A GENERALISED PRICE-SCORING MODEL FOR TENDER EVALUATION

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ABSTRACT

This paper proposes a generalised price-scoring model derived from behavioural features of prospect theory for use in a tender evaluation program. It has the potential to overcome the limitations of existing price models by allowing the within-price attribute score variations to be adjusted by a preference factor. It improves selectivity and incorporates a preference function that emphasises the evaluator and decision maker's preference in the lowest price by allowing a price-scoring curve to be derived from the skewness of the distribution of tender prices and the tender participation rate. It identifies prices that exceed the project budget and penalises them. Statistical data from a survey of tender prices was used to specify the model for practical use. Comparison between generic classes of models using price difference and price ratio functions as price gain measures is made to illustrate the model's general applicability. With the choices made available in this generalised price-scoring model, it is possible to compare the various methods of tender evaluation by computer simulation.

Keywords : *Tender Prices, Price Comparison, Price Scoring Model, Tender Price Evaluation, Judgment, Decision Making*

INTRODUCTION

The objective of tender price scoring is to assign the right values to tender prices within a project budget and to score them so that a fair evaluation that reflects the judgment and preference of the evaluator and decision maker, can be made. The rule of tender evaluation is to award a higher score to a lower price tender because a lower price has a higher value to the evaluator and decision maker. In manual evaluation methods, the price values are assigned by human judgment on an ordinal scale which implies a qualitative price-value relationship. If software decision-support tools are used in integrated tender evaluation of price and non-price attributes, price must be evaluated by a quantitative value function that converts price in monetary unit to a score on an objective numerical scale. How prices are evaluated affects the final tender ranking.

A survey of literature indicated that research on price-value relationship for the purpose of tender evaluation is scarce. One simple quantitative value function that has been used by Karsak [1] for the evaluation of flexible manufacturing systems is a price inverse model for scoring a cost-related attribute. The score synonymous with value, is derived from the price inverse, $\chi_i = x_i^{-1}$ where x_i is the price of the i^{th} tender. The model's weakness is its tendency towards indifference and not having the means to assign a degree of preference for low prices. Thus, there is no flexibility for it to fully express the judgment of the evaluator and decision maker through a preference function.

A wider search in the subject of judgment and decision making [7,8,9,10] revealed the existence of value functions in prospect theory proposed by Tversky and Kahneman [2,3] since 1979. The theory replaces the notion of utility with value which is defined in terms of gains and losses from a reference point. They suggested that the value function defined over monetary gains is $\chi_i = x_i^p$ for x_i above a reference price(0) and over losses is $-\lambda(-x_i)^p$ for x_i below the reference price. Determined from experimental data, p is 0.88 and λ is 2.25, indicating diminishing marginal value and asymmetry between gains and losses. The application of the proposed value function has received attention recently [4,7,8,9] in a number of situations in which the norms and characteristics of decision makers are modelled.

Variations in a tender's overall value are contributed by the within-attribute variations and the between-attribute variations. A value function deals only with the within-price attribute variations while the weighting functions of the tender evaluation procedures in [5,6] provide the between-attribute variations. This paper presents a generalised price-scoring model that determines the within-price attribute variations from the prices of competing tenders. Because all judgments and decisions are context-dependent [8], the credibility of a generalised price-scoring model rests on its ability to capture an evaluator or decision maker's tendency for dominance or non-dominance and on its ability to express mathematically the decision maker's judgment that is translated into a continuous continuum of strong to weak emphasis of tender price. In a realistic model, the behavioural features in prospect theory have to combine with the cost objective to achieve a wide scope of application for situations in which the decision maker's behaviour varies. Prospect theory is helpful up to this point after which the preference factor is dependent on behaviour that has to be approximated by information in a tender.

To achieve this objective, it borrows from prospect theory some behavioural features that are not found in multi-attribute utility theory and applies it to tender price judgment:

- Evaluators and decision makers are more sensitive to cost overrun than cost saving in the sense that they will reject tenders that have prices exceeding the project budget.

- If there are sufficient tender proposals to select from without cost overrun, they will view the tender's lowest price as indicative of the fair market value and the average price plus margin as the budget's upper bound.

- If the tender prices are significantly less than the project budget, they become strongly gain-seeking so as to accrue larger cost savings and to compensate for the original over-estimate.
- If the tender prices are close to but below the project budget, their gain-seeking tendency is weaker because with lesser cost saving, attention is shifted to the non-price attributes of the tender to accrue higher non-price gains.

This paper is organised as follows. First, a generalised price-scoring function of a similar form as the value function of prospect theory is proposed. Next, the project budget acting as the reference price is defined and its implications discussed. A preference function is developed for strong to weak gain-seeking, the strength of which is determined by the tender price coefficient of skewness and tender participation rate. A survey of tender prices elicits the two statistics which together with the evaluator's preference limits, are used to determine the constants of two generic classes of models. Within each generic class, comparison between price difference and price ratio functions as gain measures is made to illustrate the model's general applicability. Finally, appropriate applications of the two generic classes are suggested.

PRICE VALUE FUNCTIONS

In general, the mathematical function of a price-scoring model that captures the essence of how a score, A relates to a price gain variable, X must have the following properties.

