

Accurate Compact Flowfield-Dependent Variation Method for Compressible Euler Equations

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Abstract- Numerical simulations of compressible inviscid flows using fully implicit high order compact flowfield-dependent variation (HOC-FDV) method has been developed. The third-order accurate in time flowfield-dependent variation (FDV) scheme is used for time discretization and the fourth-order compact Pade scheme is used to approximate the first and second spatial derivatives. The solution procedure consists of a number of tri-diagonal matrix operations with considerable saving in computing time, and produces an efficient solver. The method has been tested and verified for two numerical examples, a flow over a channel flow with compression/expansion and a flow past NACA0012 airfoil. Good agreement with the analytical and other numerical solutions has been obtained in both cases. Numerical results show the validity and accuracy of the proposed method.

Keywords— Flowfield-dependent variation (FDV), Higher-order compact (HOC), Inviscid flows, Euler equations, Supersonic flow

I. INTRODUCTION

Numerical predictions of the compressible supersonic flows play very important role in computational fluid dynamics (CFD) and are very important. The FDV method was originally introduced by T. J. Chung et al. [1]. The FDV idea began from the need to address the transitions from one type of flow to another and interactions between two distinctly different flows. The approach begins by obtaining the implicitness FDV parameters from the current flowfield variables at each time step and every nodal point. These parameters are used to adjust governing equations at each regime according to the current flowfield situation. The physical interpretation of the FDV first-order parameter, s_1 , and the second-order parameter, s_2 , is the foundation of the FDV method. Large values of these parameters reflect large changes in the conservation variables. These changes may occur between adjacent nodal points within the special nodes as well as between adjacent time steps. The second-order FDV parameter, s_2 , is chosen to be exponentially proportional to the first-order FDV parameter, s_1 . This choice is based on the fact that the first-order FDV parameter tends to assure accuracy of the solution, whereas the second-order FDV parameter provides numerical stability [1-3].

A fourth-order compact Pade scheme was used to approximate the first and second order spatial derivatives. Together with the FDV technique this results in a higher-order compact flow field dependent variable (HOC-FDV) scheme. This technique was successfully applied to solve the unsteady non-linear viscous Burgers equation [6].

In this paper, the HOC-FDV scheme is implemented to solve the two-dimensional Euler equations. Two steps; along i and j directions are used for each time level. The spatial derivatives are calculated using higher-order compact approximation Pade Scheme. Each step is solved using block tri-diagonal algorithm. The problems associated with multidimensionality are solved by using the alternating direction implicit (ADI) factored algorithm which reduces the formidable matrix inversion problem to a series of small band width matrix inversion problems that have efficient solution algorithms. Cross-derivatives terms arising from the FDV derivation are excluded from the implicit side and are evaluated explicitly at the known time step.

II. GOVERNING EQUATIONS

The two dimensional Euler form of the FDV equations proposed by Chung [1] can be written in the compact factorized ADI form as:

$$\left(I + D_1 \frac{\partial}{\partial x} + D_{11} \frac{\partial^2}{\partial x^2} \right) \Delta U^* = RHS \quad (1)$$

$$\left(I + D_2 \frac{\partial}{\partial y} + D_{22} \frac{\partial^2}{\partial y^2} \right) \Delta U^{n+1} = \Delta U^* \quad (2)$$

Where

$$D_1 = \Delta t (s_1 A)^n \quad (3)$$

$$D_2 = \Delta t (s_1 B)^n \quad (4)$$

$$D_{11}^n = \left\{ -\frac{\Delta t^2}{2} s_2 A^2 \right\}^n \quad (5)$$

$$D_{22}^n = \left\{ -\frac{\Delta t^2}{2} s_2 B^2 \right\}^n \quad (6)$$

$$RHS = -\Delta t \left(\frac{\partial}{\partial x}(E) + \frac{\partial}{\partial y}(F) \right) - \frac{\Delta t^2}{2} \left(A \frac{\partial^2}{\partial x^2}(E) + A \frac{\partial^2}{\partial x \partial y}(F) \right. \\ \left. + B \frac{\partial^2}{\partial x \partial y}(E) + B \frac{\partial^2}{\partial y^2}(F) \right) \quad (7)$$

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \end{bmatrix}, E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \end{bmatrix} \text{ and } F = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \end{bmatrix} \quad (8)$$

Where A and B are the Jacobian matrices along x and y axis respectively, and s_1 and s_2 are the implicit FDV parameters.

