



**Modelling and Simulation of Mixed Queuing Server
Pharmacy System with Adjustable Parameters**

by

**Rashida Binti Abd Rashid
(1432121165)**

A dissertation submitted in partial fulfillment of the requirements for the
degree of Master of Science (Engineering Mathematics)

**Institute of Engineering Mathematics
UNIVERSITI MALAYSIA PERLIS**

2015

UNIVERSITI MALAYSIA PERLIS

DECLARATION OF DISSERTATION

Author' full name : Rashida binti Abd Rashid
Date of birth : 29th October 1985
Title : Modelling and Simulation of Mixed Queuing Server
Pharmacy System with Adjustable Parameters
Academic Session : 2015

I hereby declare that this dissertation becomes the property of Universiti Malaysia Perlis (UniMAP) and to be placed at the library of UniMAP. This dissertation is classified as:

- CONFIDENTIAL** (Contains confidential information under the Official Secret Act 1972)
 RESTRICTED (Contains restricted information as specified by the organization where research was done)
 OPEN ACCESS I agree that my dissertation is to be made immediately available as hard copy or online access (full text)

I, the author, give permission to the UniMAP to reproduce this dissertation in whole or in part for the purpose of research or academic exchange only (except during a period of _____ years, if so requested above).

Certified by:

SIGNATURE

851029-09-5126

(NEW IC NO./ PASSPORT NO.)

Date: _____

SIGNATURE OF SUPERVISOR

Dr. Mohammad Fadzli Bin Ramli

NAME OF SUPERVISOR

Date: _____

ACKNOWLEDGEMENT

Assalamualaikum

Firstly, I would like to express my gratitude towards Allah S.W.T because I am able to complete my thesis within the specified time period. I also would like to express my gratitude and appreciation to all those who gave me possibility to complete this thesis.

A special thanks to my supervisor, Dr. Mohammad Fadzli bin Ramli, who always help me, give me the suggestion and encouragement, and help me to coordinate my project especially in writing this report. He also gives his full effort in guiding me to achieve the goal as well as his encouragement to maintain my progress in track. Also, thanks to Dr. Safwati for your guidance and supporters.

I would also like to acknowledge with much appreciation the crucial role of the staff of the pharmacy unit of Hospital Tuanku Fauziah, Mr. Syafuan and Ms. Choong who help me in collecting the data. A special thanks goes to the Head of Pharmacist, Mdm A'tia that give permission for me to use the data from pharmacy unit and Dato' Dr. Ahmad Nordin, Director of Hospital Tuanku Fauziah for giving the permission to do the research at Hospital Tuanku Fauziah, Kangar, Perlis.

I would like to appreciate the guidance given by others supervisor as well as panels, especially their comments and tips to improve all lacking of me. Finally, I would thank to my husband, Khasim Bin Abdul Razak and my family for their support during my studies.

TABLE OF CONTENTS

	PAGE
THESIS DECLARATION	i
ACKNOWLEDGMENT	ii
TABLE OF CONTENT	iii
LIST OF TABLE	vii
LIST OF FIGURES	viii
ABSTRACT	x
ABSTRAK	xi
CHAPTER 1 INTRODUCTION	
1.1 Introduction to Modelling and Simulation	1
1.2 The Importance of Simulation in Healthcare Pharmacy Area	1
1.3 Organization Background	2
1.4 Problem Background	2
1.5 Research Question	3

1.6	Research Objectives	4
1.7	Research Scope	5
1.8	Expected Contribution	5

CHAPTER 2 LITERATURE REVIEW

2.1	Introduction	6
2.2	Queuing Theory	7
2.2.1	Arrival Process	8
2.2.2	Service Mechanism	9
2.2.3	Queueing Discipline	9
2.3	Healthcare Queuing System	11
2.4	Description of Model	14
2.4.1	Multi-server Queue System	14
2.4.2	Single Multiple Server Queuing Model	16
2.4.3	Parameter of $M/M/c$ Queuing Model	19
2.5	Simulation of Multiple Server by Using Discrete Event Simulation (DES)	23

CHAPTER 3 METHODOLOGY

3.1	Simulation Steps	28
3.2	Conceptual Model	31
3.3	Queuing Model of Pharmacy Unit: $M/M/2/\infty/\infty$ Model	33
3.3.1	Calculation of $M/M/2/\infty/\infty$ Queuing Model	35
3.4	Analysis of $M/M/2/\infty/\infty$ Queuing Model	37
3.4.1	Analysis of Inter-Arrival Time	37

