European Journal of Scientific Research
ISSN 1450-216X Vol.33 No.4 (2009), pp.710-717
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http://www.eurojournals.com/ejsr.htm

Numerical Solution of the Boundary Layer Flow Over an Exponentially Stretching Sheet with Thermal Radiation

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Abstract

In this paper, the problem of steady laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid with a presence of thermal radiation over an exponentially stretching sheet is investigated numerically. The governing boundary layer equations are reduced into ordinary differential equations by a similarity transformation. The transformed equations are solved numerically using an implicit finite-difference scheme known as the Keller-box method. The numerical solutions for the wall skin friction coefficient, the heat transfer coefficient, and the velocity and temperature profiles are computed, analyzed and discussed.

Keywords: Boundary Layer Flow, Exponentially Stretching Sheet, Numerical Solution, Thermal Radiation

1. Introduction

The study of flow over a stretching sheet has generated much interest in recent years in view of its numerous industrial applications such as the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries. Since the pioneering work of Sakiadis (1961) which studied the stretching flow problem, various aspects of the problem have been investigated by many authors such as Cortell (2006), Xu and Liao (2005), Hayat et al. (2006) and Hayat and Sajid (2007). Besides, Magyari and Keller (2000) also focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching surface. On the other hand, Gupta and Gupta (1997) stressed that realistically, stretching surface is not necessarily continuous. Previously, by the fact that cooling rate affects the quality of products, Ali (1995) has investigated the thermal boundary layer flow by considering the nonlinear stretching surface. Extension to that, Elbashbeshy (2001) added new dimension to the study on exponentially continuous stretching surface. A few years later, Khan (2006) and Sanjayanand and Khan (2006) studied the viscous-elastic boundary layer flow and heat transfer due to an exponentially stretching sheet.

It is worth mentioning that the studies of thermal radiation and heat transfer are important in electrical power generation, astrophysical flows, solar power technology and other industrial areas. A lot of extensive literature that deals with flows in the presence of radiation effects is now available. Elbashbeshy and Dimian (2002) analyzed boundary layer flow in the presence of radiation effect and heat transfer over the wedge with viscous coefficient. Besides that, Cortell (2008) has solved a problem on the effect of radiation on Blasius flow by using fourth-order Runge-Kutta approach. Later, Sajid and Hayat (2008) considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet by solving the problem analytically via homotopy analysis method (HAM). Recently, El-Aziz (2009) and Ishak (2009) also focused on the effects of thermal radiation in their studies.

In this paper, we investigate numerically the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet, which has been solved analytically by Sajid and Hayat (2008). By employing the similarity transformation, the boundary layer equations are solved numerically using an efficient implicit finite-difference scheme known as the Keller-box method (Cebeci & Bradshaw, 1977, 1988).

2. Mathematical Formulation

Consider the two-dimensional flow of an incompressible viscous fluid bounded by a stretching sheet in which the x-axis is taken along the stretching sheet in the direction of the motion and y-axis is perpendicular to it. Under the usual boundary layer approximations, the flow and heat transfer in the presence of radiation effects are governed by the following equations (Sajid & Hayat, 2008):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y}, \tag{3}$$

where u and v are the velocities in the x- and y- directions, respectively, ρ is the fluid density, ν (=μ/ρ) is the kinematic viscosity, μ is the dynamic viscosity, T is the temperature, k is the thermal conductivity, c_p is the specific heat and q_r is the radiative heat flux. The boundary conditions are given by

$$\begin{gathered} u(0)=U_0 e^{x/L}, \quad v(0)=0, \quad T(0)=T_\infty+T_0 e^{x/2L}, \\ u\to 0, \quad T\to 0 \quad \text{as } y\to\infty, \end{gathered} \tag{4}$$

where U_o is the reference velocity, T_o is the temperature at the plate and T_∞ is the temperature far away from the plate while L is a constant. Employing Rosseland approximation of radiation for an optically thick layer one has (Sajid & Hayat, 2008)

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where k^* is the mean absorption coefficient and σ^* is the Stefan- Boltzmann constant. T^4 is expressed as a linear function of temperature, hence

$$T^4=4T_\infty^3 T-3T_\infty^4 \tag{6}$$

Invoking Equations (3), (5) and (6) one can write

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k+\frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

Introduce the following transformations:

$$u=U_0 e^{x/L} f'(\eta),\ v=-\sqrt{\frac{\nu U_0}{2L}}e^{x/2L}\left\{f(\eta)+\eta f'(\eta)\right\},\ T=T_0 e^{x/2L}\theta(\eta),\ \eta=\sqrt{\frac{U_0}{2\nu L}}e^{x/2L}y \tag{8}$$

