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Simulation of Multivariable System via Simulink

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ABSTRACT

The common problem in a boiler system is to control and maintain the level of water. The flow of boiler system must operate correctly where the quantity of water exited are not too low nor too high. Otherwise, another compartment sitting in front of it might be seriously affected. The aim of this study is to analysis performance drum and reheater of the boiler system. The data been analyze using MATLAB programming and shown in simulation and Simulink. Based on the simulation conducted, the results show that it has a similar pattern between the reheater operation with drum outflow.

Keywords: State Space, Simulation, Simulink, Boiler, Multivariable System.

1 INTRODUCTION

State Space Model (SSM) is a type of probabilistic graphical model that depicts the probabilistic dependence between the latent state variable and the observed measurement. The measurement or the condition can be continuous or discrete. In the 1960s, the phrase "state space" was coined in the field of control engineering. SSM is a framework for investigating both stochastic and deterministic dynamic systems that are monitored or observed using a stochastic process [1]. The SSM framework has been used to handle a wide range of dynamical systems problems in engineering, statistics, computer science, and economics. SSM are also known as hidden Markov models (HMMs) and latent process models. The Kalman SSM is the most well-studied SSM [2].

Based on the typical drum-type boiler, the feedwater is fed to the drum where the water is evaporated in a standard drum-type boiler [3, 4]. The water flows into downcomers, after which the furnace is utilized to raise the temperature of the water, resulting in evaporation. Water and steam mixtures are circulated in the drum, downcomers, and risers in this manner [5]. The steam from the risers is split in the drum before flowing through the superheater and into the high-pressure turbines. It can be recycled to the reheater's boiler, where its energy content will be boosted [6, 7].

Meanwhile, reheater is used in big steam power plant units that is above 100MW to increase Rankine cycle efficiency [8]. The temperature of the steam leaving high pressure stage of the turbine is increased again by the reheater before entering the low pressure (or intermediate pressure) stage of the turbine. A reheater is a superheater that superheats the outflow steam from turbine. The reheated steam is then sent to the low-pressure stage of the turbine. By reheating steam between high- and low-pressure turbine it is possible to increase the electrical efficiency of a power plant [9]. However, the reheat cycle is usually used in large power boilers since it is only economically feasible in larger power plants. The mathematical model for a reheater is like superheater. The difference is the steam at tempo ration flow is not considered. Hence, the parameters used in the development of a SSM for the reheater are input, output, and state parameters. The mathematical model of the state space of reheater contains system of differential and algebraic equations. Lastly, model reduction is obtained from the development of reheater state space equation [10].

2 STATE SPACE

SSM is a type of probabilistic graphical model [11] that depicts the probabilistic dependence between the latent state variable and the observed measurement. The study of multivariable system is regarded as a modern theory. According to [12], any multivariable systems with n inputs, m outputs and p state variables can be a described as sets of first order differential equations. In the state space approach, the system model is arranged in a vector matrix form. SSM are also known as hidden Markov models (HMMs) and latent process models [13]. The mathematical representation of the system can be described as follow:

$$\dot{x}(t) = \vec{A}\vec{x}(t) + \vec{B}\vec{u}(t) \tag{1}$$

$$\vec{y}(t) = \vec{C}\vec{x}(t) + \vec{D}\vec{u}(t) \tag{2}$$

where $\vec{A}$ is the state matrix, $\vec{B}$ is the input matrix, $\vec{C}$ the output matrix and $\vec{D}$ is the direct transmission matrix. If $\vec{D}$ = 0, this implies that there is no direct connection between the input $\vec{u}$ and the output $\vec{y}(t)$. Equation (1) is known as state equation whereas equation (2) is the output equation [3].

3 MODEL EQUATIONS

In this research, the state space equation of the compartment boiler systems which is Reheater and Drum is obtained from [14, 15, 16].