3.4.2	Analysis of Waiting Time	42
3.4.3	Analysis of Service Time	44
3.5	Calculation of Pharmacist as Parameter of $M / M / 2$ Queuing Model	45
3.6	Development of Computer Model by Using Promodel Software	49
3.7	Structural Element In ProModel	49
3.7.1	Location	53
3.7.2	Entities	55
3.7.3	Path Networks	56
3.7.4	Processing	59
3.7.5	Arrivals	61
3.7.6	Arrival Cycle	63
3.8	Output of the Current Model	65
3.9	Verification of the Current Model	68
3.10	Validation of The Current Model	69

CHAPTER 4 SIMULATION OF EXPERIMENT ADDITIONAL PHARMACIST

4.1	Additional One Pharmacist ($M / M / 3$ Queuing Model)	77
4.2	Additional Two Pharmacist ($M / M / 4$ Queuing Model)	80
4.3	Additional Three Pharmacist ($M / M / 5$ Queuing Model)	82
4.4	Output Analysis and Model Proposal	84
4.4.1	Output of the Model	85
4.5	Research Finding	90

CHAPTER 5 CONCLUSION

5.1	Conclusion	91
5.2	Recommendation for Future Work	93

REFERENCES	94
-------------------	----

APPENDIX A	97
-------------------	----

APPENDIX B	100
-------------------	-----

APPENDIX C	101
-------------------	-----

APPENDIX D	102
-------------------	-----

APPENDIX E	105
-------------------	-----

©This item is protected by original copyright

LIST OF TABLES

NO.		PAGE
2.1	Summary of literature review	24
3.1	Summary of manual calculation	34
3.2	Number of arrival in 1 hour period	38
3.3	Number of arrival per 30 minutes period	39
3.4	Average quantity patient enter Counter 2	41
3.5	Average of waiting time for every counter	42
3.6	Distance and Interfaces for each Path Networks	58
3.7	Table for Patient_Arrival in arrival cycle	65
3.8	Summary percentage of 4 type of operation	67
3.9	Table of scoreboard for output result model development	68
3.10	Summary of raw data for average of total time and waiting time of the system	68
3.11	Comparison of actual and model output measures	71
3.12	Comparison of actual and model output measures of waiting time	73
4.1	Summary of counter and server for four scenario experiment queuing model	84
4.2	Scoreboard result for each model simulation	86
4.3	Entity states after additional one pharmacist	87
4.4	Entity states after additional 2 pharmacist	88
4.5	Entity states after additional 3 pharmacist	88
4.6	Comparison of queuing performance for every cases	89

LIST OF FIGURES

NO.		PAGE
2.1	Multi server queuing system	15
2.2	Multiple single servers queuing system	16
2.3	Four simulation scenario develop by Grit et al. (2013)	18
3.1	Simulation step involve in this study	30
3.2	Flow chart of the real system	32
3.3	Histogram number of arrival per 1 hour period	38
3.4	Histogram of average number of arrival per 30 minutes period	40
3.5	Normal distribution for average quantity patient Counter 2	41
3.6	Normal distribution for average of patient waiting time at Counter 2	43
3.7	Exponential distribution for patient service time	44
3.8	Poisson arrival distribution in 30 minute period	45
3.9	Layout of pharmacy unit	51
3.10	Grid with the layout	52
3.11	Distance per feet	52
3.12	Location edit table	53
3.13	Entities edit table	56
3.14	Path Networks editor	57
3.15	Process edit table and routing edit table	60
3.16	Summary of process edit table	60
3.17	Summary of routing edit table for patient Counter 2	61
3.18	Arrival edit table	62

3.19	Arrival cycle edit table	64
3.20	Entity States – Baseline output	66
4.1	Location edit table after addition one pharmacist	77
4.2	Layout after additional 1 pharmacist	78
4.3	Path network R3	78
4.4	Routing for patient Counter 3 and 4	79
4.5	Layout after additional 2 pharmacists	80
4.6	Location edit table after additional 2 pharmacists	81
4.7	Routing for patient Counter 4 and 5	81
4.8	Layout after additional 3 pharmacists	82
4.9	Routing for patient Counter 5 and 6	83