Equation (1) is automatically satisfied and Equations (2) and (7) reduce to

$$f'''-2f'^2+ff''=0, \tag{9}$$

$$\left(1+\frac{4}{3}K\right)\theta''+\Pr\left[f\theta'-f'\theta+Ef''^2\right]=0, \tag{10}$$

with the boundary conditions (4) become

$$\begin{gathered} f(0)=0,\ f'(0)=1,\ \theta(0)=1, \\ f'\to 0,\ \theta\to 0\ \text{ as } \eta\to\infty. \end{gathered} \tag{11}$$

In the above equations, $\Pr=\mu c p/k$, $E=U_o^2/T_o c_p$ and $K=4\sigma^* T_\infty^3/k^* k$ are the Prandtl, Eckert and radiation numbers, respectively, and prime denotes differentiation with respect to η.

3. Results and Discussion

Equations (9) and (10) subject to the boundary conditions (11) are solved numerically using the Keller-box method as described by Cebeci and Bradshaw (1977, 1988). From the results, it is seen that variations in Prandtl number Pr, radiation number K and Eckert number E do not affect the value of the wall skin friction coefficient due to the decoupled equations. The unique value obtained is 1.28180. Tables 1, 2 and 3 present the values of the heat transfer coefficient $-\theta'(0)$ for various values of the Pr, K and E, respectively. It is shown that as Pr increases, the heat transfer coefficient increases. Contradictory to Pr, increasing of parameters K and E decreases the value of heat transfer coefficient.

Table 1: Values of the heat transfer coefficient, $-\theta'(0)$ for various values of K and E with Pr =1, 2, 3

K	***E* = 0**			***E* = 0.2**			***E* = 0.9**		
	Pr=1	**Pr=2**	**Pr=3**	**Pr=1**	**Pr=2**	**Pr=3**	**Pr=1**	**Pr=2**	**Pr=3**
0	0.9548	1.4714	1.8691	0.8622	1.3055	1.6882	0.5385	0.7248	0.8301
0.5	0.6765	1.0735	1.3807	0.6177	0.9654	1.2286	0.4101	0.5869	0.6964
1	0.5315	0.8627	1.1214	0.4877	0.7818	1.0067	0.3343	0.4984	0.6055

Table 2: Values of the heat transfer coefficient, $-\theta'(0)$ for various values of Pr and E with K = 0, 0.5, 1

Pr	**E = 0**			**E = 0.2**			**E = 0.9**		
	***K*=0**	***K*=0.5**	***K*=1**	***K*=0**	***K*=0.5**	***K*=1**	***K*=0**	***K*=0.5**	***K*=1**
1	0.9547	0.6765	0.5315	0.8622	0.6177	0.4877	0.5385	0.4101	0.3343
2	1.4714	1.0735	0.8627	1.3055	0.9654	0.7818	0.7248	0.5870	0.4984
3	1.8691	1.3807	1.1214	1.6882	1.2286	1.0067	0.8301	0.6964	0.6055

Table 3: Values of the heat transfer coefficient, $-\theta'(0)$ for various values of Pr and K with E = 0, 0.2, 0.9

Pr	***K* = 0**			***K* = 0.5**			***K*= 1**		
	***E*=0**	***E*`=0.2**	***E*=0.9**	***E*=0**	***E*=0.2**	***E*=0.9**	***E*=0**	***E*=0.2**	***E*=0.9**
1	0.9547	0.8622	0.5385	0.6765	0.6177	0.4101	0.5315	0.4877	0.3343
2	1.4714	1.3055	0.7248	1.0735	0.9654	0.5869	0.8627	0.7818	0.4984
3	1.8691	1.6882	0.8301	1.3807	1.2286	0.6964	1.1214	1.0067	0.6055

Figures 1 illustrates the effects of Prandtl number Pr = 1, Eckert number E = 0.2 and radiation number K = 1 on the velocity profile $f'(\eta)$, $f(\eta)$ and temperature profile $\theta(\eta)$. It is shown that the velocity profile $f'(\eta)$ and $f(\eta)$ are inversely proportional to each other. The velocity profile is unique for all values of Pr, E and K due to the decoupled Equations (9) and (10). Further, Figs. 2, 3 and 4 show the effects of Pr, E and K on the temperature profiles $\theta(\eta)$, respectively. It is shown in Fig. 2 that the increase in Pr causes the decrease in temperature profiles $\theta(\eta)$ and the thermal boundary layer thickness. Physically, if Pr increases, the thermal diffusivity decreases and these phenomena lead to the decreasing of energy ability that reduces the thermal boundary layer. On the other hand, it has been observed in Fig. 3 that the temperature profiles and the thermal boundary layer thickness increase slightly with an increase of the Eckert number E. From Fig. 4, it is seen that the trend of the profiles is similar to the effect of E that is as K increases, the temperature profiles and thermal boundary layer thickness also increase.

