Fuzzy Variable Structure Control with Reduced-order Observer for Micro Satellite Stabilization in Space

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Abstract- **This paper presents a hybrid form of control which has both Fuzzy Logic Control (FLC) and Variable Structure Control (VSC) integrated. This proposed Fuzzy Variable Structure Control (FVSC) approach combines the power of approximate reasoning of FLC with the switching synthesis used in VSC. The FVSC is applied to control and stabilize a micro satellite in space. A model of satellite is used in this study in order to ensure the applications of FVSC in space. The reduced-order observer was introduced in this system to construct the unmeasurable state variable. The simulation results indicate that the fuzzy logic can be advantageously used to minimize the chattering and improve the time response in the satellite system.**

I. Introduction

The Variable Structure Control (VSC) is a high-speed switching feedback control. VSC consists of a set of continuous subsystems together with a suitable switching logic. The performance of this method results in changing the system-structure based on the switching logic. Due to switching delays, it is rare that ideal sliding will occur. Furthermore, chattering will exist in VSC and this has to be reduced [1,2] for minimizing the hardware fatigue. Sliding mode control (SMC) has emerged to be a robust control technique for satellite attitude control, due to its insensitiveness to system uncertainty and external disturbance when sliding motion occurs [3]. The Fuzzy Logic Control (FLC) [4] has been successfully used in several of complex systems. The FLC is robust for parameter variations and system disturbances. FLC requires only input-output variables and it is based on rules of human decision making. The reduced-order observer will estimate only those states that are not measured and its easier to implement and hence faster. Since we shall effectively used the measured values of some of the states, and only use estimated values of the others.

This paper is organized as follows. Section II discusses on the basically unstable satellite model. Brief description about the VSC, introduction of switching function and the reduced-order observer are proposed in Section III. The basic form of FLC is discussed in Section IV. The performance of hybrid form of FVSC, the fuzzy variable structure control, with different input signals are illustrated in Section V. Section VI presents some conclusions. MATLAB SIMULINK [5] is extensively employed in this work.

II. Satellite Model

Several literatures have discussed on an effective model to represent the performance of an uncontrolled satellite system in space. The satellite when not controlled is basically unstable in space. A double integrator model (with two poles at the origin of s-plane) is quite fitting to be the model of satellite deployed in the space [6].

Reference [7] states that the satellite is assumed to be in a frictionless environment. If *T(t)* is the system input torque (created by magnetic torquers) and $\theta(t)$ is the system angular output, then

$$T(t) = J\frac{d^2\theta(t)}{dt^2} \tag{1}$$

where *J* is the moment of the inertia of the satellite.

Normalizing, we define the control function as

$$u(t) = T(t)/J \tag{2}$$

and thus obtain the model

$$\ddot{\theta}(t) = u(t) \quad \text{or} \quad \frac{\theta(s)}{U(s)} = \frac{1}{s^2} \tag{3}$$

wherein U(s) is the Laplace transform of u(t). This is considered as a suitable model of a rigid satellite in a frictionless environment and is adopted throughout this paper in studying the performance aspects of satellite.

III. Variable Structure Control

VSC with sliding mode control was first proposed and elaborated in 1950's in the Soviet Union by Emelyanov and several co-researchers [8].

The basic idea of VSC relies on the fact that the controller is allowed to change its structure, i.e. to switch at any instant from one to another member of a set of possible continuous functions of the state [9]. A VSC system is shown by a second-order system as in Fig. 1(a). The concept of the basic VSC is that of generating an additional input that is based on a function of system output, x_1. The switch, in Fig. 1, switches to $+4x_1$ when x_1 value is more than +4 or to $-4x_1$ if x_1 is less than -4 and otherwise to zero. The system is given a step input with zero initial conditions. Fig. 2(a) to 2(d) shows the system responses. Here, the state variables are defined as

$$x = x1 = x_1 \quad \text{and} \quad y = x2 = x_2 \tag{4}$$

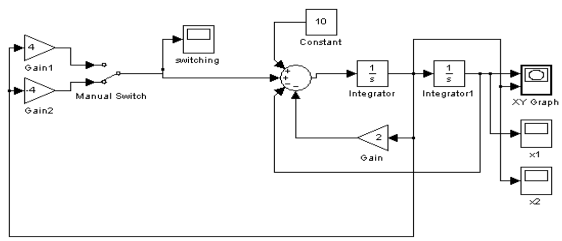
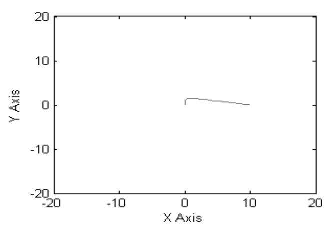