©This item is protected by original copyright

Modelling and Simulation of Mixed Queuing Server Pharmacy System with Adjustable Parameters

ABSTRACT

This dissertation involves discrete event simulation (DES) as a type of computer based modelling that imitates a real world system of pharmacy unit. Queuing theory that is used to model and analyse the characteristic of queuing system at the pharmacy unit of Hospital Tuanku Fauziah, Kangar. The input of this model is based on statistical data collected for 20 working days in June 2014. Currently patient waiting time of pharmacy unit is more than 15 minutes. The actual operation of the pharmacy unit is a mixed queuing server with $M/M/2$ queuing model where the adjustable parameter is refer to the pharmacist. Discrete event simulation method and ProModel software is used to simulate the queuing model and to propose the improvement for queuing system at pharmacy unit. Waiting time for each counter is analysed and found out that Counter 3 and 4 has the highest waiting time which is 16.98 and 16.73 minutes. Three scenarios; $M/M/3$, $M/M/4$ and $M/M/5$ are simulated by using 'What If Analysis' and waiting time for actual queuing model and experimental queuing model are compared. The simulation results show that by adding the server (pharmacist) will reduce patient waiting time. Almost 50% average patient waiting time is reduced when one pharmacist is added to the counter. However, it is not necessary to fully utilize all counters because eventhough $M/M/4$ and $M/M/5$ produced more reduction in patient waiting time, but it is ineffective since Counter 5 is rarely used and current average patient waiting time for Counter 6 is 1.99 minutes.

Pemodelan dan Simulasi pada Sistem Farmasi Campuran Barisan Pelayan dengan Parameter Boleh Laras

ABSTRAK

Disertasi ini melibatkan simulasi peristiwa diskret (DES) sebagai jenis pemodelan berasaskan komputer yang meniru sistem dunia sebenar system farmasi. Teori beratur digunakan untuk menganalisis ciri-ciri sistem beratur di unit farmasi Hospital Tuanku Fauziah, Kangar. Input model ini adalah berdasarkan kepada data statistik yang dikumpul selama 20 hari bekerja pada bulan Jun 2014. Masa menunggu pesakit di unit farmasi adalah lebih daripada 15 minit. Operasi sebenar unit farmasi adalah model barisan bercampur dengan $M/M/2$ model beratur di mana parameter boleh laras adalah merujuk kepada ahli farmasi. Kaedah simulasi peristiwa diskrit dan perisian ProModel digunakan untuk mensimulasikan model giliran dan mencadangkan penambahbaikan untuk sistem beratur di unit farmasi. Masa menunggu untuk setiap kaunter dianalisis dan didapati bahawa Kaunter 3 dan 4 mempunyai masa menunggu yang tertinggi iaitu 16.98 dan 16.73 minit. Tiga scenario model beratur iaitu $M/M/3$, $M/M/4$ dan $M/M/5$ disimulasi dengan menggunakan 'What If Analysis' dan masa menunggu dibandingkan antara model beratur sebenar dan model beratur eksperimen. Keputusan simulasi menunjukkan bahawa dengan menambah pelayan (ahli farmasi) akan mengurangkan masa menunggu pesakit. Hampir 50% purata masa menunggu pesakit telah dikurangkan apabila menambah satu ahli farmasi ke kaunter. Walaubagaimanapun, ia tidak perlu menggunakan sepenuhnya semua kaunter kerana walaupun model $M/M/4$ dan $M/M/5$ menghasilkan lebih pengurangan terhadap masa menunggu pesakit, tetapi ia adalah suatu pembaziran kerana Kaunter 5 jarang digunakan dan purata masa menunggu pesakit semasa bagi Kaunter 6 adalah 1.99 minit.

CHAPTER 1

INTRODUCTION

1.1 Introduction to Modelling and Simulation

Modelling is the process of producing a mathematical model that represents the construction and working of the system interest. Simulation is the imitation of a real world process or system over time (Banks, 2005). It is a tool to evaluate the performance of the system under different configuration over a long period of real time. The purpose of a simulation model is to enable the analyst to predict the effect of changes in the system. It must be precise as much as possible when compared to the real system. The objective of simulation study is to reduce the chance of failure to meet specification, to remove unexpected bottleneck and to optimize the system performance.

