SEIR Model Simulation and Dynamical Behaviour for Rabies Transmission in Dog Populations

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ABSTRACT

Dogs are the primary source of human rabies deaths, contributing to up to 99% of all human rabies transformations. The infection has caused tens of thousands of deaths every year all around the world. This paper discussed rabies transformation in dog populations and how the absence and presence of vaccination will transform the disease dynamics of rabies. Different control mediums like vaccination, culling and birth rate of dog populations are considered to reduce rabies transmission. By applying both the SEI and SEIR models, stability analysis for disease-free and endemic equilibrium points is determined by means of Maple and MATLAB software. The findings suggested that the spread of rabies can be reduced by increasing dog vaccination, expanding dog culling within acceptable ranges, and reducing annual newborn puppies simultaneously. In this regard, high vaccination rates must be implemented to control and, in turn, eliminate rabies transmission.

Keywords: *SEI* model, *SEIR* model, Rabies, Simulation, Stability analysis.

1   INTRODUCTION

Rabies is a zoonotic virus transmitted by the saliva of infected animals. The virus is spread through foxes, dogs, bats, raccoons, and cats scratching or biting [1]. However, dogs are the main spreaders of this virus. The infection produced by viruses of the genus Lyssavirus, family Rhabdoviridae, and order Mononegavirales [2] can impact the central nervous system, resulting in brain inflammation, paralysis, and death [3].

Rabies occurs in about 150 countries and areas worldwide, and its prevalence is particularly high in emerging regions such as Africa and Asia (about 95% of human deaths occur). Due to encounters with domestic animals such as dogs, rabies is more prevalent in rural areas with low socioeconomic status. Every year, 55,000 people die from rabies worldwide, which is expected to rise due to a lack of appropriate monitoring and reporting on country development. The number of children bitten by infected animals is more than 40% because they are more inclined to play with animals [4].

Rabies was first recorded in China around 556 BC and is still a major public health issue today. From 1950 through 2015, China reported 130,494 human rabies cases, an average of 1977 cases per year. Dog attacks cause an estimated 95% of human rabies cases in mainland China [1]. Furthermore, China comes in second only to India in the number of persons killed by rabies each year [5].

Although rabies is almost as deadly, it is easily prevented with the available tools. Vaccination is highly effective for rabies pre-exposure prophylaxis (PrEP) in humans and animals, and immediate post-exposure prophylaxis (PEP) with vaccination plus rabies immune globulin (RIG) can effectively prevent infections in humans [6]. According to Schneider et al. [7], rabies can be prevented through effective and structured campaigns for regularly vaccinating vulnerable dogs and humans. This tendency will help to avoid disease transfer from infected to exposed dogs.

Mathematics has allowed us to understand and control the spread of infectious diseases, and it is a powerful tool for analysing and forecasting the dynamics of the phenomenon. Studies on rabies in various countries using various mathematical models have existed for some time. Demirci [8] conducted a study on the transmission of rabies within a single population and investigated the stability of the solution by using an integer order susceptible-infected-recovered (*SIR*) model and a fractional order model. Meanwhile, Musaili and Chepkwony [3] also developed a susceptible-infectious-recovered-susceptible (*SIRS*) mathematical model using a system of ordinary differential equations to investigate the transmission dynamics of rabies by incorporating the impact of public health education to eradicate rabies among the dog population. The findings indicate that public health education is an effective method of controlling rabies.

The susceptible-exposed-infectious (*SEI*) model, on the other hand, has previously been used by many researchers to study rabies. Bornaa et al. [9] used the model to investigate the dynamics of rabies infection in dogs and the human population exposed to dog bites. In contrast, Leung and Davis [10] investigated estimating vaccination targets for stray dogs in controlling the spread of canine rabies when the dog population consists of stray, owned free-roaming, and owned confined dogs. Furthermore, Masud et al. [11] applied the *SEI* model to investigate the role of vaccination and dog population management in rabies control among free-roaming street dogs (FRSDs) in Dhaka, Bangladesh. The outcomes of this research indicate that immunizations alone are insufficient to control rabies in FRSs, and that food waste management should be considered an additional control method.