Fig. 1. Model basic VSC

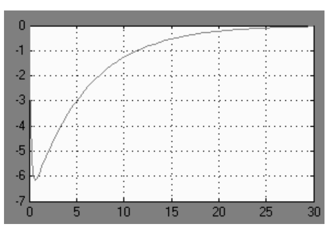
This simple VSC results in delayed response. Instead of simple On-Off form of switching, a switching equation can be derived and incorporated. This creates a sliding line in the state space of the VSC system. The state response that reaches the sliding line will slide along the line to reach the steady state value. One such switching equation is given by

$$s = (x_1 - a)\,x_2 + \left[m\left(x_1 - a\right)^2\right] \tag{5}$$

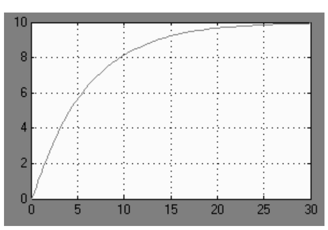
where a is a selected constant depending on the settled value of x_1 and m is the slope of sliding line. We consider m = 0.5 in this paper.



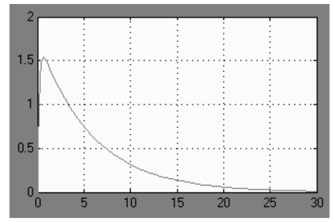
(a) the xy plot



(b) the switching (w) response



(c) the x_1 response



(d) the x_2 response

Fig. 2. Model system and responses basic VSC

Fig. 3. describes the system of Fig. 1. incorporated with the switching equation. Fig. 4.(a) till Fig. 4(d) show various responses for a step input of magnitude a = 10.

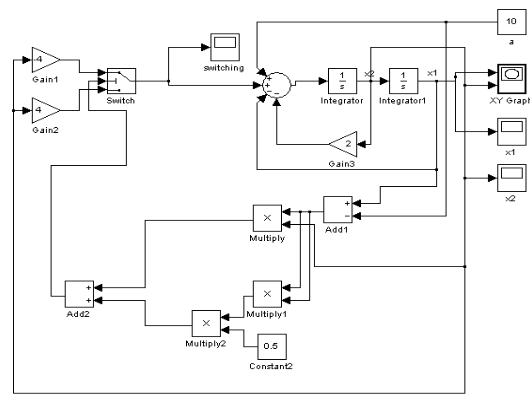
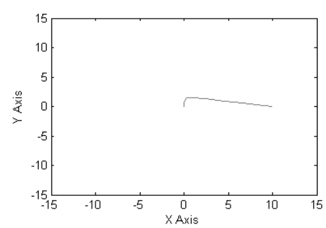
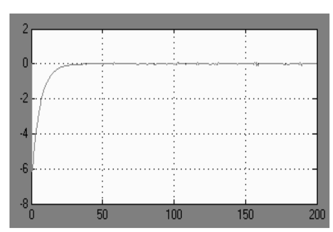


Fig. 3. Model VSC with switching equation

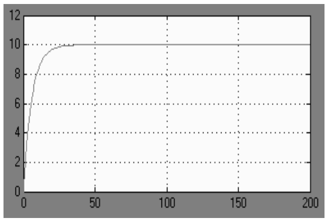
The xy plot (Fig. 4(a)) shows that the response slides along the switching line. The x_1 of this VSC system settles at value a =10. (Fig. 4(d)). The response switches at the switching line only once. The natural tendency of response having a single switching at the switching line is to be slow.



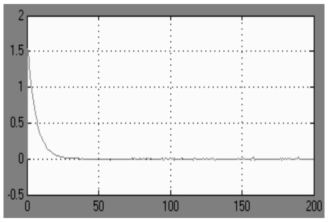
(a) the xy plot



(b) the switching (w) response



(c) the x_1 response



(d) the x_2 response

Fig. 4. Model and responses of VSC with switching equation

In the above descriptions, we have used x_1 and x_2 for feedback. In real satellite system, x_2 is inaccessible. x_2 can be estimated by an observer. Here, we use the reduced-order observer to estimate or measure the missing state variable, x_2. The system is found to be controllable and observable before the introduction of reduced-order observer. Reference [7] states that the complete description of the system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} a_{11} & a_{1e} \\ a_{e1} & A_{ee} \end{bmatrix} \begin{bmatrix} x_1 \\ x_e \end{bmatrix} + \begin{bmatrix} b_1 \\ b_e \end{bmatrix} u \qquad (6)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_e \end{bmatrix} \qquad (7)$$