3.1 State Space Equation of Reheater System

State equation of reheater systems [14]

$$\begin{pmatrix} \dot{T_{rh}} \\ X_{RH} \end{pmatrix} = \begin{pmatrix} -\frac{K_{rh}W_{ro}^{0.8}}{M_r C_{rh}} & 0 \\ \frac{K_{rh}W_{ro}^{0.8}}{V_{rh}} & -\frac{W_{ro}}{V_{rh}\rho_{rh}} \end{pmatrix} \begin{pmatrix} T_{rh} \\ X_{rh} \end{pmatrix} + \begin{pmatrix} \frac{1}{M_r C_{rh}} & 0 & \frac{K_{rh}T_r}{M_r C_{rh} W_{ri}^{0.2}} \\ 0 & \frac{W_{ri}}{V_{rh}} & -\frac{K_{rh}T_r}{V_{rh}W_{ri}^{0.2}} \end{pmatrix} \begin{pmatrix} Q_{rs} \\ h_{ri} \\ W_{ri} \end{pmatrix} \tag{3}$$

Output equation of reheater systems

$$\begin{pmatrix} P_{ro} \\ T_r \end{pmatrix} = \begin{pmatrix} 0 & R_r \left[\frac{h_{ro}-h_{ref}+C_{pr}T_{ref}}{h_{ro}C_{pr}}\right] \\ 1 & 0 \end{pmatrix} \begin{pmatrix} T_{rh} \\ X_{RH} \end{pmatrix} \tag{4}$$

Matrices A, B, and C in the state space equations of Reheater

$$A = \begin{bmatrix} -\frac{k_{rh}w_{ri}}{M_{rc}C_{rh}} & 0 \\ \frac{k_{rh}w_{ri}}{V_{rh}} & -\frac{w_{ro}}{\rho_{rh}} \end{bmatrix} \quad B = \begin{bmatrix} -\frac{1}{M_{rc}C_{rh}} & 0 \\ 0 & \frac{w_{ri}}{V_{rh}} \end{bmatrix} \quad C = \begin{bmatrix} 0 & \left(\frac{h_{ro}-h_{ref}+c_{pr}T_{ref}}{c_{Pr}}\right) \\ 1 & 0 \end{bmatrix}$$

State Vector Input-Output Vector

$$\bar{x}(t) = \begin{pmatrix} T_{rh} \\ X_{RH} \end{pmatrix} \qquad \bar{u}(t) = \begin{pmatrix} Q_{rs} \\ h_{ri} \\ W_{ri} \end{pmatrix}$$

$$\bar{y}(t) = \begin{pmatrix} P_{ro} \\ T_r \end{pmatrix}$$

3.2 State Space Equation of Drum Systems

State equation of Drum systems [15, 16]

$$\begin{pmatrix} \dot{m_{dL}} \\ x_{D1} \end{pmatrix} = \begin{pmatrix} -\frac{V_{dw}}{V_L} & 0 \\ 0 & -\frac{V_{dw}}{V_L} \end{pmatrix} \begin{pmatrix} m_{dL} \\ x_{D1} \end{pmatrix} + \begin{pmatrix} 1 & 1-x \\ h_e & (1-x)h_{wr} \end{pmatrix} \begin{pmatrix} W_e \\ W_r \end{pmatrix} \tag{5}$$

Output Equation of Drum Systems

$$\begin{pmatrix} h_w \\ w_d \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{V_L\rho_w} \\ \frac{V_{dow}}{V_L} & 0 \end{pmatrix} \begin{pmatrix} m_{dL} \\ x_{D1} \end{pmatrix} \tag{6}$$

Matrices A, B, and C in state space equations of Drum

$$A = \begin{bmatrix} -\frac{V_{dw}}{V_L} & 0 \\ 0 & -\frac{V_{dw}}{V_L} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1-x \\ h_e & (1-x)h_{wr} \end{bmatrix} \quad C = \begin{bmatrix} 0 & \frac{1}{V_L\rho_w} \\ \frac{V_{dow}}{V_L} & 0 \end{bmatrix}$$

State Vector Input-Output Vector

$$\bar{x}(t) = \begin{pmatrix} m_{dL} \\ x_{D1} \end{pmatrix} \quad \bar{u}(t) = \begin{pmatrix} W_e \\ W_r \end{pmatrix} \qquad \bar{y}(t) = \begin{pmatrix} h_w \\ w_d \end{pmatrix}$$

4 PARAMETER AND SIMULATION

Measure data from boiler of power plant were used for parameter estimation purpose. Below is the parameters and units for parameter Reheater and Drum.

Table 1: Parameter of Reheater systems.