Furthermore, Zhang et al. [5] and Ruan [1] proposed the susceptible-exposed-infectious-recovered (*SEIR*) model to investigate rabies virus transmission between dogs as well as from dogs to humans. Zhang et al. [5] performed some sensitivity analysis and compared the effects of culling and immunization of dogs. Simultaneously, Ruan [1] used the *SEIR* model to simulate human rabies data in China from 1996 to 2010 and proposed prevention and control measures for rabies transmission on the Chinese mainland. Ega et al. [4] also formulate the *SEIR* model for rabies transmission in humans and animals in and around Addis Abeba, Ethiopia, by incorporating a dog vaccination programme. The authors also improved Zhang et al. models to include the livestock population. Many other researchers have used the *SEIR* model to study rabies transmission in dogs and humans for several years, including [2,12,13]. Taib et al. [14] investigated the dog to human rabies transmission in Sarawak by employed the susceptible-exposed-infectious-recovered-susceptible (*SEIRS*) model introduced by Zhang et al. [5] and implementing dog vaccination. In contrast, Renald et al. [15] developed the susceptible-exposed-infectious-vaccinated (*SEIV*) model while also taking the control methods of vaccination and mass-culling into account.

Many researchers have done their studies on rabies transmission, focusing on both dog and human populations using the *SEIR* model. At the same time, there was a lack of discussion on analyzing the dog population only in most previous research. Therefore, this study will only focus on rabies transmission in dog populations using the *SEIR* model. This paper aims to demonstrate the spread of rabies in dog populations in the absence and presence of vaccination, as well as to investigate the impact of vaccination, culling, and birth rate on dog populations in reducing rabies transmission. The study's findings will give a better understanding of the disease to society and health agencies in controlling the spread of rabies. The following is the outline for this paper: Section 2 presents the formulation of *SEI* and *SEIR* models and the determination of disease-free and endemic equilibrium points. Section 3 discusses the stability analysis results and the dynamic behaviour of a dog population when some chosen parameters are varied. Section 4 highlights the conclusions and recommendations.

2   MATERIAL AND METHODS

This section highlights the mathematical modeling formulation of rabies transmission in dog populations in the absence and presence of vaccination strategies.

2.1   *SEI* Model Formulation

The first mathematical model is the *SEI* model, in which no vaccination treatment is provided to the dog population. This model was based on Taib et al. [14] research, but the recovered dog classes were ignored since there were no vaccinations involved in this model. Furthermore, the human population compartments are also excluded since this research only focuses on analyzing the dog population. And thus, Figure 1 depicts the *SEI* model compartment of susceptible, exposed and infected dog populations.
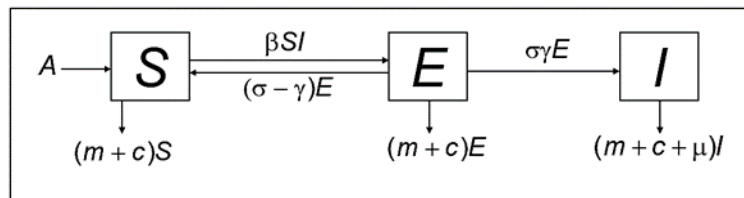

Figure 1: Rabies transmission among dog populations in the absence of vaccination.

For the *SEI* model, the population of dogs are classified into three groups, which are susceptible (S), exposed (E) and infected (I). It is assumed that only susceptible dogs give birth to susceptible newborns at the rate A. Susceptible dogs contract the disease at a proportional rate to the number of infected dogs. They enter the exposed stage before becoming infectious, in which they are infected but still not infectious.

All parameter is considered to be positive. σ represents the length of time infected dogs stay infectious, and γ is the clinical outcome risk factor for exposed dogs. Therefore, $\sigma\gamma E$ denotes exposed dogs that develop clinical rabies, whereas $(\sigma - \gamma)E$ represents dogs that do not get clinical rabies and revert to the susceptible category. The rabies transmission by interactions between infected and

susceptible dogs is expressed in βSI. In contrast, c is the dog culling rate since culling infected dogs is necessary to control the outbreak [5]. Finally, the natural mortality rate is denoted by m, while the disease-related death rate is defined by μ. Through these assumptions, the mathematical model is expressed by a set of differential equations as follows:

$$\frac{dS}{dt} = A + (\sigma - \gamma)E - \beta SI - (m + c)S$$

$$\frac{dE}{dt} = \beta SI - (m + \sigma + c)E \tag{1}$$

$$\frac{dI}{dt} = \sigma\gamma E - (m + c + \mu)I .$$

By employing the next generation matrix, the basic reproduction number, R_0 is represented by

$$R_0 = \frac{\beta\sigma\gamma A}{(m+\sigma+c)(m+\mu+c)(m+c)} .$$

R_0 value depicts an average of the infectious dog that transfers to another number of n susceptible dog population. If the value of $R_0 > 1$, rabies occurs. Meanwhile, for $R_0 < 1$, rabies does not occur. In other words, rabies dies out. For $R_0 = 1$, rabies occurs, and the disease remain constant for one infected dog transmit to another one susceptible dog.