The dynamics of the unmeasured state variables are given by

$$\dot{x}_e = A_{ee} x_e + \underbrace{a_{e1} x_1 + b_e u}_{\text{known input}} \qquad (8)$$

Since $x_1 = y$, measured dynamics are given by the scalar equation

$$\dot{x}_1 = \dot{y} = a_{11} y + a_{1e} x_e + b_1 u \qquad (9)$$

$$\dot{y} - a_{11} y - b_1 u = a_{1e} x_e \qquad (10)$$

Then reduced-order observer equation is obtained,

$$\begin{aligned} \dot{\hat{x}}_e &= A_{ee} \hat{x} + a_{e1} y + b_e u + \\ & m(\dot{y} - a_{11} y - b_1 u - a_{1e} \hat{x}_e) \end{aligned} \qquad (11)$$

if we define the estimation error as

$$\tilde{x}_e = x_e - \hat{x}_e \qquad (12)$$

The dynamics of error are given by subtracting equation (11) from equation (8):

$$\tilde{x}_e = (A_{ee} - m a_{1e}) \tilde{x}_e \qquad (13)$$

Its characteristic equation is given by

$$\left| sI - (A_{ee} - m a_{1e}) \right| = 0 \qquad (14)$$

The equation can be rewritten as

$$\begin{aligned} \dot{\hat{x}}_e &= (A_{ee} - m a_{1e}) \hat{x}_e + (a_{e1} - m a_{11}) y \\ & + (b_e - m b_1) u + m \dot{y} \end{aligned} \qquad (15)$$

The fact that the reduced-order observer requires the derivative of $y(t)$ as an input appears to present a practical difficulty. It is known that differentiation amplifies noise, so if y is noisy, the use of $\dot{y}$ is unacceptable. To get around this difficulty, we define the new state as

$$x'_e \overset{\Delta}{=} \hat{x}_e - m y \qquad (16)$$

Then, in terms of this new state, the implementation of the reduced-order observer is given by

$$\begin{aligned} \dot{x}'_e &= (A_{ee} - m a_{1e}) \hat{x}_e + (a_{e1} - m a_{11}) y \\ & + (b_e - m b_1) u \end{aligned} \qquad (17)$$

The VSC system with reduced-order observer is illustrated in Fig. 5. Fig.6 was shown the reduced-order observer structure and the responses are given in Fig. 7(a) till Fig 7(d).

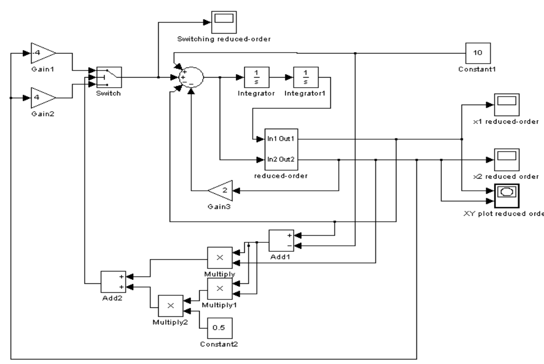


Fig. 5. The VSC system with reduced-order observer

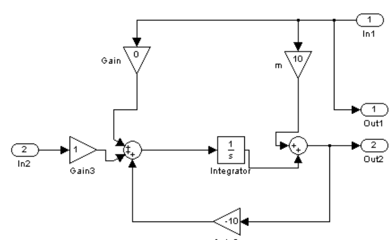
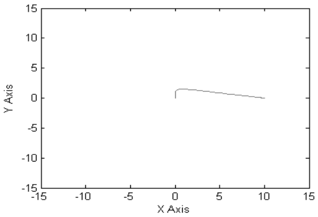
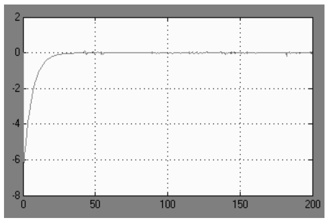


