



**GLOBAL DYNAMICS IN NEURO SYMBOLIC
INTEGRATION USING ENERGY MINIMIZATION
IN MEAN FIELD THEORY**

by

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LIST OF ABBREVIATIONS

ABM	Agent Base Modelling
AI	Artificial Intelligent
ANN	Artificial Neural Network
BM	Boltzmann Machine
CAM	Content Addressable Memory
CNF	Conjunctive Normal Form
FPGA	Field-Programmable-Gate-Array
GUI	Graphic Unit Interface
HNN	Hopfield Neural Network
HTAF	Hyperbolic Tangent Activation Function
MFT	Mean Field Theory
NC	Number of Clauses
NN	Number of Neurons
RBFNN	Radial Basis Function Neural Network
SCN	Symmetric Connectionist Network
TSP	Traveling Salesman Problem
VLSI	Very Large Scale Integrated
WAM	Wan Abdullah's Method

Global Dinamik Dalam Integrasi Neuro Simbolik Menggunakan Minimum Tenaga Dalam Teori Medan Purata

ABSTRAK

Pengaturcaraan logik dan rangkaian neural adalah dua aspek penting dalam kecerdasan buatan. Tesis ini adalah sebahagian daripada usaha ke arah integrasi rangkaian neural dan pengaturcaraan logik. Matlamat dalam melaksanakan pengaturcaraan logik berdasarkan skim pengurangan tenaga ialah untuk mencapai global minimum yang terbaik. Walau bagaimanapun, tidak ada jaminan untuk mencari nilai minimum yang terbaik di dalam rangkaian tersebut. Untuk mencapai matlamat ini, sebuah algoritma pembelajaran berasaskan konsep Mesin Boltzmann (MB) dan Fungsi Pengaktifan Tangen Hiperbolik (FPTH), telah dibangunkan bagi mempercepatkan prestasi dalam pengaturcaraan logik bagi Rangkaian Neural Hopfield (RNH) dengan menggunakan Teori Medan Purata (TMP). Pengaturcaraan logik untuk klausa tertib rendah (sehingga tertib tiga) dan klausa tertib tinggi (sehingga tertib lapan) telah dibangunkan untuk TMP. Prestasi kaedah ini dibandingkan dengan kaedah yang sedia ada untuk melakukan pengaturcaraan logik dalam RNH (MB dan FPTH). Keberkesanan kaedah ini dinilai menggunakan nisbah global minimum, jarak Hamming dan masa pengiraan. Seterusnya, Model Berasaskan Ejen (MBE) telah dibangunkan dengan menggunakan Netlogo. MBE membenarkan pembangunan pesat model, memudahkan penambahan ciri-ciri dan pengendalian yang lebih mesra pengguna dan pengekodan. Model yang dibangunkan diuji dengan menggunakan set data kehidupan sebenar dan set data simulasi. Keputusan simulasi yang diperolehi menyokong pembelajaran algoritma seperti yang dicadangkan. Prestasi dalam melakukan pengaturcaraan logik menggunakan TMP terbukti lebih baik daripada prestasi MB dan FPTH.

Global Dynamics in Neuro Symbolic Integration Using Energy Minimization in Mean Field Theory

ABSTRACT

Logic program and neural networks are two important aspects in artificial intelligence. This thesis is part of an endeavour towards neural networks and logic programming integration. The goal in performing logic programming based on the energy minimization scheme is to achieve the best ratio of global minimum. However, there is no guarantee to find the best minimum in the network. To achieve this, a learning algorithm based on the Boltzmann Machine (BM) concept and Hyperbolic Tangent Activation Function (HTAF) was derived to accelerate the performance of doing logic programming in Hopfield Neural Network (HNN) by using Mean Field Theory (MFT). Logic programming for lower order (up to third order clauses) and higher order clauses (up to eight order clauses) have been developed for MFT. The performance of this method is compared with the existing methods of doing logic programming in HNN (BM and HTAF). The global minima ratio, hamming distances and computational time were used to measure the effectiveness of the proposed method. Then, Agent Based Models (ABM) were developed by using Netlogo. ABM can allow rapid development of models, easy addition of features and a user-friendly handling and coding. Later the developed models are tested by using real life and simulated data sets. The simulation results obtain agreed with the proposed learning algorithm. The performance of doing logic programming using MFT proved to be better than the BM and HTAF.