Parameter	Definition	Units
p_ri	pressure of steam at the inlet to reheater	kg/s
w_ri	flow of steam at the inlet to the reheater	kg/s
T_ri	inlet steam temperature	$°K$
Q_rs	heat flow from the furnace	J/s
h_ri	specific enthalpy of inlet steam	J/kg
T_rh	reheater metal tube temperature	$°K$
p_ro	outlet steam pressure	Pa
T_r	reheater steam temperature	$°K$
h_ro	specific enthalpy of outlet steam	J/kg
Q_rh	heat transferred to the steam	kg/m^3
rho_rh	steam density in the reheater	kg/m^3
x_RH1	h_ro * rho_rh	
w_ro	flow of steam at the outlet from the reheater	kg/s
k_rh	experimental heat transfer coefficient	$J/(kg * °K)$
V_rh	reheater volume	m^3
M_r	reheater mass	m^3
C_rh	heat capacitance of superheater tubes	$J/(kg * °K)$
Cp_ref	ideal gas reference specific heat	$J/(kg * °K)$
T_ref	ideal gas reference temperature	$°K$
h_ref	ideal gas reference specific enthalpy	J/kg
rho_rh	steam density in the reheater	kg/m^3
T_rh	reheater metal tube temperature	$°K$
X_RH1	h_ro * rho_rh	
w_ro	outlet steam mass flow	kg/s

Table 2: Parameter of Drum systems.

Parameter	Definition	Units
h_e	specified enthalpy of water from the economizer	J/kg
v_dow	volumetric water flow to the downcomer	m^3/s
w_e	water flow from the economizer	kg/s
Q_ir	heat flow from the furnace	J/s
w_v	steam flow to the superheater	kg/s
p_v	drum outlet steam pressure	Pa
rho_v	drum outlet steam density	kg/m^3
h_v	drum outlet steams specific enthalpy	J/kg
h_r	liquid-vapour mixture specific enthalpy	J/kg
T_rt	risers metal tube temperature	$°K$

w_r	liquid vapour mixture mass flow from the risers	kg/s
rho_w	drum water density	kg/m^3
w_d	water mass flow to the downcomer	kg/s
m_dl	drum liquid mass	kg
L	drum water level	m
x_qa	steam quality	$-$
T_w	drum water temperature	$°K$
V	volume of the drum	m^3
k_ec	evaporation coefficient	$kg/(°K * s)$
r	drum radius	m
w_ec0	steady-state evaporation constant	kg/s
k_r	experimental heat transfer coefficient	$J/(s * °K^3)$
M_r	mass of riser's metal tubes	kg
C_rt	metal specific heat	$J/(kg * °K)$
tau_r	mass flow time constant	s
m_dl	drum liquid mass	kg
x_D1	h_w*m_dl	
x_D2	rho_v*v_v where V_v	
T_rt	risers metal tube temperature	$°K$
w_r	liquid-vapour mixture mass flow from the risers	kg/s

Figure 1 shows the outlet steam pressure (P_{ro}) of reheater system is increases from 0 Pa to 7.33×10^6 Pa at t=0 until t=60. Then, the outlet steam pressure becomes constant at 7.33×10^6 Pa starting at t=60. While the reheater steam pressure (T_r) increases from 0 °K to 1022.31 °K at t=0 to t=55. Then, the reheater steam pressure becomes constant at 1022.31 °K starting at t=55. The result obtained in Figure 1 is similar with the output at [15].