For further observation, the effects of E and K with fixed Pr = 1 can be found in Fig. 5. It is shown that as E and K increase, the temperature profiles also increase and the effects of K are more pronounced than the effects of E. On the other hand, the effects of E and Pr, with fixed K = 1, are illustrated in Fig. 6. It is shown that their effects are the opposite, in which the increase in E and the decrease in Pr lead to the increase in the temperature profiles. Finally, the effects of K and Pr with fixed E = 0.5 are displayed in Fig. 7. It is shown again that although both K and E have the same effects on the temperature profiles, in contrast to the effects of Pr, the effects of K are more pronounced than the effects of E.

Figure 1: Velocity profile $f'(\eta), f(\eta)$ and temperature profile $\theta(\eta)$

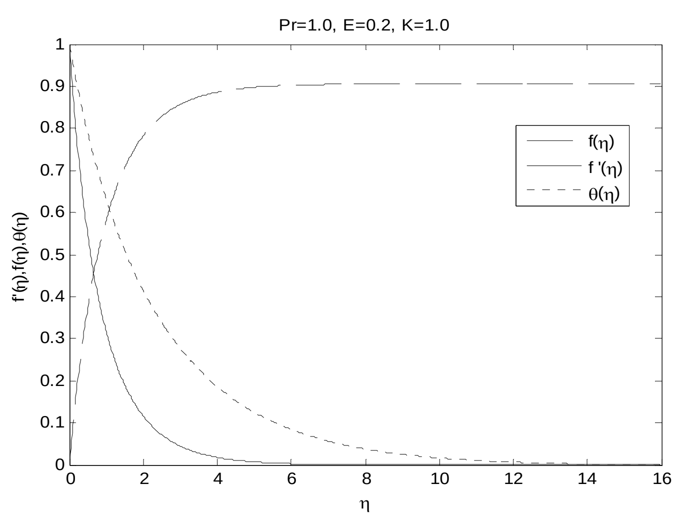


Figure 2: Effects of Pr on the temperature profiles $\theta(\eta)$