(i) A monotonically increases with X

(ii) $0 \leq \mathbf{A} \leq 1$ for $0 \leq X \leq 1$

(iii) $\dfrac{dA}{dX} \geq 0$ for $0 \leq X \leq 1$

From prospect theory [2,3], the value/score, A_i derived from the price gain X_i, of the ith tender price out of m tender prices is suggested below for two generic classes of models.

Un-normalised class: $A_i = X_i^{\,p}$ (1a)

Normalised class: $A_i = \dfrac{X_i^{\,p}}{\sum_{i=1}^{m} X_i^{\,p}}$ (1b)

where generally $-\infty < p < \infty$. Equation 1b also ensures that $\sum_{i=1}^{m} A_i = 1$ as a result of normalisation. When p is negative, a price closer to the maximum price will result in a higher score because the negative root of a small price gain produces a score close to unity. Because this is contrary to the tender evaluation rule of low price-high score, the price-scoring model must not consider values of p that are negative. With this exclusion, the preceding properties are still satisfied but additional properties are required to define the variable, p for various behavioural states as follows.

(i) $A(X)$ is constant for $p = 0$

(ii) $A(X)$ is proportional to X for $p = 1$

(iii) $A(X \neq 1) = 0$ for $p = \infty$

(iv) For Equation 1a, $0 \leq p \leq 1$

The evaluator or decision maker's behaviour can be approximated by the preference factor, p which defines the gain-seeking tendency in the shape of the price-scoring curve. With Equations 1a and 1b, there is no effect caused by price gain on the scores when $p = 0$ i.e. they all have the same unity score for the un-normalised class and $\frac{1}{m}$ for the normalised class. When p increases from 0, the model starts to exhibit a lower price-higher score characteristic, initially still having the tendency to be price indifferent. When $p = 1$, it scores linearly with price gain and is not adjusted by preference (judgment). With Equation 1b, when $p = \infty$, the lowest price attains the maximum score of 1 unit while the rest of the prices are scored zero irrespective of their gain value. Thus, as p moves from 0 to ∞, the price-scoring characteristic moves from one that is insensitive to price gain, hence eliminating price competition, to one that exhibits the strongest preference for the lowest price, hence creating the stiffest competition that results in only one possible contender. In other words, the price-scoring model inherently allows for a range of effects to be accommodated, namely indifference, linear and non-linear dependence on price gain through the specification of p.

THE PROJECT BUDGET AS A REFERENCE PRICE

Before tenders are called, a value of the project cost $\hat{x}$ is estimated. Based on this value, a project budget is given taking into account an assigned contingency cost which is provided to mitigate situations of cost overrun caused by residual project risk. If the contingency cost allocated is c, the project budget, x_B is $\hat{x} + c$. When tender submissions are received, their prices are compared with either the project cost estimate or the budget allocation. Evaluators prefer not to risk cost overrun and will be reluctant to recommend award of a contract whose price exceeds the project cost estimate or the budget. As a means of control, a price-scoring model must assign the lowest price score, usually zero, to prices exceeding the budget i.e. the value function for losses is set to zero for exclusion purpose and is restricted to coding of price gains. The budget is a reference price on an objective scale and has the same meaning as that of prospect theory discussed in [2,3,4,7].

If cost overrun is limited by x_B, then the price gain has a range $x_B - x_{min}$. If the tender prices are ordered in ascending value in the sequence, $x_{(1)}, x_{(2)}, .., x_{(j)},.., x_{(m-1)}, x_{(m)}$, then $x_{min} = x_{(1)}$. Any price, $x_{(q)}$ that exceeds x_B should either be assigned an evaluation-adjusted value equal to x_B or should be automatically excluded from the evaluation. If $x_{(1)}$ exceeds x_B, it will be the only price allowed to participate in the evaluation with gain equal to zero. This requirement ensures that in the event that all tender prices exceed the budget value, the lowest price tender will be the only alternative worthy of any consideration as far as tender price evaluation is concerned. By accurate estimation, an under-budget situation can be avoided most of the time.

PRICE-SCORING CURVES

The exponent, p in Equations 1a and 1b determines the shape of the price-scoring curve. Strong gain-seeking means a high preference ($p > 1$) for low price. In zero (neutral) preference cases ($p = 1$), it would exhibit a straight line obtained by joining the lowest price score with the score at x_B over the price difference range, $x_B - x_{min}$. Weak gain-seeking means a tendency to de-emphasise price preference ($0 < p < 1$). At the limit when $p = 0$, all prices are treated equally and the scores are impartial to price difference.

To illustrate the price-scoring curve, the highest score, A_{max} from Equation 1b is given by

$$A_{max} = A_{(1)} = \frac{X_{(1)}^p}{\sum_{i=1}^{m} X_i^p} = \frac{(1-\|P_{(1)}\|)^p}{\sum_{i=1}^{m}(1-\|P_i\|)^p} \tag{2}$$

where the normalised price is for simplicity, expressed as $P_i = 1 - X_i$.

If the highest score $A_{(1)}$, is set to unity, $A_{(1)}$ is expressed as follows:

$$A_{(i)} = \frac{A_{(i)}}{A_{max}} = \left(\frac{1-\|P_{(i)}\|}{1-\|P_{(1)}\|}\right)^p \tag{3}$$

A continuous plot of $A_{(i)}$ against P_i is shown in Figure 1 for various value of $p > 0$ to illustrate the shape of the price-scoring curves. Using the curves, the evaluator and decision maker can selected the degree of preference to match their judgment.