CHAPTER 1

INTRODUCTION

1.1 Introduction

The global dynamics in neuro symbolic integration depend on the effectiveness of the energy minimization scheme. In this thesis, neuro symbolic integration is mainly the logic programming in HNN. Thus, the energy minimization derived from the integration with logic programming technique proposed by Abdullah (1993) is vital to generate the optimum solutions in term of global minimum solutions. Hence, several optimization schemes are developed to reduce the problem in generating optimum solution in logic programming. The drawback of BM and HTAF in energy minimization can be observed when the network gets larger. Hence, the global dynamics of the network will experience neuron oscillations that resulting in local minimum solutions. To solve the problem, the MFT is proposed in this thesis. The neurons states update after energy minimization corresponds to the effectiveness of the model for logic programming. MFT was selected due to the accelerating mechanism in doing logic programming in HNN. In addition, the hybridization of BM and HTAF in MFT will enhance the energy minimization to produce some better results in term of global minima solutions. Therefore, MFT is capable to withstand the complexity when the network is getting larger. The performance is measured by using global minima ratio, Hamming distance and computation time. The MFT is expected to outperform the BM and HTAF in lower order and higher order logic programming in HNN. The global dynamics of energy minimization will be enhanced by MFT.

1.2 Motivation and Problem Statement

This thesis is a part of an endeavour towards neural-logic integration which hopefully would also allow exploration of new perspectives of knowledge (Leonid, 2007 & Mehmed, 1997). The purpose of this thesis is to propose a technique, based on the BM and HTAF known as MFT. The technique also will be extended to higher order clauses (1st-8th order clauses). In this sense, the network could be "programmed" by its input to achieve the global solutions which are the models for the clauses.

The energy minimization deals with the capability of generating the optimum solution based on global dynamics of the logic programming in HNN. The flexibility of the optimization scheme must comply with the complexity of the network. HTAF scheme proposed by Nawi (2014) has successfully reduced the local minimum solutions for the lower order logic programming Hopfield network. On the similar vein, BM proposed by Sathasivam (2011) of doing logic programming in HNN has been proven to generate global minimum solutions during simulation for the lower order logic programming in HNN. The problem arises in the energy minimization by integrating BM and HTAF when the network is getting larger and more complex. Thus, BM and HTAF will generate more local minimum solutions. The drawbacks of BM and HTAF in energy minimization lead to newly proposed method known as MFT. The effectiveness of MFT in this thesis utilized efficiency property of HTAF and the stochastic interpretation of BM for doing logic programming in HNN. The newly proposed MFT is expected to generate more global minimum solutions in doing lower order and higher order logic programming in HNN.

1.3 Objective

The objectives in this research are as follows:

- (i) To derive a new learning algorithm in doing programming in HNN.
- (ii) To combine the advantages of BM, HTAF and MFT and developed the new learning algorithm as in (i).
- (iii) To develop ABM models for the new learning algorithm (MFT).

1.4 Scope and Limitation of Research

In this thesis, HNN is focused in doing logic programming. It is because, this network is widely being used in pattern recognition and Content Addressable Memory (CAM) problems. Since in real life problems, the data selected are from pattern recognition problems, so HNN is selected.

Furthermore, in this thesis also real-life problems are considered up to fifth order clauses only. This is because, as the network gets larger, the complexity of the network also gets larger (Sathasivam, 2007). It often yields suboptimal solutions and oscillations. This thesis is limited to propositional logic programming only. The proposed method is unable to consider other variant of logic programming such as predicate logic or fuzzy logic. This is due to the nature of the logic programming that only comply symmetric connectionist network.