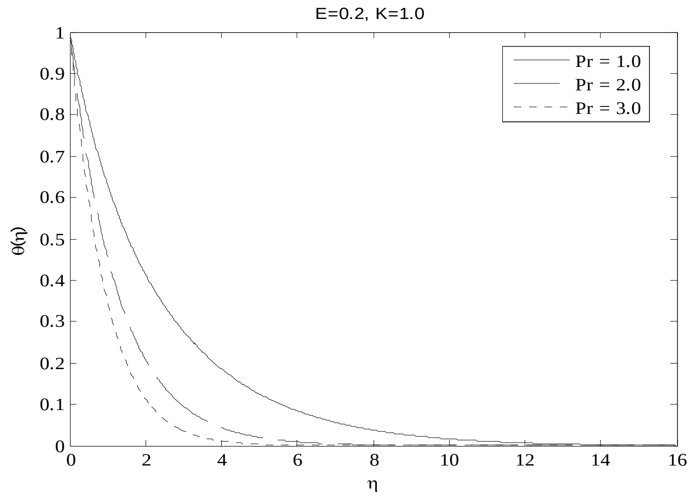


Figure 3: Effects of *E* on the temperature profiles $\theta(\eta)$

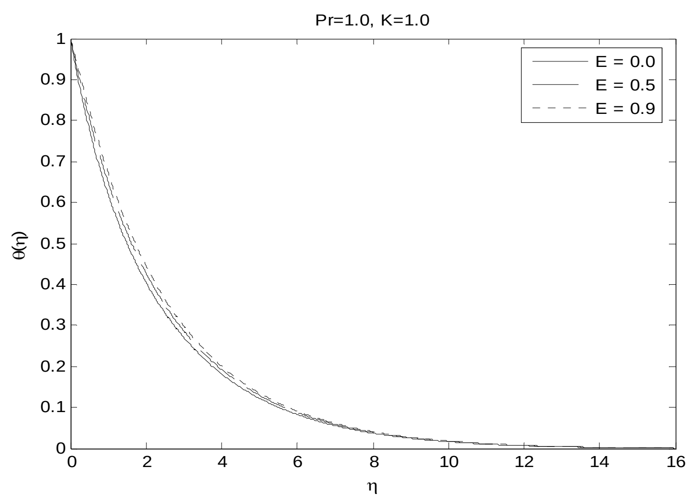


Figure 4: Effects of *K* on the temperature profiles $\theta(\eta)$

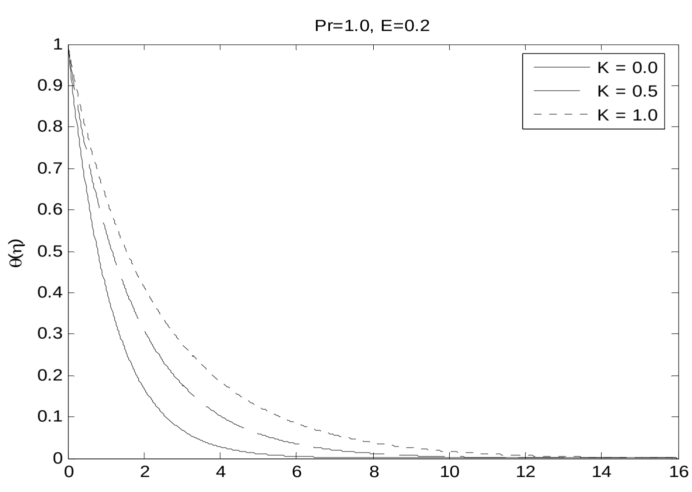


Figure 5: Effects of E and K on the temperature profiles $\theta(\eta)$ with Pr = 1

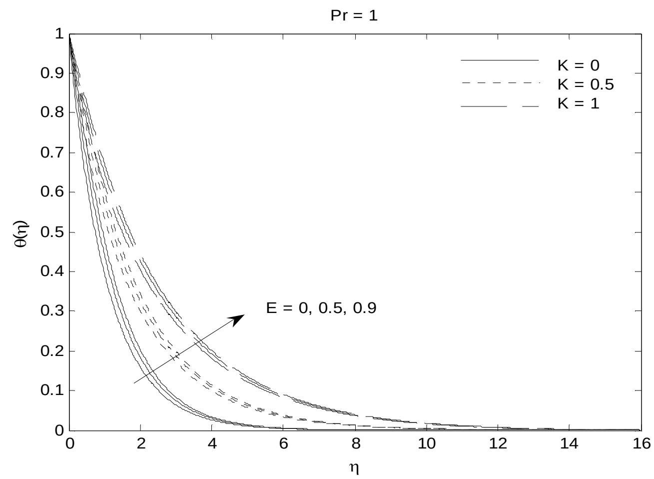


Figure 6: Effects of E and Pr on the temperature profiles $\theta(\eta)$ with K = 1

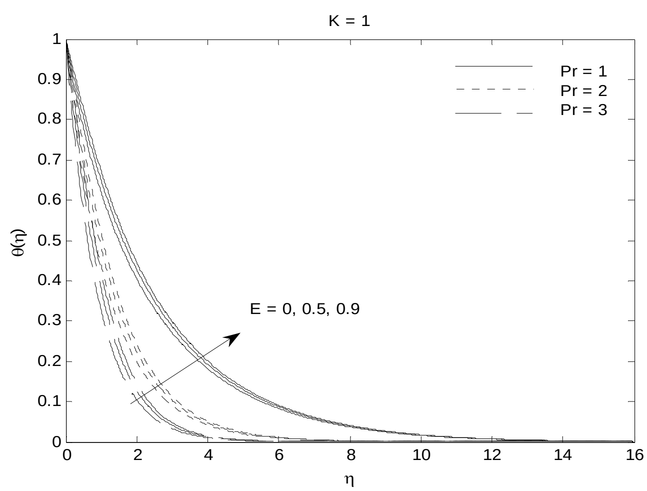
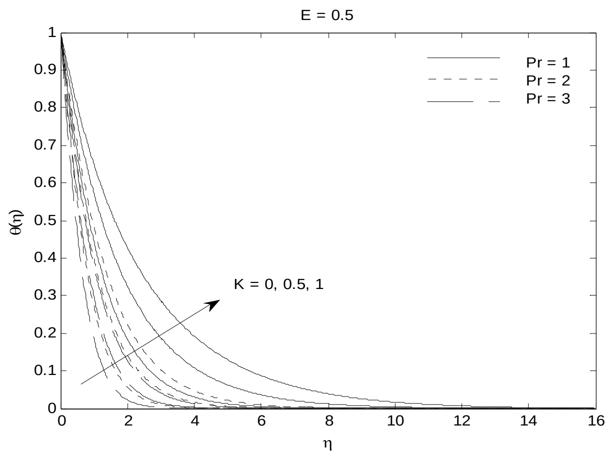


Figure 7: Effects of *K* and Pr on the temperature profiles $\theta(\eta)$ with *E* = 0.5



Acknowledgement
The authors would like to acknowledge the financial support received in the form of the Research University Grant (GUP) from the Universiti Kebangsaan Malaysia.

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