To find the equilibrium points, the *SEI* model (1) assumable to be equal to zero. There are at most two equilibrium points in the system:

i)   Free-disease equilibrium point, $P_0 = \left(\frac{A}{m+c}, 0, 0\right)$. This equilibrium point indicates an
        entirely healthy dog population in the absence of rabies.

ii)   Endemic equilibrium point, $P_1 = (S^*, E^*, I^*)$ where

$$S^* = \frac{(c + m + \sigma)(c + m + \mu)}{\beta\gamma\sigma}$$

$$E^* = \frac{A\sigma\gamma\beta - c[c^2 + 3m(m + c) + (\mu + \sigma)(2m + c) + \mu\beta] - m(\mu + \sigma)(m + \mu)}{\sigma\gamma\beta(\sigma\gamma + c + m)}$$

$$I^* = \frac{A\sigma\gamma\beta - c[c^2 + 3m(m + c) + (\mu + \sigma)(2m + c) + \mu\beta] - m(\mu + \sigma)(m + \mu)}{\beta(c + m + \mu)(\sigma\gamma + c + m)}$$

This equilibrium point describes the presence of rabies in dog populations in the absence of vaccination.

The Jacobian matrix is then used to analyse the stability of these two equilibrium points:

$$J_{SEI} = \begin{bmatrix} -\beta i - c - m & \sigma(1-\gamma) & -\beta S \\ \beta i & -m - \sigma - c & \beta S \\ 0 & \sigma\gamma & -c - m - \lambda \end{bmatrix} \qquad (2)$$

By substituting these two equilibrium points into the Jacobian matrix (2), the eigenvalues of each equilibrium point are then determined. If the sign of the eigenvalues is negative, the system is asymptotically stable. Otherwise, the system is unstable if the sign of the eigenvalues is positive. Whereas an unstable saddle occurs when the eigenvalues have the opposite sign.

2.2 *SEIR* Model Formulation

This section discusses the *SEIR* model with the presence of vaccination in the dog population. This model is an extension of the *SEI* model (1) by adding another compartment, the recovered classes (*R*). In this model, the vaccination rate, v, is introduced, and the parameter of λ represents the loss rate of vaccination immunity. Figure 2 depicts a compartment model of rabies transmission among dogs in the presence of vaccination.
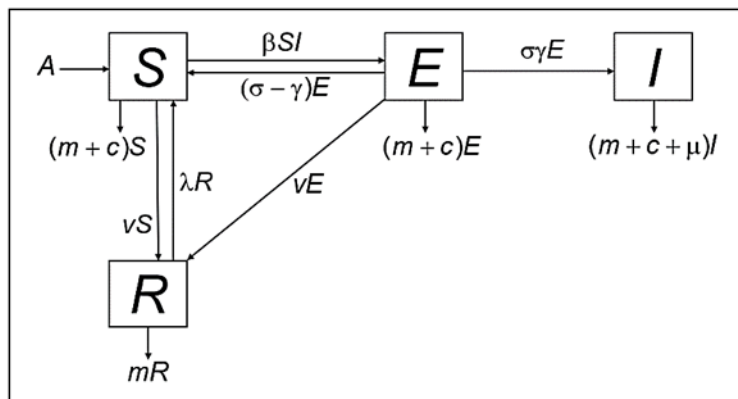

Figure 2: Rabies transmission among dog populations in the presence of vaccination.