The gradient of the RHS terms are evaluated by high-order compact schemes proposed by Hirsh [4] while the derivatives in the implicit part are approximated by second order central schemes. The higher-order approximations to the first order derivatives in the residual part are obtained by the cell-centred fourth-order compact scheme as follows:

$$E'_{i-1} + 4E'_i + E'_{i+1} = \frac{3}{h}(E_{i-1} + E_{i+1}) \quad (9)$$

$$F'_{i-1} + 4F'_i + F'_{i+1} = \frac{3}{h}(F_{i-1} + F_{i+1}) \quad (10)$$

$$\text{Where } E' = \left(\frac{\partial E}{\partial x_i} \right) \text{ and } F' = \left(\frac{\partial F}{\partial x} \right)$$

The second order derivatives in the residual term, E'' and F'' are approximated using the following equations [4]:

$$E''_{i-1} + 10E''_i + E''_{i+1} = \frac{12}{h^2}(E_{i-1} - 2E_i + E_{i+1}) \quad (11)$$

$$F''_{i-1} + 10F''_i + F''_{i+1} = \frac{12}{h^2}(F_{i-1} - 2F_i + F_{i+1}) \quad (12)$$

$$\text{Where } E'' = \left(\frac{\partial^2 E}{\partial x^2} \right) \text{ and } F'' = \left(\frac{\partial^2 F}{\partial x^2} \right)$$

III. NUMERICAL EXAMPLES

The two dimensional Euler FDV equations have been tested for two numerical examples, a channel flow with compression/expansion and a flow past NACA0012 airfoil. The examples include formation of oblique and expansion waves and their reflections and interactions.

A. Channel flow

The case presents a supersonic laminar flow entering a channel with a compression corner and an expansion corner located at the lower surface and a straight upper surface. The computational grid of 241×131 has been generated by algebraic grid generation routine. Clustering at the compression corner, expansion corner, the lower surface, and the upper surface has been implemented. The inflow

boundary conditions are $M_\infty=3$, $T_\infty=300$ K, and $\rho_\infty=1.25$ kg/m³.

Mach number contours of the converged solutions are shown in Fig. 1. The comparisons of the pressure and Mach number along the lower surface with the analytical solutions are shown in fig. 2 and fig. 3 respectively. The figures show good agreements between the current computational results and the analytical solutions. Fig. 4 shows the contours of the first order convection variation parameter, s_1 . Note that $s_1 = 0$ away from the shockwaves and becomes nearly unity at locations of high gradients. Note that contours resemble the flowfield itself.

B. Flow Past NACA 0012 Airfoil

External flow calculations around a NACA 0012 airfoil are performed using HOC-FDV scheme for Mach number 0.8 and angle of attack 1.25. The Reynolds number is 3×10^6 . The grid is an O-type grid with 161×33 points that is generated by an elliptic grid generation procedure described by Hoffman and Chiang [5]. The outer boundary of the grid is located 10 chord lengths from the airfoil. The Flows are initially set to free stream conditions and the Flow tangency boundary condition is applied at the airfoil surface.

Fig. 6 shows a qualitative comparison of Mach number contours over the airfoil surface with the results given by high order compact of reference [7]. The contours are very identical and the figure indicates that the transonic flow has a shock wave on the upper surface of the airfoil. Table 1 shows the quantitative comparison of lift and drag coefficients in of the present results with results taken from Mawlood [7]. It is clear from this table that the present results are in good agreement with high order compact results obtained using high order compact scheme. The absolute errors history of the numerical solution of the non-dimensional density is plotted in Fig. 7.