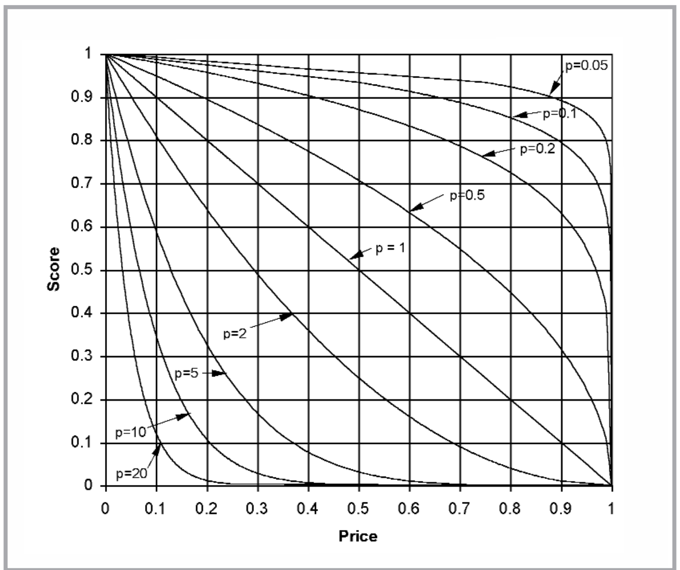

Figure 1: Normalised score against normalised relative price

DISTRIBUTION OF PRICES

Information about the tender can be extracted from the price distribution statistics. In an ideally competitive environment, the coefficient of variation and the variance are indicative of the degree of dissimilarity of the pricing characteristics of the tenderers. Price skew is an asymmetry arising out of either legitimate reasons or collusion [11,12]. It is not the purpose here to determine its cause but to use it to determine preference. The existence of a low price located to a distant left of a central bunch of prices (Figure 2a) gives rise to left asymmetry and a negative coefficient of skewness. If the prices are ideally symmetrical about their mean (Figure 2b), then the coefficient of skewness is zero. The existence of a high price located to the distant right of a central bunch of prices (Figure 2c) gives rise to a right asymmetry and a positive coefficient of skewness.

The coefficient of skewness, υ is proposed as a measure of the degree of gain-seeking. It qualifies by having the following properties.

(i) Its magnitude monotonically increases with skew

(ii) For a positive (right) asymmetry, it is positive and it must represent a strong gain-seeking characteristic that favours the lowest price from the bunch of prices by making the scores reduce very quickly to zero for exceptional prices that are located much higher than the lowest price.

(iii) For symmetry of prices, it is zero and it must not contribute to any preference by being neutral.

(iv) For a negative (left) asymmetry, it is negative and it must represent a weak gain-seeking characteristic that still favours the lowest price but the scores of the bunch of prices are not reduced too quickly.

The lowest price is distant from the central bunch of prices because the pricing of the lowest tender is either made under a different condition or with a different strategy from those of the central bunch. The lowest tenderer may have a legitimate advantage over the rest because of technology, available capacity, resources and location which can be evaluated outside the price domain as in [4,5]. Or in an attempt to win the contract, the lowest price tenderer may resort to higher risk-

