Theoretical Study Of The Structural, Electronic Structure, Fermi Surface, Electronic Charge Density and Optical Properties of the of LnVO$_4$ (Ln= Sm, Eu, Gd and Dy)

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We present first-principles calculations of the electronic structure, Fermi surface, electronic charge density and optical properties of LnVO$_4$ (Ln= Sm, Eu, Gd and Dy) based on density-functional theory using the local density approximation (LDA), generalized-gradient approximation (GGA) and the Engel Vosko GGA formalism (EV-GGA). The calculated band structure and density of states show that LnVO$_4$ compound is metallic. The total DOS at Fermi level $N(E_F)$ were 1.49, 0.95, 1.93 and 4.69 states/eV unit cell, and the bare electronic specific heat coefficient ($\gamma$) are 0.26, 0.16, 0.33 and 0.81 $\text{mJ/mol-K}^2$ for SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$ respectively. The Fermi surface of EuVO$_4$ compound is composed of one band and SmVO4, GdVO$_4$ and DyVO$_4$ composed of two bands crossing along the $\Gamma$–$A$ direction of Brillion zone. The bonding features are analyzed by using the electronic charge density contour in the (1 0 1) crystallographic plane. We found that Ln-O and V-O atoms forms a strong covalent. For further insight information about the electronic structure, the optical properties are derived and analyzed.

**Keywords:** FPLAPW calculations; Electronic structure; Fermi surface; Charge density; Optical properties; DFT:

1. INTRODUCTION

Despite of their technological importance, which covers a number of applications comprising their use as thermo phosphors, cathodoluminescent materials and scinillators, until in recent times there has been a drought of high precision structural information for arthovanadate compounds of the
form LnVO$_4$. This situation was cured, at least in fraction, by the fresh report by Chakounmkos et al. [I] of the bursting profile Rietveld structural scrutiny of neutron powder-diffraction data for the zircon-structure LnVO$_4$ compounds with Ln = Sc, Cc, Y, Pr, Nd, Tb, Yb, Er, He, Tin and Lu. Preceding to this investigation, high-precision X-ray diffraction single-crystal structure results were only existing for CeVO$_4$ [2] and NdVO$_4$[3] whereas lower precision refinements, centered on neutron powder diffraction data, had been accounted for TbVO$_4$, DyVO$_4$, ErVO$_4$, HoVO$_4$ and YbVO$_4$ [4-6].

Whereas the full-profile Rietveld assessments of Chakoullakos et al. [I] granted accurate crystallographic data for numerous of the rare-earth orthovanadate compounds (plus ScVO$_4$ and YVO$_4$), neutron-powder diffraction data could not be attained for the lanthanide compounds EuVO$_4$, SmVO$_4$, EuVO$_4$ and GdVO$_4$, due to their soaring cross sections for neutron absorption.

Neither experimental nor theoretical details regarding the electronic structure, Fermi surface, electronic charge density and optical properties are available in the literature. The reasons mentioned above motivate us to perform these calculations, using the full potential linear augmented plane wave (FP-LAPW) method in order to provide reference data for the experimentalists and to complete existing theoretical works on this material.

The rest of the paper has been divided in three parts. In Section 2, we briefly describe the computational techniques used in this study. The most relevant results obtained for the electronic structure, Fermi surface, electronic charge density and optical properties for EuVO$_4$, SmVO$_4$, EuVO$_4$ and GdVO$_4$ compounds are presented and discussed in Section 3. Finally, in Section 4 we summarize the main conclusions of our work.

### 2. CALCULATION METHODOLOGY

The crystal structure of EuVO$_4$, SmVO$_4$, EuVO$_4$ and GdVO$_4$ are shown in Fig.1. The scrutinize compounds have the tetragonal symmetry with space group I41/amd (No. 141). The lattice constant for EuVO$_4$ are $a=7.2647(9)$ Å and $c=6.384(1)$ Å, for SmVO$_4$ are $a=7.2358(7)$ Å and $c=6.362(1)$ Å, for EuVO$_4$ are $a=7.2122(7)$ Å and $c=6.346(2)$ Å and for GdVO$_4$ are $a=7.1429(8)$ Å and $c=6.300(2)$ Å. The atomic position in sequence for EuVO$_4$, SmVO$_4$, EuVO$_4$ and GdVO$_4$ are given in Table 1.