1.5 Significance of the Study

The closest way to represent the actual human thinking is by mapping the biological human thought process with a set of rules and logic. These set of rules and logics must behave in the dynamic manner where the rules can be change according to the behaviour of the perceived knowledge. The main drawback of the model for logic programming in HNN by Abdullah (1992), Sathasivam and Abdullah (2008a), Hamadneh, Sathasivam and Choon (2012), Kasihmuddin, Shareduwan, Mansor, Asyraf, and Sathasivam, (2016a) and Mansor and Sathasivam (2016) are the usage of logical rule that map the important knowledge. The order of the propositional logic in the mentioned study only consider lower order clause programming. In real life simulation, higher order clause will portray information about the behaviour of the data set.

HTAF has been proven to increase the efficiency of the logic programming in HNN (Nawi, 2014). This is due the nature of HTAF that reduce the complexity of the neuron during simulation. On the other hand, BM proposed by Sathasivam (2011) of doing logic programming in HNN has been proven to reduce local minima during simulation. The advantages of both models (HTAF and BM) leads to newly proposed method known as MFT. MFT in this thesis utilized efficiency property of HTAF and the stochastic interpretation of BM for doing logic programming in HNN. The newly proposed MFT is expected to outperform the existing model (HTAF and BM). Again, the proposed MFT will be tested for higher order clause. The newly proposed MFT is considered as a new contribution on neural network and logic programming research.

The novel ABM has been proposed by Sathasivam and Fen (2013) to do logic programming in HNN. ABM is one of the best way to simulate the actual flow of the network. Each variable or factors in the system can be controlled directly by the user

without changing the nature of the network. In this thesis, the proposed MFT will be developed in ABM. The nature of the proposed MFT in doing simulated data set and real-life data set will set the first benchmark in logic programming and neural network research.

1.6 Thesis Outline

This thesis is organized as follows. Chapter 1 introduces the topic in this thesis, methodology, objectives and others. In Chapter 2, discussion about theories related to logic programming in HNN is carried out. Chapter 3 deals with theory of BM, HTAF and MFT. Chapter 4 delivers introduction to Netlogo and ABM. Meanwhile in Chapter 5 the simulation results and discussions are included. Real life data sets testing are discussed in chapter 6. Finally, Chapter 7 consist of the conclusion and future work regarding this thesis.

CHAPTER 2

LITERATURE REVIEW

Due to the disadvantages in standalone neural networks and logic programming, researchers have combined these two paradigms in the aspect of neuro symbolic integration. Thus, the BM and MFT are also integrated with neuro symbolic paradigm. Hence, the work will be carried out on ABM due to better Graphic User Interface (GUI). In this section, the notable works on HNN, logic programming, HTAF, BM, MFT and ABM will be discussed in detail from the development and recent applications.

2.1 Hopfield Neural Network

The emergence of HNN has produced a lot of research since decades ago. Originally, HNN was proposed by John Hopfield, a scientist based in University of California, Berkeley in 1982. Thus, Hopfield (1982) proposed an associative computational model that made a tremendous breakthrough in the Artificial Intelligent (AI) field. Pursuing that, the HNN was implemented to the optimization and constraint satisfaction problems (Hopfield & Tank, 1985; Wen, Lan, & Shih, 2009). Since then, tremendous modification and improvements have been applied to the HNN architecture to solve any optimization problems. In theory, the HNN comprises of a simple recurrent network that has an efficient associative memory and resembled the biological brain. For instance, Wen et al. (2009) proposed that the HNN is one major neural network specialized and crafted for solving constraint optimization or mathematical programming problems. The main benefit of HNN is that its structure can be realized on an electronic circuit, possibly on a

very large-scale integration circuit, for an on-line solver with a parallel-distributed process. The structure of HNN utilizes three common methods, penalty functions, Lagrange multipliers and primal dual methods to construct an energy function (Pinkas, 1991). Moreover, the HNN minimizes Lyapunov energy by utilizing the physical Ising spin of the neuron states (Joya, Atencia & Sandoval, 2002). On top of that, the network produced global output by minimizing the network energy. Pinkas (1991) and Abdullah (1992) described a bi-directional mapping between logic and energy function of symmetric neural network. Besides, both methods are the building blocks for a corresponding logic program. Due to the effectiveness of energy changes in HNN, several researchers have combined the idea of logic programming with HNN. Several standard models were developed by Sathasivam and Abdullah (2011) and Sathasivam and Fen (2013).