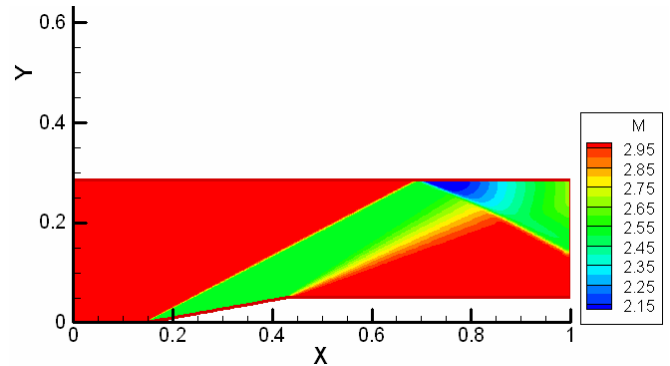


Fig. 1 Steady state Mach number contours

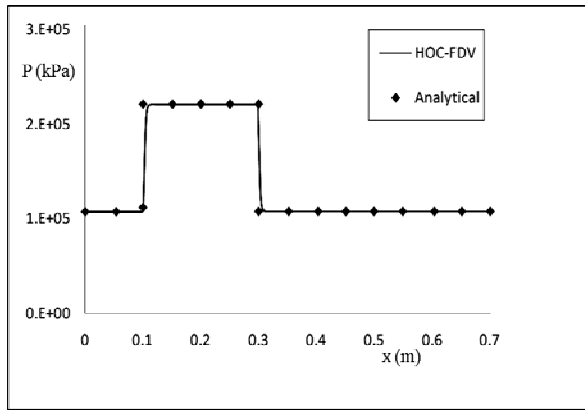


Fig. 2 Pressure distribution along the wall

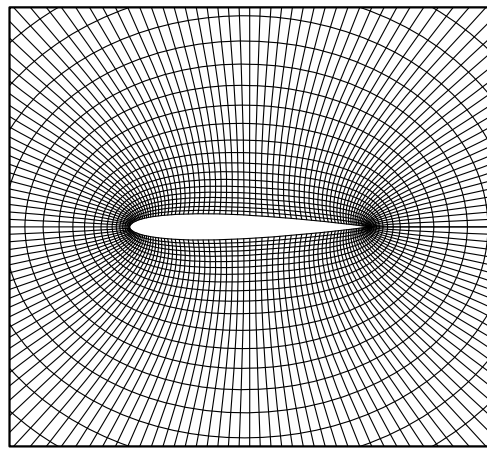


Fig. 5 A close view of the O-grid for NACA 0012 airfoil

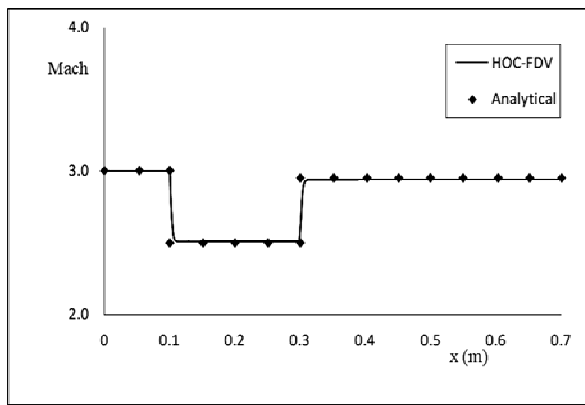
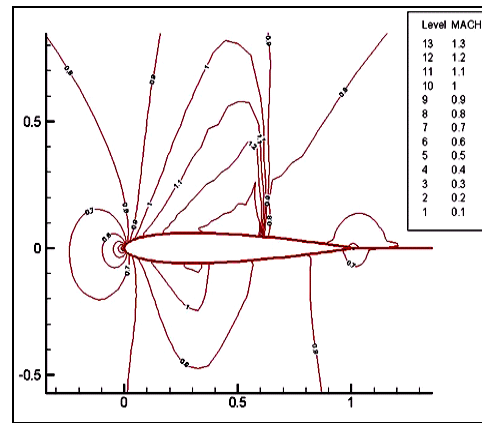