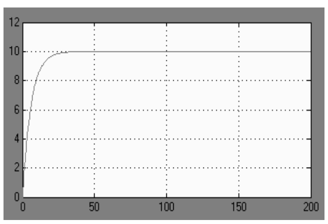
Fig. 6. The reduced-order observer



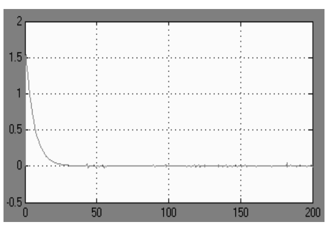
(a) the xy plot



(b) The switching response



(c) The x_1 response



(d) The x_2 response

Fig. 7. Responses of VSC system with reduced-order observer

In order to evaluate the performance of the satellite model for multiple step changes of input, a signal generator that produces a different input function is introduced. The square wave input function with amplitude, 1.0 unit and frequency, 0.06 Hertz is now applied to the VSC system with switching equation. This system is given in Fig. 8. The resulting responses are illustrated in Fig. 9(a) till 9(d).

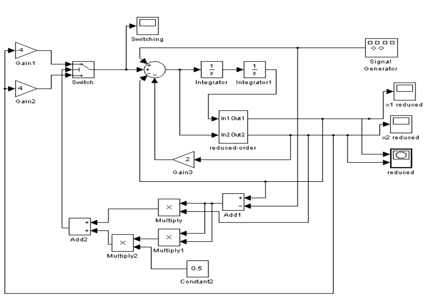
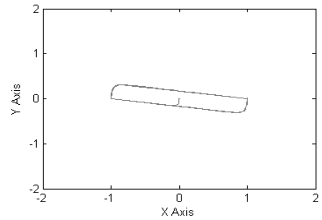
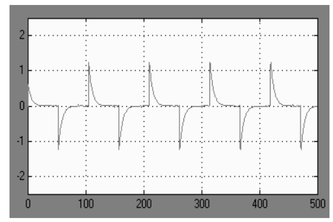


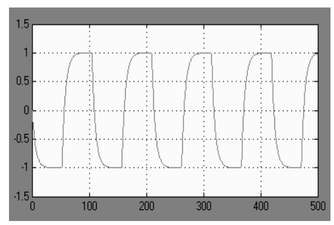
Fig. 8. System with signal generator



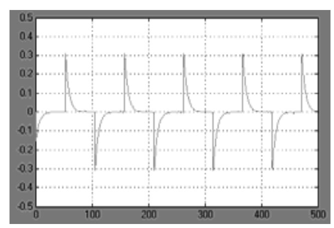
(a) the xy plot



(b) the switching response



(d) the x_1 response



(e) the x_2 response

Fig. 9. Responses of system with square wave input function

The (x,y) response of the system clearly shows the sliding characteristics. However, all these responses portray severe oscillations along the switching line. This shows that the VSC with switching equations demonstrate a kind of high speed chattering along the switching line. These chattering, though small in magnitude, is ever repeating in the satellite response thus creating a fatigue to the system hardware and may end with unrecoverable deterioration in responses. Besides, the time responses are rather slow taking almost 25 s to settle, thus restricting the use of higher speed step sequence control.

IV. Fuzzy Logic Control

Though, the responses are satisfactory in terms of performance, the property of chattering occurs, sometimes severe with unacceptable amplitudes. This chattering has to be minimized or removed. A simple gain tuning can be obtained by the help of the VSC theory [10] However, a new proposal of using FLC is advanced in this work. Triangular fuzzy sets with a Mamdani type FLC formulation which is known to be faster than other formulations is used in this work [11]. The Fuzzy Variable Structure Control (FVSC) is depicted in Fig.10.

There are three linguistic labels used in both for x_1 and error, Fig.10(a) and 10(b). The "error" is defined as (reference input-x_1). For x_1, the fuzzy variables are: Small (S), Medium (M), and Large (L); and for error, they are Negative (N), Zero (Z) and Positive (P). The output from FLC, z, is divided as Very Low (VL), Low (L), Normal (N), High (H) and Very High (VH), Fig.10(c). Fuzzy Logic control is achieved through the Fuzzy Associative Memory defined as in Table 1.

