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A NUMERICAL ALGORITHM FOR COMPUTING THE SOLUTION OF FUZZY DIFFERENTIAL EQUATIONS

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$$\begin{cases} X'(t) = \frac{1}{2}X + 2\sin(3t \\ X(0) = (-1, 0, 1), \end{cases}$$

PRODUCT DESCRIPTIONS

- A new computational algorithm in fuzzy environment
- A new fuzzification process
- A hybrid approach

PROBLEM STATEMENT

Let us consider the following initial value problem:

$$\begin{cases} x'(t) = f(t, x(t)), & t \in [t_0, T], \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where $f : [t_0, T] \times \mathbb{R} \to \mathbb{R}$ is a real-valued function and $x_0 \in \mathbb{R}$. Suppose that the initial value in (1) is uncertain and replaced by a fuzzy interval, then we have the following fuzzy initial value problem:

$$\begin{cases} X'(t) = \hat{f}(t, X(t)), & t \in [t_0, T], \\ X(t_0) = X_0, \end{cases} \tag{2}$$

where $\hat{f} : [t_0, T] \times \mathcal{F}(\mathbb{R}) \to \mathcal{F}(\mathbb{R})$ is a fuzzy-valued function and $X_0 \in \mathcal{F}(\mathbb{R})$.

DERIVATION OF ALGORITHM

Suppose we have the initial value problem in (1). By integrating both sides over $[t_i, t_{i+1}]$, we have

$$\int_{t_i}^{t_{i+1}} x'(t)dt = \int_{t_i}^{t_{i+1}} f(t, x(t))dt, \tag{3}$$

which gives

$$x(t_{i+1}) = x(t_i) + \int_{t_i}^{t_{i+1}} f(t, x(t))dt. \tag{4}$$

Using integration by parts, we see that

$$x(t_{i+1}) = x(t_i) + (t_{i+1} - t_i)f(t_i, x(t_i)) + \int_{t_i}^{t_{i+1}} (t_{i+1} - t)f(t, x'(t))dt. \tag{5}$$

By setting $h = t_{i+1} - t_i$, we have

$$x(t_{i+1}) = x(t_i) + hf(t_i, x(t_i)) + \int_{t_i}^{t_{i+1}} (t_{i+1} - t)f(t, x'(t))dt. \tag{6}$$

We truncate (6) at the $\int_{t_i}^{t_{i+1}} (t_{i+1} - t)f(t, x'(t))dt$ term. Therefore, we have the following general equation for Euler method:

$$x(t_{i+1}) = x(t_i) + hf(t_i, x(t_i)), \quad i = 0, 1, 2, ..., N-1. \tag{7}$$

For simplicity, we rewrite (7) as

$$x_{i+1} = x_i + hf(t_i, x_i), \quad i = 0, 1, 2, ..., N-1. \tag{8}$$

Let $m(h, t_i, x_i) = x_i + hf(t_i, x_i)$. Now, (8) becomes

$$x_{i+1} = m(h, t_i, x_i), \quad i = 0, 1, 2, ..., N-1. \tag{9}$$

where m is a real valued function. If m accepts fuzzy intervals X_i as arguments, then from Zadeh's extension principle we have

$$X_{i+1} = m(h, t_i, X_i), \quad i = 0, 1, 2, ..., N-1. \tag{10}$$

where m becomes a fuzzy interval valued function. The membership function of m is defined as:

$$m(h, t_i, X_i)(y) = \begin{cases} \sup\limits_{x \in m^{-1}(h, t_i, y)} X_i(x) & , \text{if } y \in \text{range}(m), \\ 0 & , \text{if } y \notin \text{range}(m). \end{cases} \tag{11}$$

If $[X_{i+1}]^{\alpha} = [x^{\alpha}_{i+1,1}, x^{\alpha}_{i+1,2}]$, then (10) becomes

$$x^{\alpha}_{i+1,1} = \min\{m(h, t_i, u) \mid u \in [x^{\alpha}_{i,1}, x^{\alpha}_{i,2}]\}, \tag{12}$$

$$x^{\alpha}_{i+1,2} = \max\{m(h, t_i, u) \mid u \in [x^{\alpha}_{i,1}, x^{\alpha}_{i,2}]\}. \tag{13}$$

The Eqs. (12) and (13) will generate the approximate solution of (2) at each level of α-cut.

NUMERICAL EXPERIMENT

Consider the following fuzzy initial value problem:

$$\begin{cases} X'(t) = \frac{1}{2}X + 2\sin(3t), & t \in [0, 4], \\ X(0) = (-1, 0, 1). \end{cases} \tag{14}$$

The analytical solution of (14) is given by

$$x_1^{\alpha}(t) = -\frac{24}{37}\cos(3t) - \frac{4}{37}\sin(3t) + \left(-(1-\alpha) + \frac{24}{37}\right)e^{\frac{1}{2}t},$$

$$x_2^{\alpha}(t) = -\frac{24}{37}\cos(3t) - \frac{4}{37}\sin(3t) + \left((1-\alpha) + \frac{24}{37}\right)e^{\frac{1}{2}t}.$$

The numerical solutions of (14) are given as follows:

Table: Errors

α	$x_1^{\alpha}(4)$	$x^{\alpha}_{4,1}$	$E^{\alpha}_{4,\text{left}}$	$x_2^{\alpha}(4)$	$x^{\alpha}_{4,2}$	$E^{\alpha}_{4,\text{right}}$
0.0	-3.085512	-3.099599	0.014087	11.692601	11.389693	0.302908
0.1	-2.346606	-2.375134	0.028528	10.953695	10.665229	0.288466
0.2	-1.607700	-1.650670	0.042970	10.214789	9.940764	0.274025
0.3	-0.868795	-0.926205	0.057410	9.475884	9.216300	0.259584
0.4	-0.129889	-0.201740	0.071851	8.736978	8.491834	0.245144
0.5	0.609016	0.522724	0.086292	7.998072	7.767370	0.230702
0.6	1.347922	1.247189	0.100733	7.259167	7.042906	0.216261
0.7	2.086828	1.971653	0.115175	6.520261	6.318441	0.201820
0.8	2.825733	2.696118	0.129615	5.781356	5.593976	0.187380
0.9	3.564639	3.420583	0.144056	5.042450	4.869512	0.172938
1.0	4.303544	4.145047	0.158497	4.303544	4.145047	0.158497

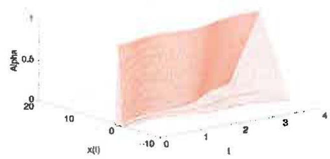

Figure: The analytical solution of (14)

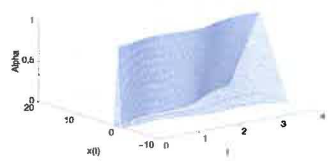

Figure: The numerical solution of of (14)

NOVELTIES

- The dependency problem has been considered in developing the algorithm
- An efficient computational algorithm has been proposed in order to guarantee the convexity of fuzzy solution on the time domain

ADVANTAGES

- Easy to implement
- Numerical solution converges to real solution

COMMERCIAL POTENTIALS

- Fuzzy Calculator
- Broad applications in Science and Engineering fields

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PUBLICATION

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