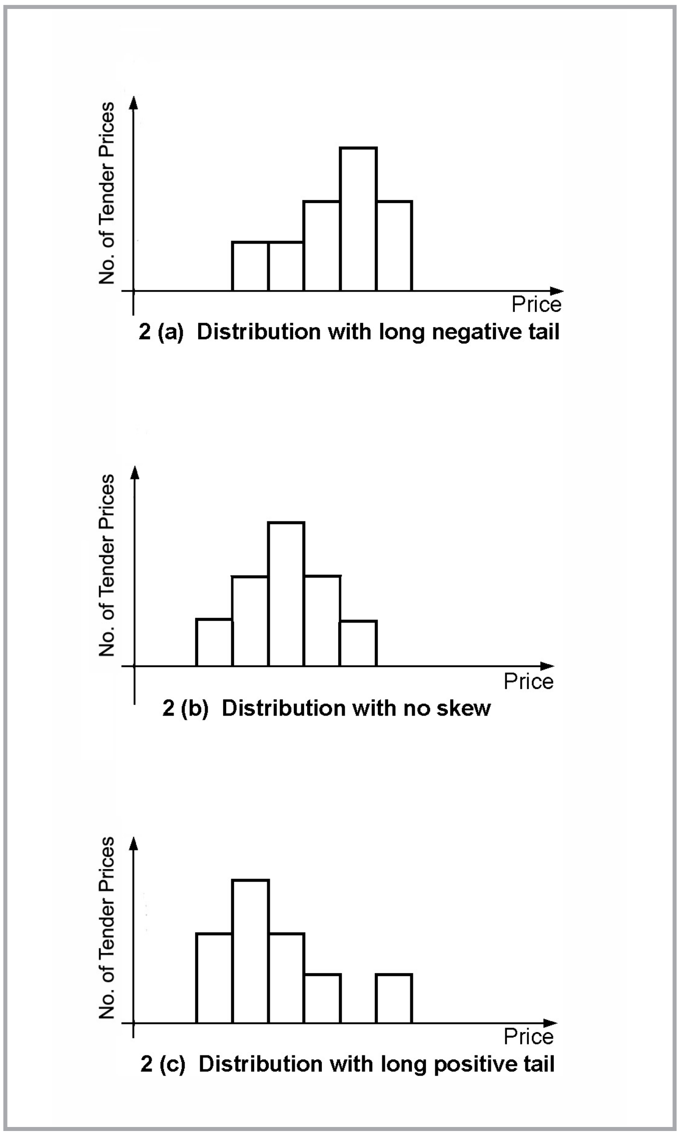

Figure 2: Illustration of skewness in frequency distribution

taking and cutting profit margin, a strategy that the others may not be willing to adopt. If the determination of the project budget is accurately done, such extreme price deviation from the cost estimate must signal the need for extra caution during the evaluation and to uncover the hidden project risks. While giving the highest score, the evaluator is risk-averse and must prudently limit the emphasis of the lowest price. A similar effect may also be created from strategies that defeat fair competition by many forms of cartel collusion. Non-competing cartel prices are increased and the cartel-promoted tenderer's price is raised to just below the next-lowest price [12,13] as illustrated in the comparison between Figures 2a and 2b. If such strategies are revealed in the skew, they are penalised through exclusion by the budget limit as well as by price de-emphasis. When automatic action could not be taken effectively by the software program, pre-assignment of p (<1) is the way to counter this problem.

If behaviour is expressed in the preference factor p, the distribution's coefficient of skewness υ, is a behaviour variable that induces a preference such that the relation between p and υ meets the following:

(i) $p > 0$ for $-\infty < \upsilon < \infty$
(ii) $p = \infty$ if $\upsilon \to \infty$
(iii) $p = 1$ if $\upsilon \to 0$
(iv) $p = 0$ if $\upsilon \to -\infty$

A possible general polynomial expression that satisfies these properties is the skewness factor, k_1 given as follows:

$$k_1 = \sum_{i=1}^{L1} \alpha_{2i-1} \upsilon^{2i-1} \tag{4}$$

where α_{2i-1}'s are positive constants to be determined for L_1 terms. υ is zero if the number of tenders evaluated is less than 3.

TENDER PARTICIPATION RATE

If there are sufficient tender proposals to select from without incurring cost overrun, the price-scoring model should allow for increase in its price selectivity under a situation of high tender participation rate. When υ is very negative, p is near zero. The scores tend to equalise and lose their discriminative ability. In Equation 1b, high tender participation rate decreases the absolute scores by the effect of a larger denominator. The consequence is that when the price scores are brought into the aggregation process with other non-price factors, they become less contributory from their low score values and the lack of discrimination. This effect is counteracted by increasing the value of p with tender participation rate. One way is to include in p, a separate tender participation factor, k_2 which is a function of the number of tenderers, m being evaluated. It should have p increasing monotonically with m i.e. $\frac{dp}{dm} > 0$ for $m > 0$.

It is suggested that k_2 adopts a general positive polynomial function of m in the form

$$k_2 = \beta_0 + \sum_{j=1}^{L2} \beta_j m^j \tag{5}$$

where β_0 is a constant and β_j's are positive constants to be determined for L_2 terms. When $m = 1$, there is no competition and k_2 can be set to zero thus, $\beta_0 = -\sum_{j=1}^{L2} \beta_j$.

GENERAL PREFERENCE FUNCTION

How k_1 and k_2 should be combined to obtain p does not have a unique approach. It must be application dependent and satisfy practical and intuitive requirements. One approach is to add the effects of distribution skew, k_1 and the effects of tender participation rate, k_2 i.e. $k_1 + k_2$. An additive effect has advantage over the multiplicative one ($k_1 \bullet k_2$) because it would not nullify the effect of tender participation rate if the skewness were zero. However, addition alone cannot satisfy all the properties of p. A non-nullifying and property-complying expression for p as a function of k_1 and k_2 is suggested as follows:

$$p = \exp(k_1 + k_2) \tag{6}$$

This expression restricts the value of p to > 0 and thus, satisfies the required properties of p. k_1 contributes to p by ensuring the possibility of specifying a strong preference for low price with positive skew or a de-emphasis of preference for low price with negative skew. It ensures that when $\upsilon = 0$, zero (neutral) preference results from it. The tender participation rate, m contributes positively to p through k_2. Exponentiation ensures a positive p all the time and it amplifies the combined effects, thus making the influence of p stronger. Flexibility is given such that without the aid of these two factors, the evaluator and decision maker can still assign the value of p independently.

At this point, it is seen that the price-scoring model combines the ideas of price gain measure, X and of the degree of preference p, which is a function of the skewness of the price distribution and the number of tenders evaluated, to calculate the price scores, either un-normalised or normalised respectively.

ANALYSIS OF TENDER PRICES

If the purpose of tender evaluation is to select an optimum tender, then the information that reveal price relativities should be put into good use for decision making. A survey was conducted to gather data on tender participation rate and tender prices with the aim of obtaining statistics on tender participation rate and on tender coefficients of variation and skewness. The range of values of m and υ can then be established for use in the price-scoring model. From a total of 75 data sets from past tender exercises, the coefficients of variation (deviation/mean) and skewness for each data set were calculated. From their distributions, the coefficients' means, medians and deviations were obtained from statistical analysis and are summarised in Table 1. The coefficient of skewness is expected to vary from negative values to positive values while the coefficient of variation is always positive.