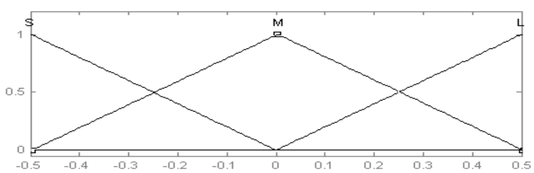
The very common method of defuzzification adopted in this work is that of finding the FLC output by using the concept of centroid [12]. A discrete form of equation is employed to obtain the defuzzified control signal.

$$z^* = \frac{\int \mu(z)\ z\ dz}{\int \mu(z)\ dz} \tag{18}$$

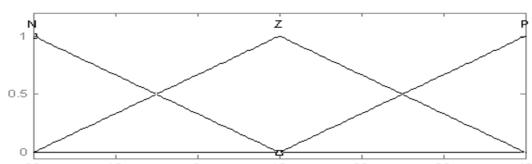
where

z : the output fuzzy variable of FLC
$\mu(z)$: the membership function of z
z^* : the defuzzified value of z

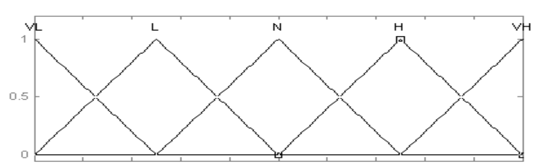
This defuzzification function is available in the MATLAB Fuzzy Logic Tool Box.



(a) the membership function for input x_1



(b) the membership function for error



(c) the membership function for FLC output

Fig. 10. Fuzzy sets of FLC inputs and output.

Table 1: FAM Table

x_1 \ error	N	Z	P
S	VH	H	N
M	H	N	L
L	N	L	VL

V. Fuzzy Variable Structure Control

The fuzzy logic control illustrated in Section IV is employed in the system of Fig.5 and is brought in Fig. 11. First the system is controlled with a single step input function of magnitude 10. The responses are displayed in Fig. 12(a) till 12(d).

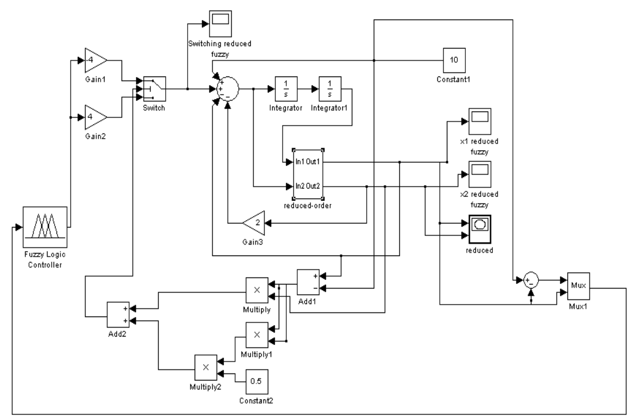
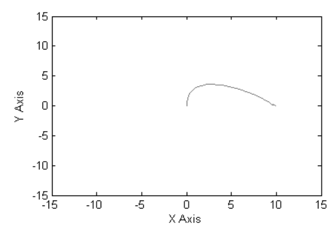
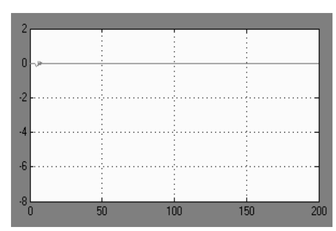
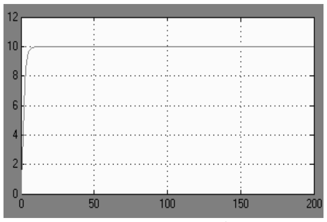


Fig.11 The FVSC System



(a) the xy plot


(b) the switching response


(c) the response of x_1

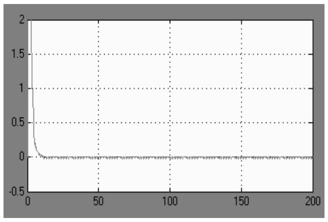
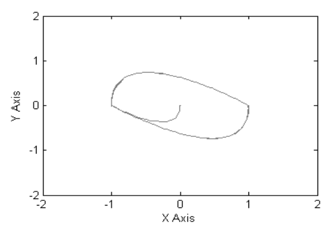
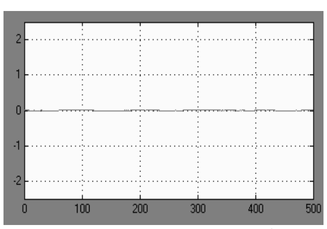

(d) the response of x_2

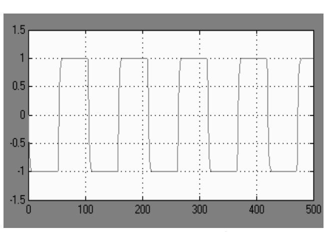
Fig. 12. FVSC system and its responses to single step input

The (x,y) plot indicates the sliding characteristic in the response. There is practically no chattering. This happens when FLC output, z falls within the switching ranges of +4z and -4z. The system output x_1 and its derivative x_2 exhibit a much faster response compared to x_1 and x_2 of Fig. 7. They settle at 5 s compared to those of Fig. 7 settling at 25 s. The FVSC is next subjected to a square wave input function. The system responses are furnished in Fig. 13.