Fig. 3 Mach number distribution along the wall



(a) Present Results

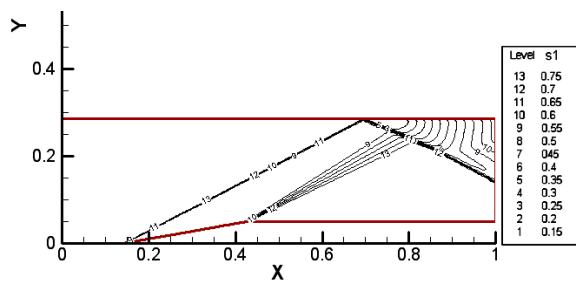
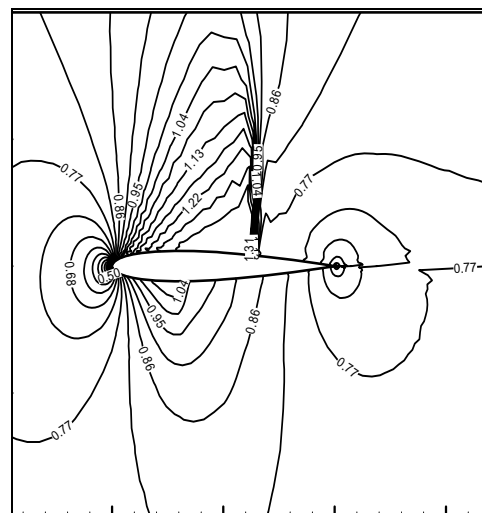


Fig. 4 First-order variation parameter, s1 contours



(b) HOC results [7]

Fig. 6 Comparison of Mach number contours for NACA 0012 airfoil ($M_\infty = 0.8$, $\alpha = 1.25$)

TABLE I
COMPARISON OF LIFT AND DRAG COEFFICIENTS FOR THE FLOW PAST
NACA 0012 AIRFOIL ($M_\infty = 0.8$, $\alpha = 1.25$)

parameter	HOC-FDV	Mawlood [7]
C_L	0.3735	0.37432
C_D	0.02332	0.02350

- [7] M. K. Mawlood, A high-order compact finite difference solver for the Euler and Navier-Stokes equations, Ph.D. Thesis, Universiti Putra Malaysia, Malaysia, 2004.

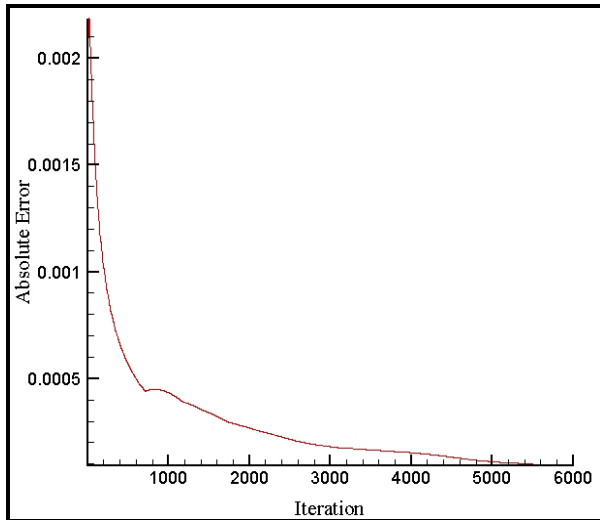


Fig. 7 Error history for NACA 0012 airfoil
($M_\infty = 0.8$, $\alpha = 1.25$)

IV. CONCLUSIONS

A higher-order compact-flowfield dependent variation method (HOC-FDV) has been developed. The numerical examples are tested to demonstrate the accuracy and resolution of the method in capturing flow physics for all regimes. The higher order FDV scheme results have been compared with analytical solution. The results are in excellent agreement with the analytical solution.

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