1.2 The Importance of Simulation in Healthcare Pharmacy Area

Healthcare management systems play an important role to have high quality service performance. Simulation is one of the tools to analyze the hospital pharmacy operations. It helps management to predict changes and make better operational

decision. Maryam et al. (2012) focused on modelling and simulation of a pharmacy delivery system in Malaysia's hospitals to determine the relationship between system components, reducing the patient service delay and increasing customer satisfaction. In their study, they used WITNESS simulation in order to decrease the queue length and dispense the medication rapidly as possible to the patients.

1.3 Organization Background

There are six main units of the Department of Pharmacy at Hospital Tuanku Fauziah, Kangar. It consists of Pharmaceutical Procurement & Supply, Pharmacy Specialist Unit, Pharmacy Inpatient Unit, Unit of Clinical Pharmacy, Pharmacy Unit Poison and Drug Information and Pharmacy Management Unit. In this study, we only focused on Pharmacy Specialist Unit as a main unit that dispenses medication to their outpatients. Based on their customer charter the patient should be served within 30 minutes after their arrival.

1.4 Problem Background

There are six specific counters in their Pharmacy Specialist Unit service. The first counter is the receptionist counter where the patient will be given the drug prescribed by the doctor to the pharmacist and then takes the queuing number of their turns. The queuing number is based on the type of drug and the quantity of drug prescription. There are special counters to serve the elderly people and disabilities. The express counter is for urgent cases or less than three drug prescription order. One of the counter is used for the staff and nurses.

In this study, we focused on modelling a queuing system with unlimited waiting rooms, constant arrival rate and c identical servers. In the pharmacy unit, there are 6 counters but only 3 pharmacists are on duty. The first pharmacist as mentioned previously waits at the receptionist counter to give the queuing number to the patient based on their drug prescription. The second counter is Server 1 where the pharmacist are served for Counter 2, 3 and 4. For Server 2, pharmacist are served for Counter 5 and 6. Both of them will dispense the medicine to the patient.

The problem arises when the patient is in the queue for a long time, which is more than 15 minutes. The average of patient waiting time at waiting area of each counter needs to be recognized before simulation run. In this study, waiting time and arrival pattern for each counter are referred as fixed parameter based on the real data distribution and the server or the pharmacist are determined as adjustable parameter. The average service time is also studied in determining the total time of a patient spends in the system.

1.5 Research Question

This study focuses on measurement of system performance of waiting time in the pharmacy system. Since the server cost is related to the budget, the server must be fully utilized. Therefore, this research tries to have some insight to the following questions:

- i. How long does a patient expect to wait in the queue before they are served?
- ii. How long do they have to wait before the service is complete?

- iii. What is the probability that a patient has to wait longer than a given time interval before they are served?
- iv. What is the average length of the queue?
- v. What is the probability that the queue will exceed a certain length?
- vi. What is the expected utilization of the server and the expected time period which it would be fully occupied?

The above question can be answered by evaluating some alternatives in improving the current situation. The alternative is:

- i. How many servers should be used?
- ii. Should priorities for certain cases of customers can be introduced?

1.6 Research Objectives

The study aims to find the suitable quantity of adjustable parameters (server/pharmacist) that will reduce patient waiting time. The objective of this study is:

- i. To determine the behaviour or characteristics of queuing system at the pharmacy unit of Hospital Tuanku Fauziah such as patient waiting time, arrival pattern, queue discipline of first in random out (FIRO), first in first out (FIFO), arrival rate, inter-arrival and service time or processing time.
- ii. To develop queuing simulation model for the mixed queuing server pharmacy system by using discrete event simulation (DES).
- iii. To propose alternative improvement for the queuing system at pharmacy unit.

1.7 Research Scope

This study contains the analysis of queuing system for arrival and departure time for patients about the amount of time spent before medication can be received at pharmacy unit of Hospital Tuanku Fauziah. The data of patient arrival and departure times are real data. It is collected over a period of 20 consecutive working days in June 2014 on 7.30 am to 5.30 pm, Monday to Friday.

1.8 Expected Contribution

Discrete event simulation (DES) permits modelling of complex queuing system. In this study, the current queuing model is $M/M/2$ and three scenario of experiment simulation which is $M/M/3$, $M/M/4$ and $M/M/5$ is investigated to find the suitable quantity of pharmacist needed to serve patients so that patients waiting time is reduce.