Subsequently, the *SEIR* model is demonstrated as follows:

$$\frac{dS}{dt} = A + \lambda R + (\sigma - \gamma)E - \beta SI - (m + v + c)S$$

$$\frac{dE}{dt} = \beta SI - (m + \sigma + v + c)E \qquad (3)$$

$$\frac{dI}{dt} = \sigma\gamma E - (m + \mu + c)I$$

$$\frac{dR}{dt} = v(E + S) - (m + \lambda)R.$$

Next, the equilibrium points for the *SEIR* model (3) are given by:

i)   Free-disease equilibrium point, $Q_0 = (S^0, 0, 0, R^0)$ such that

$$S^0 = \frac{A(m+\lambda)}{\lambda(m+c)+m(m+v+c)}$$

$$R^0 = \frac{vA}{\lambda(m+c)+m(m+v+c)}$$

ii)   Endemic equilibrium point, $Q_1 = (S^*, E^*, I^*, R^*)$ where

$$S^* = \frac{S^0}{R^0}$$

$$E^* = \frac{I^*(\mu+m+c)}{\sigma\gamma}$$

$$I^* = \frac{S^0\sigma\gamma v\lambda+\left(AR_0-S^0(m+v+c)\right)\sigma\gamma(m+\lambda)}{\sigma\beta\gamma(m+\lambda)S^0-R_0(m+\mu+c)(\sigma(1-\gamma)(m+\lambda)+v\lambda)}$$

$$R^* = \frac{vS^0}{(m+\lambda)R_0}+\frac{v(m+\mu+c)}{\sigma\gamma(m+\lambda)}$$

This equilibrium point describes the presence of rabies in dog populations in the presence of vaccination strategies.

Next, the basic reproductive number is expressed as:

$$R_0 = \frac{\beta\sigma\gamma S^0}{(m+\sigma+v+c)(m+\mu+c)}.$$

The stability of these two equilibrium points is then investigated using the Jacobian matrix:

$$J_{SEIR} = \begin{bmatrix} -\beta i - c - v - m & \sigma - \gamma & -\beta S & \lambda \\ \beta i & -m - v - \sigma - c & \beta S & 0 \\ 0 & \sigma\gamma & -c - m - \mu & 0 \\ v & v & 0 & -\lambda - m \end{bmatrix}. \qquad (4)$$

The stability analysis is carried out using Maple software, whereas MATLAB software is used to simulate and analyse the dynamical behaviour of dog populations based on models (1) and (3). The parameter values used in both models are listed in Table 1.

Table 1: The parameter values used in the *SEI* (1) and *SEIR* (3) model.

Parameter	Description	Standard value	Source
A	Annual birth rate	3×10^6	[5]
λ	Vaccination immunity loss rate in dogs	1	[5]
γ	Clinical outcome risk in exposed dogs	0.4	[5]
σ	The reciprocal of the dog incubation period	6	[5]
m	Dog natural death rate	0.08	[5]
β	Dog transmission rate	1.58×10^{-7}	[5]
v	Dog vaccination rate	0.09	[5]
μ	Dog disease-related death rate	1	[5]
c	Dog culling rate	0.05	Assumed

3 RESULTS AND DISCUSSION

3.1 Stability Analysis

This section discusses the result of stability analysis and the reproductive number of the *SEI* and *SEIR* models in further detail. Using the parameter values displayed in Table 1, the basic reproductive number, R_0, of rabies transmission in the absence of vaccination is estimated to be $R_0 = 1.26$, indicating the occurrence of an epidemic. In addition, the estimated basic reproductive number for rabies transmission in the presence of vaccination was $R_0 = 1.18$. It can be observed that the reproductive numbers are decreasing due to the vaccination programme. Therefore, it is possible to conclude that vaccination can reduce the spread of rabies in dog populations.

Next, the stability analysis was conducted for disease-free and endemic equilibrium points. To determine the eigenvalues, all the equilibrium points from the *SEI* and *SEIR* models are substituted into the Jacobian matrices (2) and (4), respectively. Thus, the stability analysis results for both models are summarized in Tables 2 and 3.

Table 2: Stability analysis results for *SEI* model.

Equilibrium point, $P = (S, E, I)$	Eigenvalues	Stability result
$P_0 = (2.308 \times 10^7, 0, 0)$	$\lambda_1 = 0.2431$	Unstable
	$\lambda_2 = -0.1230$	
	$\lambda_3 = -7.5031$	
$P_1 = (1.827 \times 10^7, 2.471 \times 10^5, 5.249 \times 10^5)$	$\lambda_{1,2} = -0.0831 \pm 0.1598i$	Stable
	$\lambda_3 = -7.3067$	

Table 3: Stability analysis results for *SEIR* model.