<table>
<thead>
<tr>
<th>Atoms</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln = Sm</td>
<td>0000</td>
<td>0.7500</td>
<td>0.1250</td>
</tr>
<tr>
<td>V</td>
<td>0000</td>
<td>0.2500</td>
<td>0.3750</td>
</tr>
<tr>
<td>O</td>
<td>0.4316(3)</td>
<td>0000</td>
<td>0.2045(4)</td>
</tr>
<tr>
<td>Ln= Eu</td>
<td>0000</td>
<td>0.7500</td>
<td>0.1250</td>
</tr>
<tr>
<td>V</td>
<td>0000</td>
<td>0.2500</td>
<td>0.3750</td>
</tr>
<tr>
<td>O</td>
<td>0.4318(4)</td>
<td>0000</td>
<td>0.2036(6)</td>
</tr>
<tr>
<td>Ln= Gd</td>
<td>0000</td>
<td>0.7500</td>
<td>0.1250</td>
</tr>
<tr>
<td>V</td>
<td>0000</td>
<td>0.2500</td>
<td>0.3750</td>
</tr>
<tr>
<td>O</td>
<td>0.4323(5)</td>
<td>0000</td>
<td>0.2028(7)</td>
</tr>
<tr>
<td>Ln= Dy</td>
<td>0000</td>
<td>0.7500</td>
<td>0.1250</td>
</tr>
</tbody>
</table>
We have executed the calculations using the full-potential linear augmented plane wave [FPLAPW] as incorporated in WIEN2K code [7]. This is an implementation of the density functional theory (DFT) [8,9] with different possible approximations for the exchange correlation (XC) potentials. The XC potential were the local density approximation (LDA) of Ceperley–Alder parameterized by Perdew–Zunger [10], generalized gradient approximation (GGA) of Perdew–Burke–Ernzerhof (PBE) [11], which is based on exchange-correlation energy optimization to calculate the total energy. Also we have used the Engel–Vosko GGA formalism (EVGGA) [12], which optimizes the corresponding potential for band structure calculations. In order to achieve energy eigenvalues convergence, the wave functions in the interstitial region were expanded in plane waves with a cut-off $K_{\text{max}} = 7/RMT$, where $RMT$ denotes the smallest atomic muffin-tin sphere radius and $K_{\text{max}}$ gives the magnitude of the largest $K$ vector in the plane wave expansion. The $RMT$ were taken to be 2.24, 2.23, 2.22 and 2.14 atomic units (a.u.) for Sm, Eu, Gd and Dy respectively and for V and O were 1.62 and 1.43 a.u. A mesh of 2000 $k$ points was used in the irreducible Brillion zone integration. In the self consistent field for electronic structure, Fermi surface and optical properties convergence was checked through self consistency. The valence wave functions inside the muffin-tin spheres are expanded up to $l_{\text{max}} = 10$, while the charge density was Fourier expanded up to $G_{\text{max}} = 12$ (a.u.)$^{-1}$.

3. RESULTS AND DISCUSSION

3.1. Electronic structure

Fig. 2 shows the calculated energy band structure of LnVO$_4$ compounds at equilibrium lattice parameters along the high symmetry directions in the Brillion zone. The valence and conduction bands overlap considerably at the Fermi level as a result these would exhibit metallic properties. More understanding can be achieved from the total and partial density of states (TDOS and PDOS) which are also depicted for the investigated compounds in Fig. 4 and 5. We displayed the calculated results by the EVGGA approximations. With the reason that this approximation gives better band splitting as well optical transition as is clear from the Fig. 4 and 5, that when we replace Sm by Eu and Gd so the peaks shift towards the lower energies and when we replace Gd by Dy again the peaks shifts towards the higher energies.

The peaks between -6.0 eV and -2.0 eV are due to Ln-s/p, V-s/p and O-p states with minimum contribution of O-s states. While the peaks around the Fermi level are due to Ln-f with small contribution of Ln-s/p. The peaks situated between 2.0 eV and 5.0 eV are due to the contribution of V-p/d states with small contribution of O-s state and the peaks from 6.0 eV to 15.0 eV are due to Ln-d and O-s states with small contribution of V-s/p states. There exists a strong hybridization between V-s and V-p in the energy range -6.0 eV and -3.0 eV. Whereas there is strong hybridization around the Fermi level between V-s, V-p and O-s states and also at 2.0 eV there is also a strong hybridization between Ln-d and O-p states.
At the Fermi level $E_F$, the values of TDOS (EVGGA) for SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$, were 1.49, 0.95, 1.93 and 4.69 states/eV unit cell, respectively. Hence one may to conclude that DyVO$_4$ is more conducting than SmVO$_4$, EuVO$_4$, and GdVO$_4$.