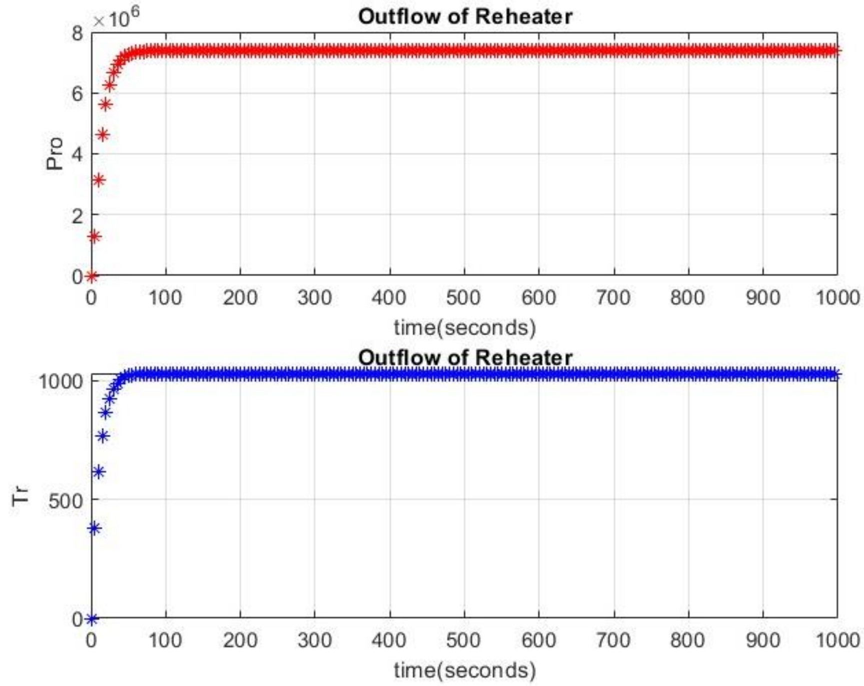

Figure 1: Outflow of Reheater.

A dynamic simulator in MATLAB/SIMULINK (MATLAB, 2017) is used to generate the output. Figure 2 and Figure 3 shows the simulation of reheater system by using SIMULINK. Both outputs from MATLAB and Block Diagram shows the same result as given before (see Figure 1, 2 and 3).

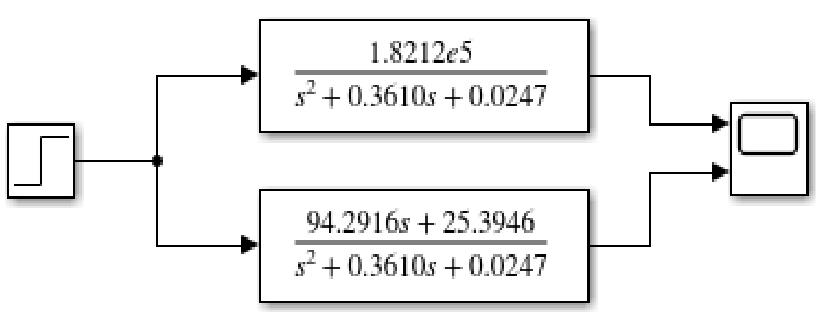

Figure 2: The simulation of Reheater system via Simulink.

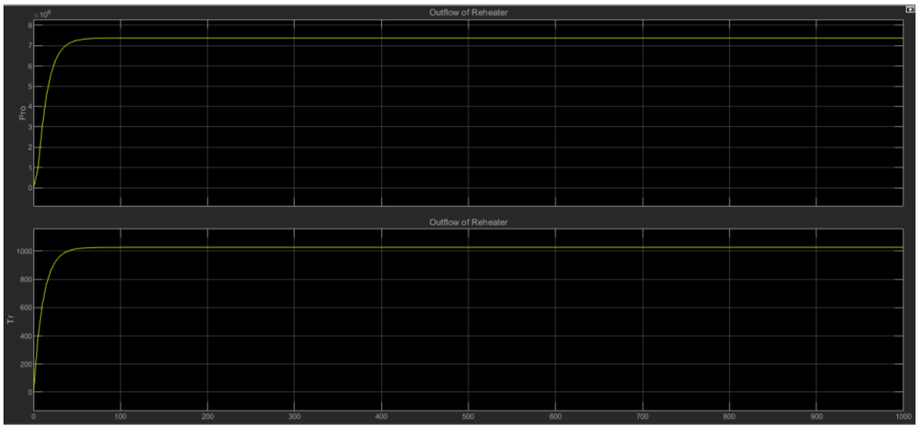

Figure 3: Outflow of Reheater via Simulink.

The first graph shows the Specific Enthalpy of Drum Water (h_w) increases from 0 Pa 9.95$\times$ 10^{-3} J/kg at t=0 until t=45. Then, it becomes constant at 9.95$\times$ $10^{-3}$$J/kg$ from t=45 to t=1000. Next, the second graph shows the Water Mass Flow Out (w_d) to the Downcomer increases from 0 Pa 9.97$\times$ 10^{-1} kg/s at t=0 until t=40. Then, it becomes constant at 9.97$\times$ 10^{-1} kg/s from t=40 to t=1000