Equilibrium point, Q= (S, E, I, R)	Eigenvalues	Stability result
$Q_0 = (2.195 \times 10^7, 0, 0, 1.829 \times 10^6)$	$\lambda_1 = 0.1722$	Unstable
	$\lambda_2 = -0.1257$	
	$\lambda_3 = -1.1743$	
	$\lambda_4 = -7.5222$	
$Q_1 = (1.853 \times 10^7, 1.84 \times 10^5, 3.909 \times 10^5, 1.60 \times 10^6)$	$\lambda_1 = -1.1744$	Stable
	$\lambda_{2,3} = -0.0763 \pm 0.1274i$	
	$\lambda_4 = -7.3848$	

As shown in Tables 2 and 3, the *SEI* and *SEIR* models represent two equilibrium points with corresponding eigenvalues. The eigenvalues of disease-free equilibrium points P_0 and Q_0, have the opposite sign, resulting in an unstable pattern. In contrast, all eigenvalues are negative for the endemic equilibrium points, P_1 and Q_1, resulting in a stable pattern. Since the basic reproductive number for both models are greater than one, we have already proved that disease-free and endemic equilibrium point stability results are unstable and stable, respectively [16].

3.2   Rabies Transmission in Dog Populations in the Absence and Presence of Vaccination

This section discusses the numerical results of the *SEI* and *SEIR* models in the absence and presence of vaccination treatment. This section aims to compare the intervention effects of these two models for 20 years. For the *SEI* model, rabies transmission is controlled solely through the culling of dogs. However, the *SEIR* model controls rabies through dog culling and vaccination. The simulation results are represented in Figure 3.
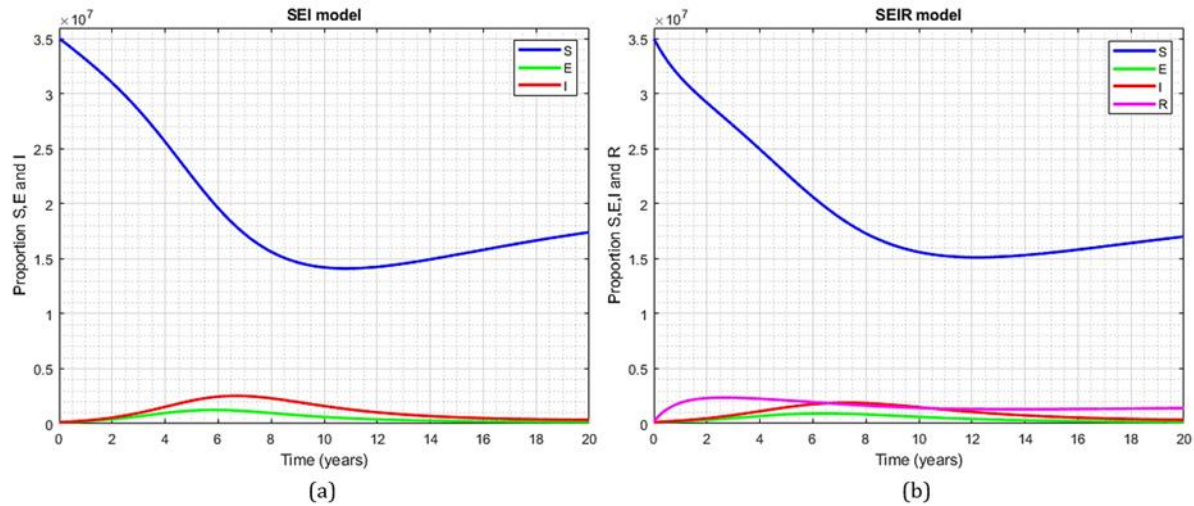
Figure 3: The *SEI* and *SEIR* model simulations in the (a) absence and (b) presence of vaccination.

Figure 3(a) demonstrates that the susceptible dog population decreases slightly over 11 years, from 3.5×10^7 to 1.4×10^7 dogs, before growing gradually for the remaining years. In contrast, Figure 3(b) shows that the susceptible dog population decreases sharply in the first 3 years, from 3.5×10^7 to 2.7×10^7 dogs, then gradually declines to 1.6×10^7 dogs in the next 9 years and begin to increase for the rest of years. This situation arises due to incorporating vaccination strategies into the *SEIR* model. Thus, the number of recovered dogs increased for the first 3 years, beginning to decline slightly and remaining constant at 0.5×10^7 dogs in subsequent years.

