vii) in the recession curve of all the hydrographs, the simulated results always show faster recession and underestimation. This is due to the problem in the laboratory setting in that water through the spray nozzles could be not stopped instantaneously, upon closure of the control valve. The residual water in the pipes would contribute quite a big volume of water onto the small-scale lab runoff basin. This being in contrast to the mathematical model that assumes instantaneous cutoff.

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NOTATION

Q	*discharge overland or in channel*
A	*wetted Area, channel flow area*
R	*hydraulic radius*
L	*element length*
V	*flow velocity*
q	*lateral inflow*
n	*roughness coefficient*
Δx	*smallest element length*
Δt	*time increment*
Δt_c	*Courant time increment*
i_{max}	*maximum rainfall intensity*
x	*horizontal distance*
y	*depth of water surface*
t	*flowing time*
g	*gravity*
S_f	*friction slope*
S	*plane slope, bed slope*
m	*coefficient*