The expected contribution of queuing theory in pharmacy system at Hospital Tuanku Fauziah is a range of queuing theory results such as reduction of patient waiting time. The healthcare process that can be viewed in this organization of queuing system is patient arrival, patient waiting time, patient service time and patient departure. The server in this pharmacy system is pharmacist. One of the advantage of reducing patient waiting time is patient satisfaction. Hopefully, the method adopted in this study can be used as a guide in analysing queuing problems in others pharmacy unit.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Computer simulations are very popular in healthcare area because of the complexity of the system and their existing problems. One of the example of complex system in healthcare is emergency department (ED). David et al. (2014) used discrete event simulation (DES) and space syntax analysis (SSA) to facilitate decision making related to ED to lead to high performance of ED design and extends to broad application in healthcare. Mohamed et al. (2009) also used simulation to design a decision support tool for the operation of an emergency department at a government hospital in Kuwait. In this case, simulation is used to evaluate the impact of various staff levels on service efficiency. The results show that the simulation models generates optimum allocation of staff increase by 28% in processing of patients and average 40% reduction in waiting time for patients. Therefore simulation is important in making decision in complex system such as emergency department.

Computer simulation is able to change different option and test various scenarios by using the same empirical condition. The output is used to observe the outcomes and to examine the interaction and relationship between the system. Bo et al. (2013) created a computer simulation to imitate the current operation in clinic. The result shows that by adding a number of psychiatrists and extending 2 hours of daily clinic hours gives incremental decreases in the number of patients during service time and outside clinic hour.

2.2 Queuing Theory

Queuing theory is the analysis of queues or waiting lines where the customers wait to receive a service. It deals with actual waiting times. It is a powerful tool because it required relatively little data and can be evaluated quickly and compare various alternatives for providing better service. Smaranda et al. (2015) used queuing theory in analysing the improvement of hospital bed occupancy and resource utilization. They focus in 'What if' analysis as a tool to find the effect on the outcomes of the queueing system of hospital bed occupancy and resource utilization by using real data from a geriatric department of hospital in London. Generally, there are three components of queue system:

- i. Arrival process
- ii. Service mechanism
- iii. Queue discipline

2.2.1 Arrival Process

Arrival may originate from one or several sources referred to as calling population. Calling population can be finite or infinite. It shows how a customer arrive either single arrival or group's arrival and how the arrivals are distributed in time. Arrival process is the probability of time between successive arrival. It also called as inter-arrival time distribution. The inter-arrival time for patient hospital emergency department is 19.2 hour as mentioned by Peter et al. (2014) in their paper. That means average patient flow is 5 patients every 4 days. 87% of patient arrive during daytime are high priority status and the remaining patient is low priority status. The information of the arrival process is important before modelling the simulation for the stroke patient care pathway.

There are many assumptions of inter-arrival, τ_j such as typical assumption is independent and identically distributed and other possible assumption likes bulk arrival, balking and correlated arrivals. Suppose that the customer arrives at times t_1, t_2, \dots, t_j . The inter-arrival time is the time taken between first and second arrival where:

$$\tau_j = t_j - t_{j-1} \quad (2.1)$$

Poisson arrival process corresponds to arrival at random. The inter-arrival times are independent and identically distributed. It is exponential distribution where it has the same constant of time interval between successive arrival. It is described by a single parameter which is the average arrival rate, $\frac{1}{\lambda}$.

Christopher et al. (2014) used $M/G/c$ queuing model to model health care service tasks such as x-ray, computer tomography, and others service that form queues

in order to reduce patient waiting time in hospital with patient arrivals modelled by using Poisson distribution.

2.2.2 Service Mechanism

Service mechanism of queuing system is specified by number of servers or a prescription of the resources needed for the service to begin such as staff, devices, counter, medical units and etc. It describes how the service is provided and how long the service will take. Each server has its own queue and probability. The server might be in series or in parallel. Series server means each server has a separate queue and parallel service has one queue for all servers. Service time may be constant or random and usually is exponential distribution. In service mechanism, an exception is allowed when a server stop processing a customer to deal with another priority or emergency customer.