While the exposed dog population in Figure 3(a) shows a slight increase to 0.5×10^7 dogs in the first 6 years before steadily declining for the rest of the years, the exposed dog population in Figure 3(b) increased very slowly. It nearly flattened within 4 to 8 years before steadily declining for the rest of the years.

Furthermore, Figure 3(a) shows that the infected dog population increased to 0.27×10^7 in the first 7 years before gradually declining for the next 13 years. Conversely, Figure 3(b) shows that the infected dog population increased to 0.2×10^7 for 8 years before gradually declining for the remaining 12 years. In summary, the influence of vaccination on the dog population revealed a decline in the number of exposed and infected dogs in the *SEIR* model.

3.3 Dynamical Behaviour of Susceptible, Exposed, Infected and Recovered Dog Populations

This section presents the numerical results of the *SEIR* model, where the vaccination strategies are treated in the susceptible dogs. We conduct sensitivity analysis on several model parameters by simulating the model for 50 years while the parameter values are varied. We aim to illustrate and discuss the impact of varying vaccination rates (v), culling rates (c), and birth rates (A) on infected dogs. The numerical simulations and analyses are predicated on the standard values shown in Table 1. The results are shown in Figures 4, 5 and 6.
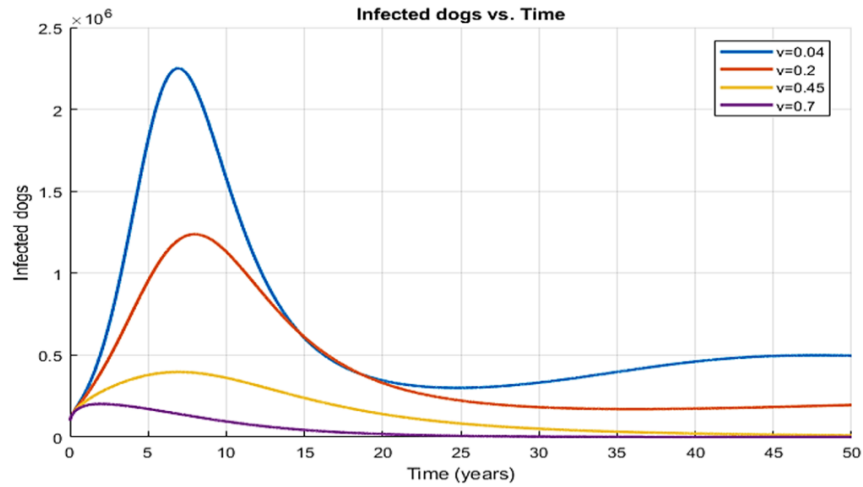
Figure 4: The effect of varying vaccination rate, v on the infected dogs, $I(t)$.

First, we explore the influence of vaccination parameter, v on infected dogs as illustrated in Figure 4. The varying vaccination rates of $0.04, 0.2, 0.45$ and 0.7 are considered for observing behavioural changes in infected dog populations. These values represent the proportion of total susceptible dogs that have been vaccinated. It can be observed that as vaccination rates increase, the number of infected dogs reduces. A second rabies outbreak will probably occur if the vaccination rate is less than 20%. When the dog vaccination rate is at least 45%, the number of infected dogs will decrease after the initial outbreak and will be eradicated in the years ahead. Furthermore, an earlier eradication of the disease is possible if vaccination rates of susceptible dogs can be increased to at least 70%. This finding agrees with the World Health Organization's vaccination coverage guidelines [17]. As a result, vaccination appears to be the most effective method for preventing the spread of rabies.
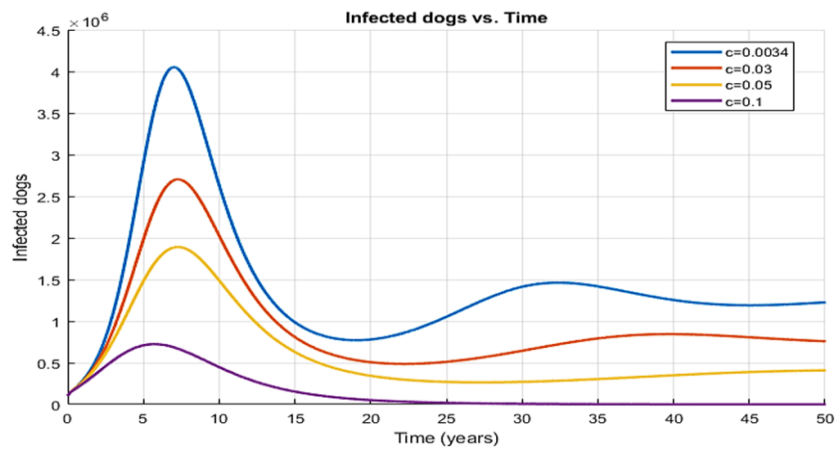

Figure 5: The effect of varying culling rate, c on the infected dogs, $I(t)$.