(a) the xy plot


(b) the switching response of z


(c) the response of x_1

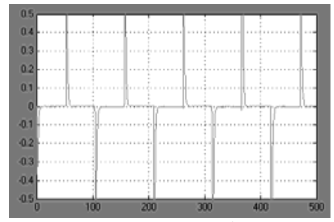

(d) the response of x_2

Fig. 13. FVSC system and its responses

Two main advantages of FVSC responses can be observed when compared to VSC responses given in Fig. 5. One is the improved speed of response and the other is minimizing the chattering along the switching line in the phase plane. Both of these are beneficial to the micro-satellite system for a faster control and stabilization.

CONCLUSION

This paper discusses the concepts of variable structure control designed for micro satellites. The micro satellite that is deployed in space is basically unstable and has to be stabilized and controlled as fast as possible. A basically unstable model is adopted in this work to represent such a satellite and is controlled for its attitude stabilization. The scheme of basic variable structure control (VSC) is applied first and is systematically developed to a VSC with a sliding mode and then to a form of fuzzy variable structure control (FVSC). which integrates a fuzzy logic in the feed back loop. The reduced-order observer is applicable to estimate the missing state variable in all these cases. The VSC systems exhibit slow responses and high speed chattering The FVSC is shown to be faster with minimized or no chattering. It is envisaged that the proposed FVSC approach is suitable for stabilizing and controlling a micro-satellite that is deployed in space.

REFERENCES

[1] H.Borhan, Gh.Vossoughi, “Improvement of Sliding Mode Control Performance Using Fuzzy Gain Scheduling”, School of Mechanical Engineering, Sarif Universiti of Technology, Tehran, Iran.

[2] Ping Guan, Xiang-Jie Liu, Ji-Zhen Liu, “Adaptive Fuzzy Sliding Mode Control for Flexible Satellite”, Engineering Applications of Artificial Intelligence 18, 2005, pp. 451-459.

[3] X.J.Liu, P.Guan, J.Z.Liu, “Fuzzy Sliding Mode Attitude Control of Satellite”, Proceedings of The 44th IEEE Conference on Decision and Control, and the European Control Conference, 2005, pp.1970-1975.

[4] Xinghuo Yu, Zhihong Man, Baolin Wu, Design of Fuzzy Sliding-mode Control Systems”, Fuzzy Sets and System 95, 1998, pp. 295-306.

[5] Kevin J.Walchko, Dr.Paul C.Mason, “Development of a Fuzzy Sliding Mode Control for Satellite Attitude Control”, GSRP Final Report, Mechanical Engineering, Universiti of Florida, 2000.

[6] Gene F. Franklin, J. David Powell and Abbas Emami-Naeini, *Feedback Control of Dynamic System*, Pearson Education Ltd., 006.

[7] M Gopal, *Digital Control and State Variable Methods Conventional and Neuro-Fuzzy Control Systems*, Tata McGraw-Hill, 2003.

[8] John Y.Hung, weibing Gao, James C.Hung, "Variable Structure Control : A Survey," IEEE Transactions on Industrial Electronics, Vol.40, No.1, February 1993, pp. 2-22.

[9] Gamal M.Aly and Mohammed A.Tayeb, "Fuzzy Logic-based Variable Structure Controllers," Journal of Microcomputer Applications (1995) 18, pp. 115-125.

[10] H.X.Li, H.B.Gatland, A.W.Green, "Fuzzy Variable Structure Control", IEEE Transactions on Systems, Man.and Cybernetics – Part B: Cybernetics, Vol.27, No.2, April 1997, pp 306-312.

[11] R Nagarajan, Paulraj M P, Sazali Yaacob, Zaridah Mat Zain, Rosfariedzah Rusli, "An Adaptive-Predictive Fuzzy Logic Controller for the Attitude Control of a Micro-Satellite" ,International Association for Modeling and Simulation Technique in Enterprise(AMSE Journal). (to be publish in 2009) Improvement of Sliding Mode Control Performance Using Fuzzy Gain Scheduling

[12] Timothy J. Ross, *Fuzzy Logic with Engineering Applications*, McGraw-Hill, Inc., 1997.