The electronic specific heat coefficient ($\gamma$), which is a function of the density of states, can be calculated using the expression $\gamma = \frac{1}{3} \pi^2 N(E_F)K_B^2$, where $N(E_F)$ is the DOS at the Fermi energy $E_F$ and $K_B$ is the Boltzmann constant. The calculated density of states at the Fermi energy $N(E_F)$ enables us to calculate the bare electronic specific heat coefficient, which is 0.26, 0.16, 0.33 and 0.81 mJ/mol-$K^2$ for SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$, respectively.

Electron charge density denotes the nature of the bond character. In order to predict the chemical bonding and also the charge transfer in SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$ compounds, the charge-density behaviors in 2D are calculated in the (1 0 1) plane and displayed in Figs 6. The result shows that the substitution of the Sm by Eu, Gd and Dy leads to redistribution of electron charge density i.e. the bond distances increase. As an atom is interchanged by the others the VSCC (valence shell charge carrier) properties changes (Badar et. al.). By investigating the influence of replacing Sm by Eu, Gd and Dy in SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$ compounds, it has been found that the charge density around ‘Sm’ and ‘Eu’ is greater than both of Gd and Dy atoms. From the calculated electron density it is clear that Ln and O, and also V and O atoms forms a strong covalent bond.

3.2. Fermi surface

It is well established that electronic states crossing the Fermi level are primarily responsible for the Fermi surface (FS) structure and always considered as a key quantity to understand the electronic structure of any metallic material. The conventional way to find out the Fermi surface is to measure energy distribution curves (EDC) for distinct k-points of the Brillouin zone and to ascertain the k-locations where bands traverse the Fermi energy. The electrons adjacent to the Fermi level are responsible for conductivity, the electronic structure of any metallic material is understandable from the Fermi surface (FS). Fermi surfaces can also be inspected by using de Haas van Alphen (dHvA) experiments, the exact determination of the shape of the FS depends on models using easy geometries to define the magnetic field angle dependency of the measured frequencies, but the accurate shape of the FS can of course be much further complex. Therefore we figured the FS of SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$ using the FPLAPW method outlined above. The results are shown in Fig 7.

The Fermi surface of EuVO$_4$ compound is composed of one band while SmVO$_4$, GdVO$_4$ and DyVO$_4$ composed of two bands crossing along the Γ−A direction of Brillion zone. The SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$ Fermi surface consist of a set of holes and electronic sheets, where the empty areas contain holes and shaded areas electrons.

In the four compounds a considerable differences there exists in the FS topology, due to changes in inter-atomic distances, bonding angles and as well as to the degree of band filling (Fig. 7). In the Fermi surface the color changes occur due to the change in electron velocity. The red and violet color shows fast and the slow velocity of electrons while the remaining colors have the intermediate
velocity of electron. The structure of the four compounds are considerably the same but the changes occur in the color which resulting the change in velocity of electron.

3.3. Optical Properties

Study the optical properties of solids has been an interesting research topic in basic research as well as for industrial applications. In basic research the origin and nature of different excitation processes are of fundamental interest and in the industrial applications can make use of them in many optoelectronic devices. Therefore optical properties bring together experiment and theory. The calculation of $\varepsilon_2(\omega)$ requires energy eigen-values and electron wave functions. These are natural outputs of a band structure calculation. The interband transitions of the dielectric function can be split into direct and indirect transitions. We overlook the indirect inter-band transitions involving scattering of phonons that are expected to give a small contribution to $\varepsilon(\omega)$ [13]. To compute the direct inter-band contributions to the imaginary part of the dielectric function $\varepsilon_2(\omega)$, it is essential to operate summation up all likely transitions from the occupied to the unoccupied states. These compounds are crystallized in the I41/amd (#= 141) space group. This symmetry group has two dominant independent components of the dielectric tensor. The corresponding dielectric functions are $\varepsilon_{xx}(\omega)$ and $\varepsilon_{zz}(\omega)$ corresponding to the applied electric field (corresponding to light polarization) parallel and perpendicular to the crystallographic c-axis. Taking the appropriate transition matrix elements into account, we calculated the imaginary part of the dielectric functions $\varepsilon_2(\omega)$ using the expressions given in the Refs. [14–16]:

$$\varepsilon_{xx}(\omega) = \frac{8}{m\omega^2} \int \sum_{\text{BZ}} \frac{\left| P_{xx}^{n'n'}(k) \right|^2}{\nabla \omega_{nn'}(k)} dS_k \quad \ldots \ldots \ (1)$$

$$\varepsilon_{zz}(\omega) = \frac{12}{m\omega^2} \int \sum_{\text{BZ}} \frac{\left| P_{zz}^{n'n'}(k) \right|^2}{\nabla \omega_{nn'}(k)} dS_k \quad \ldots \ldots \ (2)$$

The above terms are written in atomic units with $\varepsilon^2 = 1/m=2$ and $\hbar=1$, where $\omega$ is the photon energy. $P_{xx}^{n'n'}(k)$ and $P_{zz}^{n'n'}(k)$ are the X and Z component of the dipolar matrix elements between initial $|n'k\rangle$ and final $|n'k\rangle$ states with their eigenvalues $E_n(k)$ and $E_{n'}'(k)$ respectively. $\omega_{nn'}(k)$ is the energy difference, where $\omega_{n'n'}(k) = E_n(k) - E_{n'}'(k)$ and $S_k$ is a constant energy surface with $S_k = \{k; \omega_{nn'}(k) = \omega\}$. The integration is done over the first irreducible Brillion zone (IBZ).

Since the investigated compounds have metallic nature thus we have to include the intra band transitions (Drude term). Therefore both of the intra-band and inter-band transitions are contribute:

$$\varepsilon_2(\omega) = \varepsilon_{2\text{inter}}(\omega) + \varepsilon_{2\text{intra}}(\omega) \quad \ldots \ldots \ (3)$$

where

$$\varepsilon_{2\text{intra}}(\omega) = \frac{\omega_{n'n'}^\tau}{\omega(1 + \omega^2 \tau^2)} \quad \ldots \ldots \ (4)$$

$$\varepsilon_{2\text{intra}}(\omega) = \frac{\omega_{n'n'}^\tau}{\omega(1 + \omega^2 \tau^2)} \quad \ldots \ldots \ (5)$$
where $\tau$ and $\omega_p$ [51] is mean free time and mean frequency amongst the collisions.

$$\omega_p^2 = \frac{8\pi}{3} \sum_{kn} \theta_{kn} \delta(e_{kn}) \ldots (6)$$

As $e_{kn}$ is $E_n(k) - E_F$ and $\theta_{kn}$ is the velocity of electron.

The calculated values of $\hbar\omega_p$ corresponding to two polarizations parallel and perpendicular to $c$-axis are given in Table. 2.

**Table 2.** The calculated plasma energy in eV of the LnVO$_4$ compound.

<table>
<thead>
<tr>
<th>Compounds</th>
<th>$\hbar\omega_p^{xx}$</th>
<th>$\hbar\omega_p^{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SmVO$_4$</td>
<td>0.40</td>
<td>8.21</td>
</tr>
<tr>
<td>EuVO$_4$</td>
<td>0.68</td>
<td>8.27</td>
</tr>
<tr>
<td>GdVO$_4$</td>
<td>0.42</td>
<td>8.32</td>
</tr>
<tr>
<td>DyVO$_4$</td>
<td>0.65</td>
<td>0.73</td>
</tr>
</tbody>
</table>