95% of the tenders has a participation rate below 10 and the median is 4. The price coefficient of variation's mean and median are close to each other at 0.1274 and 0.1221 respectively while its deviation is small at 0.0752. The survey data are indeed from tenderers who shared similar localised

Table 1: Analysis of tender participation rate and tender prices

Tender Price Coefficients' Statistics	Tender Participation Rate, m	Price Coefficient of Variation, σ/μ	Price Coefficient of Skewness, υ
Mean	4.933	0.1274	0.2828
Median	4	0.1221	0.4682
Deviation	2.124	0.0752	0.9764
5th Percentile	3	0.02637	-1.5570
95th Percentile	9.3	0.2850	1.6301

characteristics [13] and quoted prices with small deviations from the mean price. 95% of the coefficients of variation is below 0.2850. A budget up to 1.285 times the project cost estimate will capture most tender prices for evaluation. The mean of the coefficient of skewness, μ_υ is 0.2828 and its deviation, σ_υ is larger at 0.9764. 90% of its variations lies within the range of –1.5570 and 1.6301 with a median of 0.4682. The large σ_υ is an indication of the sensitivity of skewness to extreme values and υ can be suitably employed to vary the preference over a wide range.

SPECIFIC PREFERENCE FUNCTION

A specific preference function for the price-scoring model adopts the first order functions of $k_1 = \alpha\upsilon$ from Equation 4 and of $k_2 = \beta(m-1)$ from Equation 5, where α and β are the positive constants in the expression of p as follows:

Thus, $$p = \exp[\alpha\upsilon + \beta(m-1)] \tag{7a}$$

or $$q = \ln p = \alpha\upsilon + \beta(m-1) \tag{7b}$$

The price-scoring model is to work within the specified limits determined by:

- The maximum tender participation rate, m_{max} and the minimum, which is 1.
- The upper and lower percentiles of the coefficient of skewness (υ_{max} and υ_{min}) of the tender distribution.
- The upper and lower limits (p_{max} and p_{min}) of the preference factor

The tender participation rate can be estimated from the number of tender documents collected and thus a maximum can be set. A minimum of 1 is set to ensure that the effect of tender participation rate could be felt at 2 onwards according to Equation 7a.

Equation 7b shows that the variation of the model constants, α and β are dependent on the logarithm of the preference limits. They are less sensitive to the variations of p and can accommodate the fuzziness of subjective judgments without large changes in value. Figure 1 provides the evaluator and decision maker a means of specifying the limits by inspection.

Thus, $$q_{max} = \ln p_{max} = \alpha\upsilon_{max} + \beta(m_{max} - 1) \tag{8a}$$
$$q_{min} = \ln p_{min} = \alpha\upsilon_{min} \tag{8b}$$

Solving for α and β,

$$\alpha = \frac{q_{min}}{\upsilon_{min}} \tag{9}$$

$$\beta = \frac{q_{max}\upsilon_{min} - q_{min}\upsilon_{max}}{\upsilon_{min}(m_{max} - 1)} \tag{10a}$$

If $\beta' = \beta(m_{max}-1)$,

then, $$\beta' = \frac{q_{max}\upsilon_{min} - q_{min}\upsilon_{max}}{\upsilon_{min}} \tag{10b}$$

To ensure that α is positive (> 0), then v_{min} and q_{min} must have the same sign.

For $\beta \geq 0$, $$(q_{max}\,\upsilon_{min} - q_{min}\,\upsilon_{max})/\,\upsilon_{min} \geq 0 \tag{11a}$$

$$\frac{\ln p_{max}}{\ln p_{min}} \geq \frac{\upsilon_{max}}{\upsilon_{min}} = \kappa \tag{11b}$$

$$p_{max} \geq p_{min}\,\kappa \tag{11c}$$

$$p_{min} \geq p_{min}\,\kappa \tag{11d}$$

The choice of p is guided by the intuitive requirement that the effect of υ should be greater than that of m.

Thus, $\alpha > \beta'$ and $$p_{max} < p_{min}^{\lambda} \tag{12}$$
where $\lambda = \dfrac{1+\upsilon_{max}}{\upsilon_{min}} = \kappa + \upsilon_{min}^{-1}$.

For a given p_{min}, combining Equations 11c and 12 gives the range of p_{max}.

$$p_{min} < p_{max}^{\kappa} < p_{min}^{\lambda} \tag{13a}$$

Similarly given p_{max}, $$p_{max}^{\frac{1}{\lambda}} > p_{min} > p_{max}^{\frac{1}{\kappa}} \tag{13b}$$

From the price survey, $\hat{\upsilon}_{min}$, $\hat{\upsilon}_{max}$ can be obtained for specified percentile limits of its distribution. κ can be calculated. $\hat{p}_{min}$ is specified to calculate α. p_{max} is then selected so that β and β' are kept positive. In this paper, the range of the skewness coefficient is fixed at –1.5567 and at 1.6301 to give κ equals to –1.0472. To cater for at least 95% of tender cases, m_{max} is fixed at 11. The corresponding values of α and β' are shown in Table 2 for each pair of p_{min} and p_{max}. The effect of tender participation can be nullified ($\beta = 0$) by either p_{min} or p_{max} determined from Equations 11c and 11d. For a given p_{min}, the evaluator and decision maker can choose the strength of tender participation rate from the range of values of p_{max} determined from Equation 13a. Nullification will not occur if p_{max} is larger than the lower extreme value. With 3 parameters fixed, the preference function p, is determined from p_{min} and p_{max} for $\lambda = -1.6896$ as follows. Given $p_{min} = 0.2$, $5.395 < p_{max} < 15.167$ and given $p_{max} = 5.0$, $0.3857 > p_{min} > 0.2151$.