Meanwhile Hajar, Yousef & Mahani (2014) introduced an efficient systolic architecture for efficient implementing of digital HNNs for solving shortest path problem on Field-Programmable-Gate-Array (FPGA) chips in industry. This means, the neuro symbolic integration also been widely being used in industry. Bansal and Dixit (2016) applied the HNN in genetic algorithm whereas Kasihmuddin, Mansor and Sathasivam (2016a) implemented HNN to solve the Bezier curves satisfiability problem. Wang (2016) provided a new method to study non-autonomous dynamic systems with variable pseudo-almost periodic coefficients using fixed point theorem. He worked with higher order HNN. However, this method involves huge computation of variables. On the other hand, recent work by Zhang, Hou, Zhao, Wang, Xi and Li (2017). emphasizes on the classification by using Hopfield associative memories. The work reported the welding quality of Chernoff face image had been successfully classified even though under abnormal welding conditions.

The choice of HNN is not only due to the capability to blend with other networks but more primarily due to the power of CAM that resembles the biological intelligence system. Additionally, the related works on HNN are shown in Table 2.1.

2.2 Logic Programming

Logic programming began in the early 1970 as a foundation to many applications such as AI. The credit for introduction of logic programming goes mainly to Kowalski (1978). During early stage, Kowalski was led to the fundamental idea that logic can be used as a programming language. The ability to extract the knowledge from the logic programming makes ANN is very suitable to develop AI system (Lloyd, 2012). This leads to various methods to integrate symbolic with connectionist AI. This combination has been a major works by Utgoff (1989) where he developed an algorithm that integrates decision tree and perceptron. Blair, Dushin, Jakel, Rivera and Sezgin (1999). showed intimate relationship between logic programming and dynamical system related to self-similarity and chaos. This perspective provoked more researchers to combine logic programming with other intelligent system. The assimilation also was developed by various researchers (Gallant, 1988; Pomerleau, 1991) and successfully created a system that can produce high and low-level decisions. Towell and Shavlik (1994) insert a set of symbolic rules into a feed forward network. The network is then refined by using standard learning algorithm with specific set of learning data. The refined network is utilized to classify the raw data. While researchers in traditional symbolic AI concentrate in the development of powerful knowledge representation, Pinkas (1991) were concentrating on powerful learning mechanism.

Table 2.1: Related Literature on HNN

Author	Method	Summary and Findings
Hopfield (1984)	The computational network based on a stochastic model of McCulloch-Pitts neurons.	The computational power of HNN was inaugurated by taking into account the content addressable memories and the stochastic properties of the primitive McCulloch-Pitts neuron.
Pinkas (1991)	Energy function of HNN.	The work described a bi-directional mapping between propositional logic and energy function of a symmetric neural network.
Abdullah (1993)	Higher order HNN for Horn logic program.	The HNN has minimized the logical inconsistencies in the interpretation of the logic program. Logical contents were obtained by the synaptic strength computed of the network.
Joya et al. (2002)	The dynamic of discrete HNN for optimization.	The proposed discrete HNN was proven in the avoidance of tremendous local minima solutions obtained after the computation.
Wen et al. (2009)	HNN in mathematical programming problem.	The work pinpointed the computational ability of HNN in doing mathematical programming such as VLSI simulation.
Sathasivam and Abdullah (2011)	Logic mining by using discrete HNN.	The logic mining can be done by extracting the information entrenched in the Horn clauses. The results were supported by a better performance evaluation metrics.
Sathasivam and Fen (2013)	Logic programming in the HNN by ABM.	The values of global minima ratio and Hamming distance provide solid evidence of the effectiveness of logic programming in HNN by using ABM.
Hajar et al. (2014)	Digital HNN on Field-Programmable-Gate-Array (FPGA) chips.	The FPGA chips were memorized accurately, stable, robust and more generalized by learning through HNN compared to the existing methods.
Bansal and Dixit (2016)	The pattern recalling as CAM by HNN and genetic algorithm.	This work proves that the HNN can be blended with the other metaheuristic algorithms to accelerate the computation and memory.
Kasihmuddin et al. (2016a)	Enhanced HNN in Bezier Curves Satisfiability.	The enhanced HNN has fruitfully solved Bezier Curves Satisfiability with an excellent global minima ratio, Hamming distance and CPU time.
Zhang et al. (2017)	Hopfield associative memories.	The proposed network has classified the welding pattern according to the performance evaluation metrics recorded after the simulations.