For the metallic materials there are two contributions to $\varepsilon(\omega)$, namely intra-band and inter-band transitions. Fig. 8, illustrates the variation of the imaginary part of the frequency dependent electronic dielectric function $\varepsilon_2(\omega)$. The calculated imaginary part $\varepsilon_1(\omega)$ can be used to describe the transitions between the occupied and unoccupied bands. It can be seen from $\varepsilon_2(\omega)$ spectra that the spectral features of the optical properties for the investigated compounds are looking different that is ascribed to the fact that the band structures of these compounds acquire different dispersion as a consequence the allowed optical transition are occurs between different states, resulting in different dispersion of the spectral features of the optical susceptibilities suggesting that moving through the periodic table from left to right replacing the element Sm by Eu, Gd and Dy, show there is a considerable change in the dispersion of the spectral features of the optical properties. We should emphasize that there is a considerable anisotropy between $\varepsilon_2^{xx}(\omega)$ and $\varepsilon_2^{zz}(\omega)$ components. The peaks in the optical response are caused by the allowed electric-dipole transitions between the valence and conduction bands. In order to identify these structures we should consider the magnitude of the optical matrix elements. The observed structures would correspond to those transitions which have large optical matrix dipole transition elements. From the imaginary parts of the dielectric function $\varepsilon_2^{xx}(\omega)$ and $\varepsilon_2^{zz}(\omega)$ the real parts $\varepsilon_1^{xx}(\omega)$ and $\varepsilon_1^{zz}(\omega)$ can be calculated using Kramers–Kronig relations [17]. Our calculated real parts are illustrated in Fig. 9. Again we find a considerable anisotropy between $\varepsilon_1^{xx}(\omega)$ and $\varepsilon_1^{zz}(\omega)$.

4. CONCLUSION

In summary, we calculate the electronic structure, Fermi surface, charge density and optical properties of EuVO$_4$, SmVO$_4$, EuVO$_4$, GdVO$_4$ and DyVO$_4$ compounds by means of DFT within LDA, GGA and EVGGA. The investigated compounds possess metallic nature, the values of DOS at $E_F$ ($N(E_F)$) were 1.49, 0.95, 1.93 and 4.69 states/eV unit cell, and the bare electronic specific heat
The coefficient (γ) are 0.26, 0.16, 0.33 and 0.81 mJ/mol-K² for SmVO₄, EuVO₄, GdVO₄ and DyVO₄, respectively. The Fermi surface of EuVO₄ compound is composed of one band while SmVO₄, EuVO₄, GdVO₄ and DyVO₄ composed of two bands crossing along the Γ−A direction of Brillion zone. The bonding features are analyzed by using the electronic charge density contour in the (101) crystallographic plane. The bonding features are analyzed by using the electronic charge density contour in the (101) crystallographic plane. We found that Ln-O and V-O atoms forms a strong covalent. There exists a strong hybridization between V-s and V-p in the energy range -6.0 eV and -3.0 eV. Whereas there is strong hybridization around the Fermi level between V-s, V-p and O-s states and also at 2.0 eV there is also a strong hybridization between Ln-d and O-p states. We also present the dielectric function of SmVO₄, EuVO₄, GdVO₄ and DyVO₄ and the imaginary part $\varepsilon_2(\omega)$ of the dielectric function is discussed in detail.

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References


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Figure 1. Unit cell structure for LnVO₄ compound.
Figure 2. Calculated Band structures for LnVO₄ compound
Figure 3. Calculated combined total density of state for LnVO₄ compound (State/eV unit cell)
Figure 4. Calculated total density of state for LnVO₄ compound (State/eV unit cell)
Figure 5. Calculated Partial density of state for LnVO₄ compound (State/eV unit cell)
Figure 6. Calculated Fermi surface for LnVO₄ compound.
Figure 7. Calculated electron charge density for LnVO₄ compounds
Figure 8. Calculated imaginary part of dielectric tensor component for LnVO₄ compound \( \varepsilon_2(\omega) \)
Figure 9. Calculated real part of dielectric tensor component for LnVO₄ compound \( \varepsilon_1(\omega) \)
Fig. 2
Fig. 3

(a)
Fig. 5
Fig. 6

Fermi surface for SmVO$_4$
Fermi surface for EuVO₄

Merged 2D Band

Merged 3D Band

Fermi surface for GyVO₄
Fermi surface for DyVO$_4$

Fig. 7

SmVO$_4$
EVGGA

\[ \varepsilon^\omega(\omega) \]
\[ \varepsilon^\omega(\omega) \]

Energy(eV)
(b) EuVO\textsubscript{4} EVGGA

\[ \varepsilon_1^{\text{xx}}(\omega) \]

\[ \varepsilon_1^{\text{zz}}(\omega) \]

(c) GdVO\textsubscript{4} EVGGA

\[ \varepsilon_1^{\text{xx}}(\omega) \]

\[ \varepsilon_1^{\text{zz}}(\omega) \]