Table 2: Values of α and β' for the preference function
($\hat{\upsilon}_{max}$ = 1.6301 and $\hat{\upsilon}_{min}$ = –1.5567)

p_{min}	p_{max}	α	β' (m_{max} = 11)
0.3	5.0	0.7733	0.3489
0.25	5.0	0.8904	0.1581
0.2151	5.0	0.9871	0.0000
0.2	7.5	1.0337	0.3300
0.2	5.395	1.0337	0.0000

PRICE GAIN FUNCTIONS

Only price gains are of interest. At this point, the model excludes the coding of losses which is done in prospect theory. In an integrated tender evaluation procedure, price gain must be dimensionless. When price is in monetary unit, it must be evaluated separately because it is not compatible with other dimensionless attributes. One property of a price gain function is that for prices, $x_i > x_j > x_k$, the price gains derived from these prices must satisfy the condition $X_i < X_j < X_k$. Transitivity of price gains must be assured.

The qualitative ordinal scale commonly used cannot be adopted here because it fails to preserve the numerical information of price. Value scores must be made on quantitative scales. From experience, both price difference and price ratio have been use in tender evaluation. Price difference is a comparison of prices in an interval scale whereas price ratio represents relative value comparison in a ratio scale.

PRICE DIFFERENCE MODEL SUB-CLASS

For the price difference model, the price gain variable is a measure of the distance of a price from an upper price limit and is expressed as follows.

$$X_i = \frac{x_{max} - x_i}{x_{max} - x_{min}} \tag{14}$$

where x_{max} is the reference price on the interval scale and x_{min} is the lower price limit in the tender and x_i is the tender price of the i^{th} tender alternative. The price gain, $0 \le X_i \le 1$ is a normalised measure of price difference. It is small if x_i is close to the reference price, x_{max} and it is largest (unity) when $x_i = x_{min}$. With the project budget x_B, the price gain in dimensionless unit is adjusted to

$$X_i = \frac{x_B - x_{(i)}}{x_B - x_{min}} \tag{15a}$$

such that $x_{min} = x_{(1)}$ and if $x_{min} > x_B$, all X_i's = 0.

An un-normalised price gain in monetary unit is given by

$$X_i = x_B - x_{(i)} \tag{15b}$$

This is a straightforward relation that equates price score with price difference. It is a useful measure for evaluation methods that compare marginal benefit with price and evaluate price separately from the other criteria. Although it does not satisfy the condition $0 \le X \le 1$, it can be included in the generalised model as a special case in which the price gain value is equated with the price difference.

Correspondingly from Equation 15a and 15b, the ordered price gain sequence, $X_{(1)}, X_{(2)},.., X_{(j)},.., X_{(m-1)}, X_{(m)}$ in descending price difference, is obtained for calculating their scores, $A_{(i)}$. Thus, this method is a comparison based on ordered absolute values with reference to an artificial zero at x_B. It also satisfies the transitivity property.

PRICE RATIO MODEL SUB-CLASS

If X is allowed to assume other non-linear functions of price, it is possible to incorporate $X_i = \chi_i = x_i^{-1}$, Karsak's value function, into the generalised price-scoring model. Although p equals to unity, its scoring curve is strictly not linear. If $x_i = x_{min} + \Delta x_i$, where Δx_i is a small positive difference of the i^{th} price above the minimum price, x_{min}, then the value function is given by

$$X_i = \frac{1}{x_{min} + \Delta x_i} \approx \frac{1}{x_{min}}\left(1 - \frac{\Delta x_i}{x_{min}}\right) \tag{16}$$

If normalisation is done, the value function represents a ratio as follows:

$$X_i = \frac{x_{min}}{x_{min} + \Delta x_i} \approx \left(1 - \frac{\Delta x_i}{x_{min}}\right) \tag{17}$$

Both Equations 16 and 17, when substituted into Equation 1b produce identical effects. Both differ from the price difference function. The generalised model accepts both these two price ratio functions but Equation 16 can only be used in the normalised class of models because on its own it is not dimensionless. Cost control is implemented by excluding prices that exceed x_B from evaluation. The largest price does not necessarily have a low value because Equations 16 and 17 work in an ordered ratio scale with natural zero. Thus, a restriction to good discrimination is imposed. From the price survey, the standard deviation of price is small, making Δx_i of Equations 16 and 17 small relative to x_{min}. X_i is thus closer to unity than to zero. These two price gain functions satisfy the transitivity property but not the referencing property required in prospect theory. They represent a specific sub-class that works on ratio comparison.