He described any symbolic system requires interpreter to process the information expressed in the representation that is unfortunately absent in many connectionist networks. Pinkas considered Symmetric Connectionist Network (SCN) that includes HNN, BM, harmony theory and MFT. In addition, he described SCN can learn representations of propositional logic by examining the truth assignments that satisfy the formula. The reason why he selected SCNs is because the symmetric networks were characterized by energy function which makes it easier to specify the network's behavior. In addition, SCN able to capture the information embedded in logic formula for vital procedural control.

Abdullah (1992) proposed a method to find optimal synaptic weight. He showed the dynamical change in synaptic weight when a system learns from the logical rules. Each of the variables in the logic will be represented as neurons. Synaptic weight of the logical rules can be found by comparing the cost function (inconsistencies minimization of logical assignments) with Hopfield energy function. The synaptic weight obtained by using Abdullah method has similar knowledge content as conventional Hebbian learning (Sathasivam & Abdullah, 2008a). Sathasivam and Abdullah (2008b) showed Abdullah method outperform conventional hebbian learning rule when dealing with larger number of neurons. Both researchers introduced Horn logic as a symbolic representation in HNN. The advancement of logic programming in HNN is not limited to normal logic only. In another development, Sathasivam (2012) proposed a fuzzy logic in HNN by implementing Fuzzy C-Mean clustering to the neuron state before retrieval phase of HNN can take place. The retrieved neuron states from this model will be computed by using Lyapunov energy function. This study helps user to grade continuous value from the logic programming before it can be learned by HNN. The other side of the coin, Hamadneh et al. (2012) introduced Horn logic programming in Radial Basis Function Neural Network

(RBFNN). The proposed RBFNN proposed propositional logic programming with single step operator. His RBFNN model is able to do other variant of logic such as Circuit logic, 3 Satisfiability and knowledge based system. Table 2.2 shows list of important studies for logic programming in neural network.

2.3 Boltzmann Machine

The most celebrated milestones of AI are the neuron model of McCulloch and Pitts (1943), Rosenblatt perceptron (1958) and along with the Hebb learning (1949). The mentioned contributions in AI led to the introduction of HNN by (1982). Since then, there were many neural network practitioners interested to optimize signal processing and information theory. In particular, great optimization relevance such as the capability to learn or retrieve memories can be found by implementing genuine statistical mechanics learning (Amit et al., 1985; Hertz, Krogh & Palmer, 1991). A common goal in machine learning is to design a model that can estimate the probability distribution of its possible states (Honavar & Uhr, 1994). If a particular machine learning is unable to construct the satisfactory model, the goal of the machine learning can be achieved by the observation of a large number of samples (Coolen, Kühn & Sollich, 2005). BM is one of the most prolific machine learning in doing constrained optimization problem. BM has been designed to capture the complex statistic of arbitrary system by dividing neurons in two subsets that is visible and hidden units. Marginalizing the BM over the hidden unit allows BM to reproduce through the visible units and complex distribution of states by learning appropriate synaptic weight (Hinton, 2007a).