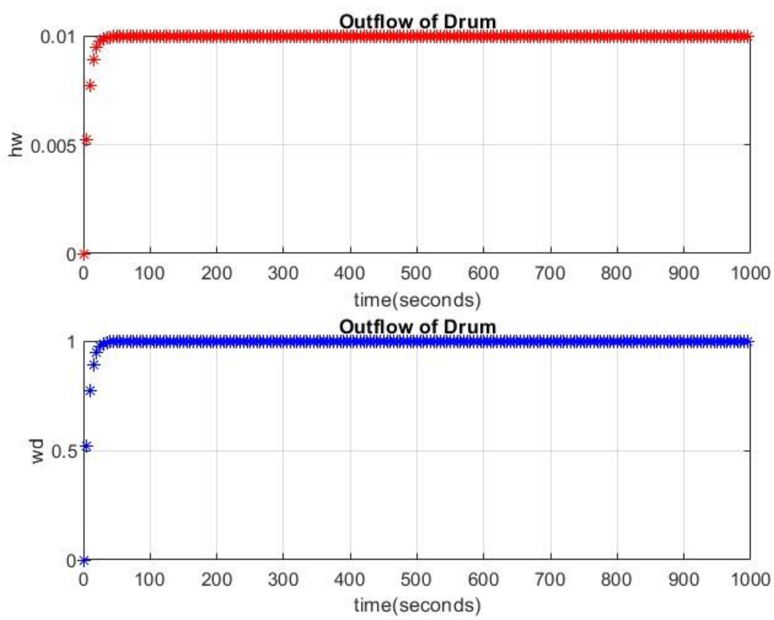

Figure 4: Outflow of Drum

Figure 2 and Figure 3 shows the simulation of Drum systems by using SIMULINK. Both outputs from MATLAB and Block Diagram shows the same result as given before (see Figure 4, 5 and 6).

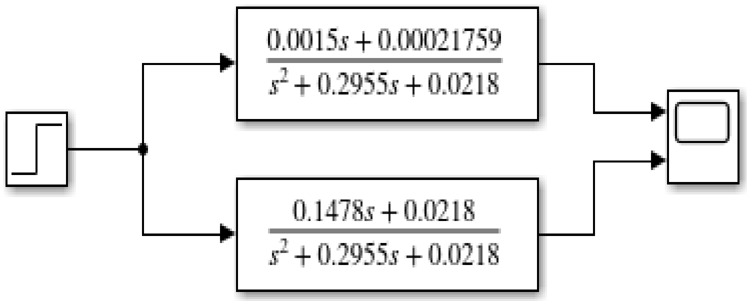

Figure 5: Drum Simulink

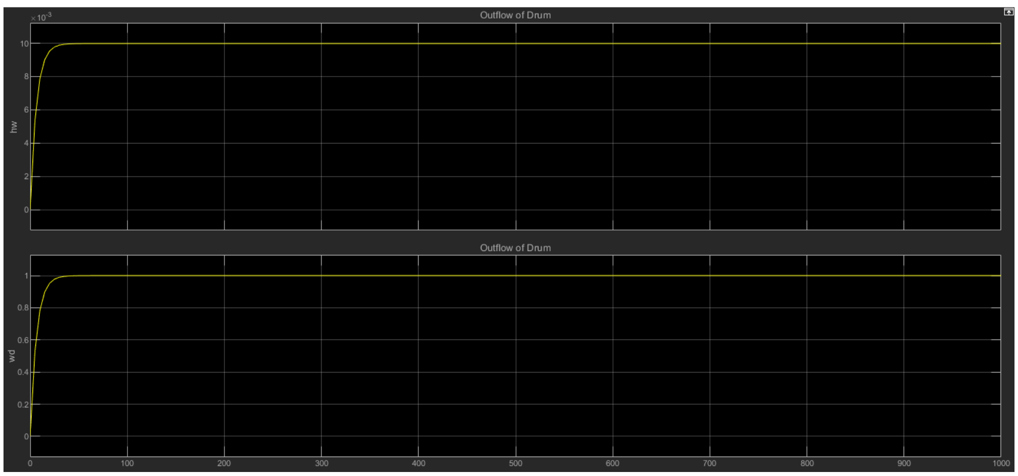

Figure 6: Outflow of Drum via Simulink.

5 CONCLUSION

This research successfully to find the level of water flow out from drum and reheater. The result shows that it has a similar pattern between the reheater operation with drum outflow. Hence this can justify the operation system of boiler are operated correctly.

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