COMPARISON BETWEEN PRICE DIFFERENCE AND PRICE RATIO MODELS

Discrimination and cost control are the bases of comparison between the two model sub-classes. One set of tender prices for low-price bunching and one set for high-price bunching are used to illustrate the characteristics of the price ratio and price difference models. The two tender sets have the same minimum and maximum prices. Their statistics are tabulated in Tables 3 and 4 for both tenders. The higher price tender naturally has the higher mean price. When the large extreme prices are eliminated by cost control, the mean prices are reduced slightly but they remain relatively robust enough to continue to indicate the market norm. The behaviours of their standard deviations and coefficient of skewness are less predictable because they are dependent on the price distribution after cost control action. A more reliable indication of the direction of change is seen in the reductions in the coefficients of variation after cost control. The tender with low-price bunching has coefficient of skewness of 1.721 for 8 prices and decreases to 1.324 for 7 prices after cost control. The corresponding values of *p* are 5.169 and 3.573 respectively. For the set with high-price bunching the coefficient of skewness is –1.170 for 8 prices and increases to –1.274 for 7 prices after cost control. The skew has

Table 3: Statistics from low-price bunching tender

Tender Prices, RM Million (Low-Price Bunching)	Statistics	Values (Cost-Controlled Values in Brackets)
5.973	Mean	6.079 (5.952)
5.713		
5.899	Standard Deviation	0.4119 (0.2201)
6.012		
6.386	Coefficient of Variation	0.0678 (0.0370)
5.926		
5.758	Coefficient of Skewness	1.721 (1.324)
6.965		

Table 4: Statistics from high-price bunching tender

Tender Prices, RM Million (Low-Price Bunching)	Statistics	Values (Cost-Controlled Values in Brackets)
6.673	Mean	6.515 (6.451)
5.713		
6.689	Standard Deviation	0.3998 (0.3846)
6.512		
6.788	Coefficient of Variation	0.0614 (0.0596)
6.226		
6.465	Coefficient of Skewness	-1.170 (-1.274)
6.965		

increased for high price bunching because the extreme (lowest) price is not eliminated by cost control in this case. The corresponding values of p are 0.3942 and 0.3538 respectively.

For a set of tender prices, 3 sets of normalised scores and 2 sets of un-normalised scores are calculated using a common preference function, p whose coefficients α and β determined earlier were adopted. The price scores from these 5 models are shown in Table 5 for low-price bunching and in Table 6 for high-price bunching. All 5 models show some degree of score bunching corresponding to price bunching. In Karsak's model, p equals to 1(α and β equal to zero) without cost control. Its scores are smaller compared with those of the price ratio model in the next column. Tables 5 and 6 show that the price ratio models do not convey significant score differences if the price differences are small when compared to their absolute values. As illustrated in Table 6, they more effectively provide information in terms of relative differences but are particularly weak in discriminating prices when there is high-price bunching. Comparatively, the price difference models are more discriminating because they enhance the scores of the low prices and reduce the scores of the high prices. As a result, they will tend to strengthen the effect of price when an overall tender evaluation is made together with the other non-price attributes.

By cost control, the over-priced scores of both the price ratio and the price difference models are set to zero, effectively eliminating over-priced tenders from consideration. To a small extent, cost control increases the normalised scores because the sum of scores is reduced by the elimination of the over-priced tender. The weakness of the price ratio model in not being able to score the budget price to zero is obvious here.

Normalisation makes the sum of scores equals to 1 and the individual scores less than 1. An un-normalised model enhances all price scores by making the score of the lowest price equal to 1. The larger score magnitude in the un-normalised price-scoring model has a greater ability to create price dominance in the evaluation because it always starts with a score of 1 irrespective of the number of tender prices.

PRICE-SCORING MODELS

From the generalised price-scoring model, a number of possible models for particular applications are derived as shown in Figure 3.

The un-normalised class using the un-normalised price-difference function with $p = 1$ is perhaps the simplest model and has attracted the widest application, especially in the 2-envelope system of tender evaluation in which price is separated from the non-price attributes. This model suits direct price comparison methods. Price difference in the mind of the evaluator is a straight forward and logical means of comparing prices in common monetary units. One would expect that a procedure by price difference is attractive because evaluators naturally judge and rank on the basis of price difference. By assigning $p = 0.88$, it becomes the original value function of prospect theory.

Table 5: Comparison among the price ratio models and the price difference models in low-price bunching case

Tender Prices, RM Million	Karsak's Model (α=0, β=0)	Normalised Price Scores Price Ratio Model (x_B = RM6.88million) (α=0.8904, β=0.01581)	Price Difference Model (x_B = RM6.88million) (α=0.8904, β=0.01581)
5.973	0.1267	0.1398 (*0.8530*)	0.1100 (*0.4063*)
5.713	0.1325	0.1639 (*1.0000*)	0.2708 (*1.0000*)
5.899	0.1283	0.1462 (*0.8918*)	0.1456 (*0.5378*)
6.012	0.1259	0.1366 (*0.8334*)	0.0940 (*0.3473*)
6.386	0.1185	0.1101 (*0.6717*)	0.0126 (*0.0464*)
5.926	0.1278	0.1438 (*0.8774*)	0.1318 (*0.4867*)
5.758	0.1315	0.1594 (*0.9724*)	0.2353 (*0.8689*)
6.965	0.1087	0.0000 (*0.0000*)	0.0000 (*0.0000*)

Note: Price scores in bracketed italic are un-normalised scores using Equation 1a.