2.2.3 Queuing Discipline

Queue discipline means the rule that a server uses to choose the next customer from the queue when server completes the service of the current customer. It also determines the order of treated priority of the customer. Commonly used in queue discipline are 'first in first out' (FIFO) or 'first come first serve' (FCFS) because it seemed to be fair to all customers. The patient will treat FIFO in order synonymous to a waiting list that is processed from top to bottom. However, in some urgent cases they do not follow FIFO. Customers are served in order of their importance on the basis of their service requirements. They call customer as 'last in first out' (LIFO) or 'last come first

serve' (LCFS). Therefore FIFO does not reflect to health care practice pattern. Changing queue discipline can reduce the congestion. The lowest service time, resulting the smallest average time of customer spend in the queue. Other possible queuing discipline is 'last come first serve pre-empt resume' (LSFS-PR), 'round robin' (RR) with finite quantum size, 'processor sharing' (PS) where RR with infinitesimal quantum size and 'infinite server' (IS). This term is used in network of queue (Daniel, 1995).

Queue behaviour refers to the action of the customer while they are queuing. The customer might be 'balking' or they decided not to join the queue if it is too long. They might be 'reneging' or the customer leaves the queue if they wait too long for service. They also might be 'jockeying' or switch between queues if they think they will get faster service by doing so. Size of the waiting room at each station also needs to be considered because when the waiting room is limited, the customer will leave. Server vacation is to referred as a period of time when servers are not available (Lakshmi et al. 2013)

Queuing models are widely used in service facilities, production and material handling system. It is useful in gaining on the impact of various priority schemes for determining service among patients. Performance measures in queue models are utilization, waiting time or number of waiting people which is average over a specific period. Utilization denotes the percentage of time server or counter unit is busy (Banks et al. 2005).

2.3 Healthcare Queuing System

There are many problems in health care system which can be solved by using queuing theory in operational research. It has advantages of producing simple mathematical models. Simulation model represents the complex and dynamic behaviour of the real queuing process. By increasing the power of computer numerical methods and simulation model, it needs to be used alongside the traditional queuing theory to understand the real life of queuing system as well as possible.

In health care, computer simulation supports analyze treatment scenarios and patient flows. Discrete Event Simulation (DES) can be used to model the waiting lists or queues, resources used, waiting time and cost. Srinivasan et al. (2014) used discrete event simulation method to simulate different vital sign measurement time and physician consultation time. They varied the percentage of patient population adopting the wearable device health monitoring system (WHMS) program and the result yields beneficial in absorbing more patient and reducing patient denials than the current primary care system. WHMS program acts as a potential solution in reducing the patient load across the primary care system.

In healthcare settings, $M/M/1$ model may prove to be useful because it is fitting well in reality. It may be a reasonable choice for modelling such as walk in clinic, pharmacy operations and patient check-in and registration service at the hospital. This situation involves multiple servers that take care of customer who queue up for a similar service. $M/M/1$ model is used to estimate capacity requirement and to keep peak period congestion within tolerate limits when the customer arrives and service rate vary in time. Amar (2009) showed how to obtain performance measure and optimal

control parameter by using $M^x/G/1$ queuing model of energetic retrial queue with vacation.

Queuing theory is used to minimize inefficiencies and delays. It provides an analytical approach to estimate long term performance. It requires assumptions on arrival and service time distributions. As the system becomes complex, simulation is used instead of this analytical approach.

Queuing theory is used extensively in pharmacy operation management. It can be used to access a multitude factor such as prescription fill time, patient waiting time, patient counselling time and staffing level. By better understanding of queuing system, service manager can improve satisfaction of the relevant group of the system such as customer, employees and management team. Ndukwe et al. (2011) described a queue characteristic exist in pharmacy area is a single server multiple queue. In their study, they found that waiting time is reduced from 167 to 55.1 minutes after re-orientate the staff.

There are some models that are used to determine the queue system. In multi-server queues, a queue has an infinite number of servers. $M/M/\infty$ model is a queue model that has an infinite number of servers. This means, every customer that arrives can immediately enter the service and no one is waiting. This model is an element in most models, but usually treated as a delay node rather than actual queuing model. Another model is $M/M/c$; multi-server queue with c identical server. Customers that arrive when a server is free can enter service immediately. If all servers are occupied customers will wait in FCFS order until someone departs and the server becomes available. When they arrive and servers are busy then the customer dropped and lost. This model is studied by Erlang in his analysis of the telephone network in 1913. It is