According to several studies [18-20], culling infected dogs is necessary for controlling the outbreak in the short term. Figure 5 illustrates the trajectory of infected dogs at various levels of culling. This analysis considers varying culling rates of 0.0034, 0.03, 0.05 and 0.1, which represents the proportion of culled dogs. The findings suggest that the culling of 10% of infected dogs each year will contribute to rabies extermination. Rabies will remain in the population if the culling rate is less than 10%. Regardless, it is not recommended to cull dogs at a higher rate because it is unacceptable to most pet owners. In addition, as stated in the GARC [21], mass dog culling is not practical for rabies control because it creates community problems. This is due to the fact that culling is frequently met with mass protests, both within and outside the local area, as the methods employed are likely inhumane [22].
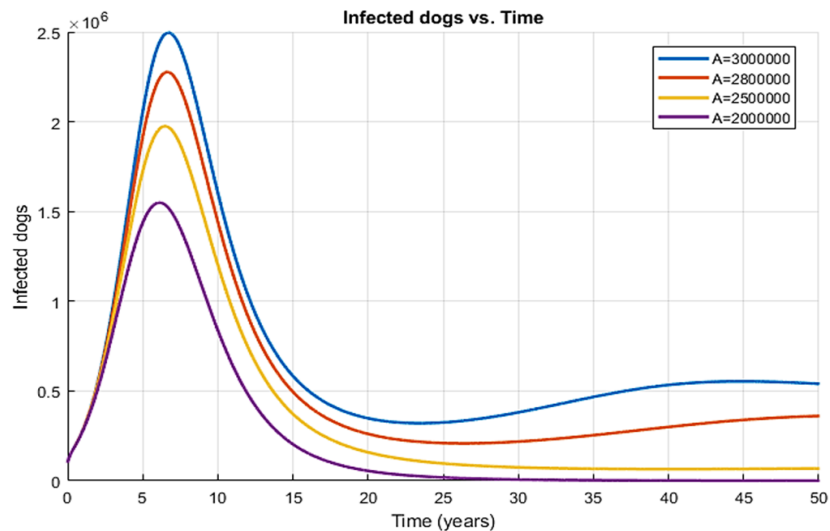

Figure 6: The effect of varying birth rate, A on the infected dogs, $I(t)$.

Based on Zhang's research, human rabies can be controlled by reducing the annual crop of newborn puppies [23]. Correspondingly, we are also interested in investigating how the number of puppies born each year affects the number of infected dogs. As depicted in Figure 6, reducing the annual number of newborn puppies would not prevent a second outbreak. Rabies could only be eradicated when the annual birth rate of puppies falls below 2×10^6. Hence, this analysis demonstrates that limiting dog birth would contribute to the eradication of rabies by decreasing the number of susceptible dogs entering the population.

4   CONCLUSION

In this paper, the *SEI* and *SEIR* models are employed to control and predict rabies transmissions in dog populations. The stability analysis of the disease-free and endemic equilibrium points of both models was presented and discussed. Our analysis suggests that rabies can be treated by increasing dog vaccination coverage, increasing dog culling according to acceptable ranges, and reducing annual newborn puppies simultaneously. To summarise, high vaccination levels must be maintained to prevent rabies transmission. Canine vaccination is thus the foundation of the global strategy to end

human rabies, which intends to eliminate dog-transmitted rabies-related human deaths by 2030 [24]. Vaccination campaigns for dogs should select an excellent strategy to achieve 70% vaccination coverage as early as possible [25]. The other strategy is to restrict dog movements into and out of infected areas or sites in order to limit the chances of potentially infected animals coming into contact with susceptible animals. This will minimize the number of vulnerable animals in an affected region, hence preventing disease transmission. This research, however, is simply based on the data and parameter values from the previous study. As a suggestion for future research, we should use actual data to analyze the rabies situation in other countries, particularly Malaysia. This is due to the low survival rate of rabies victims in Malaysia, making it a significant public concern.

ACKNOWLEDGEMENT

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