Table 6: Comparison among the price ratio models and the price difference models in high-price bunching case

Tender Prices, RM Million	Karsak's Model (α=0, β=0)	Normalised Price Scores Price Ratio Model (x_B =RM6.88million) (α=0.8904, β=0.01581)	Price Difference Model (x_B =RM6.88million) (α=0.8904, β=0.01581)
6.673	0.1216	0.1410 (*0.9465*)	0.1247 (*0.5423*)
5.713	0.1421	0.1490 (*1.0000*)	0.2300 (*1.0000*)
6.689	0.1213	0.1409 (*0.9457*)	0.1212 (*0.5271*)
6.512	0.1246	0.1423 (*0.9547*)	0.1529 (*0.6648*)
6.788	0.1180	0.1395 (*0.9364*)	0.0242 (*0.1050*)
6.226	0.1303	0.1446 (*0.9700*)	0.1874 (*0.8147*)
6.465	0.1255	0.1426 (*0.9572*)	0.1595 (*0.6936*)
6.965	0.1165	0.0000 (*0.0000*)	0.0000 (*0.0000*)

Note: Price scores in bracketed italic are un-normalised scores using Equation 1a.

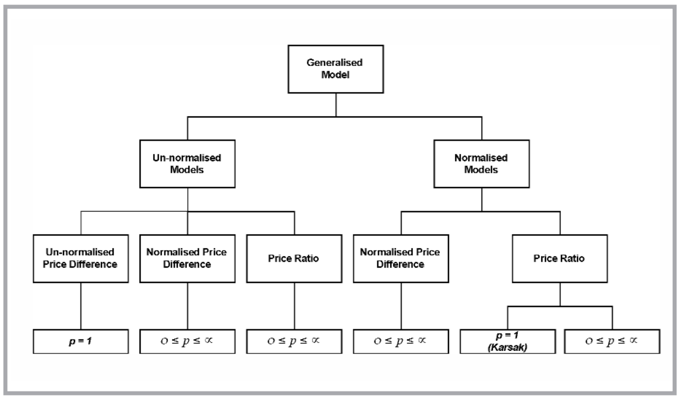

Figure 3: Particular price-scoring models from the generalised model

The un-normalised class using normalised price gain functions are suitable for integrated evaluation of both price and other non-price attributes in a computer program. Because of their larger score values compared with those of the normalised class, they have price dominant tendencies. The price-difference function can be adopted if dominance by a single price is desired. On the other hand, the price-ratio function will tend to move the scores away from single-price dominance. When price bunching occurs, collective dominance exists in the un-normalised function. The normalised class using normalised price gain functions have different characteristics compared with those of the un-normalised class of models. They produce an effect that tends not to emphasise on price nor discriminate between prices thus, allow the other attributes to play more influential roles in the evaluation. Their smaller scores are controlled by the number of tenders evaluated. The more tenders, the smaller the scores and the lesser the ability to dominate.

The choice of any of the above models should be made by matching their characteristics with the evaluation objective. If the lowest price is to dominate, then an un-normalised class of price-difference scoring model is appropriate. If price dominance is not intended, then a normalised class of price-ratio scoring model is more effective. Both the un-normalised and normalised classes that use a dimensionless price gain function based on either price difference or price ratio meets the requirement of dimensionless attribute scores in Thum's fuzzy tender evaluation model [5].

CONCLUSION

A generalised price-scoring model is developed by incorporating two price gain functions, one based on price difference on an interval scale and the other on price ratio on a ratio scale. It attempts to prevent cost overrun by eliminating over-priced tenders by comparison with the project budget. The behaviour of evaluators and decision makers is modelled in the preference factor which is a function of the tender participation rate and its price distribution skew. In the presence of unusual influences or when fair competition is threatened, the model will attempt to counteract the negative strategies adopted by tenderers, failing which a pre-assignment option built into the program allows the evaluator to make judgment outside the model.

The generalised price-scoring model produces two generic price model classes. The un-normalised class tend to produce price dominant effects while the normalised class tends to allow non-price attributes to have more influence in the overall evaluation. These two classes produce their own price ratio sub-class using a price gain derived from the ratio of the minimum tender price to a price, and their price difference sub-class derived from the price gain measure of a price with reference to the project budget. A survey of tender prices provides data for estimating the tender participation rate and the range of variations of the price distribution's coefficient of skewness to be used in the preference function. By appropriate choice of price gain function and of the preference function, 5 particular price-scoring models are derived. Price difference models are found to be better at discriminating prices than price ratio models. Price ratio models are more suitable for matching tender evaluation objectives that do not emphasise price. The combination of the value functions and the price gain functions with built-in cost control produces a diverse number of models for specific applications in a tender evaluation computer program.

Further investigation is required to test them with the non-price attributes in order to define their application in a tender evaluation procedure. The generalised price-scoring model hence, provides a means of studying the existing methods of tender evaluation by computer simulation. For example, comparisons can be made between an integrated approach in which price and other attributes are combined in a 1-stage evaluation procedure, and one in which price is evaluated after the evaluation of other non-price attributes is completed in a 2-stage evaluation procedure. ■

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