# Design of Binary Coded Pulse Trains with Good Autocorrelation Properties for Radar Communications 

Siti Julia Rosli ${ }^{1, *}$, Hasliza Rahim $^{2}$, Ruzelita Ngadiran $^{3}$, K. N. Abdul Rani ${ }^{4}$, Muhammad Imran Ahmad ${ }^{5}$ and Wee Fwen Hoon ${ }^{6}$.<br>${ }^{12356}$ School of Computer and Communication Engineering, Universiti Malaysia Perlis, Perlis<br>${ }^{4}$ Department of Electronics Engineering Technology, Faculty of Engineering Technology, Universiti Malaysia Perlis, Perlis


#### Abstract

Finite length of sequences that are modulated both in phase and amplitude and have an ideal autocorrelation function (ACF) consisting of merely a pulse have many applications in control and communication systems. They are widely applied in control and communication systems, such as in pulse compression systems for radar and deep-space ranging problems [1-5]. In radar design, the important part is to choose a waveform, which is suitable to be transmitted because the waveform controls resolution in clutter performance. In addition, it can solve a general signal problem particularly related to the digital processing. Energy ratio (ER), total side lobe energy (SLE), and peak sidelobe level (PSL) are three properties of such sequences interest. This paper presents a method using the Complementation, Cyclic Shift and Bit Addition for synthesizing and optimizing a binary sequence implemented to improve the sequences of a similar quality with the Barker sequence, particularly for lengths greater than 13. All of these methods are guided by the specific parameter with good characteristics in ACF (ER, SLE, and PSL) [6,7,8]. Such sequences can then be effectively used to improve the range and Doppler resolution of radars.


## 1 Introduction

Sequences with low autocorrelation side lobe levels are useful for channel estimation, radar and spread spectrum communication applications. Sequences achieving the minimum peak autocorrelation side lobe level one are called Barker Sequences. A Barker sequence is a finite length binary sequence with the minimum possible autocorrelation. Currently, there only eight known Barker sequences and it has been conjectured that these are the only Barker sequences that exist, shown in Table 1.

Table 1. Barker Sequences

| Code <br> length | Code elements | PSL | ER | SLE |
| :--- | :--- | :--- | :--- | :--- |
| 2 | $+1-1$ | 0.5 | 2 | 0.5 |
| 3 | $+1+1-1$ | 0.3333 | 3 | 0.2222 |
| 4 | $+1+1-1+1$ | 0.2500 | 4 | 0.2500 |
| 5 | $+1+1+1-1+1$ | 0.2000 | 5 | 0.1600 |
| 7 | $+1+1+1-1-1+1-1$ | 0.1429 | 7 | 0.1224 |
| 11 | $+1+1+1-1-1-1+1$ <br> $-1-1+1-1$ | 0.0909 | 11 | 0.0826 |
| 13 | $+1+1+1+1+1-1-1$ <br> $+1+1-1+1-1+1$ | 0.0769 | 13 | 0.0710 |

Barker sequences are then used as the best sequences in the highest ER. Binary sequences were initially investigated for the purpose of pulse compression in radar systems. This method results in better range and

Doppler resolution without the need to shorten a radar pulse nor increase the power [11,12].

## 2 The Optimization Problem

Binary sequences with a good periodic autocorrelation functions (ACFs) with low range side lobes are required for many communication applications. However, some good binary sequences have a large sequence length. The efficiency of these codes depends upon the energy content in the range side lobes of their ACF. Peak side lobe level (PSL) and integrated side lobe level (ISL) are the two performance measures for pulse compression codes. Barker sequences are known as the binary sequences that have low PSL and it is only available in length 13.

Many applications require longer codes to achieve a higher signal to noise ratio (SNR). By exhaustive computer search program [13], found all binary sequences up to length of 48 and they had developed an analytical method for generating good binary pulse compression codes.

### 2.1 Synthesis Using Element Complementation

This scheme is proposed for representing the complementation of (signed and unsigned) binary numbers in which there are sets of sequences that possibly have a good property in the ACF. In mathematics and computing, the method of complementation is a method used to change the sign of

[^0]one number from another making that value as a representation for a symmetric range (i.e. from $2^{k}$ which k is the bit integers). The method which is also known as the simplest test to distinguish between the two possibilities is the complementation test.

Figure 1 shows the flowchart to determine the variation of the different sign of that sequence. In this case, it shows the binary number system and shorthand methods in representing binary codes. With $k$ binary digits, it can represent the $2^{\mathrm{k}}$ unique patterns from $111 \ldots .1$ until -1-1-1....-1. So this method will be run as itteration until $2^{\mathrm{k}}$ unique patterns to find a good characteristics in ACF . When the signed quantities are represented in the same $k$ digits, it still has only $2^{k}$ patterns to work with. Unless it increases the number of digits available (i.e. make $k$ larger), the representation of signed numbers will involve the distribution of $2^{k}$ patterns into positive and negative portions. Sign/Magnitude is the simplest way to represent signed numbers, but the most common is the modification of a signed or magnitude called the element complementation.


Fig. 1. Complementation Method flowchart

### 2.1.1 Sign or Magnitude Notation

Sign or magnitude notation is the simplest and one of the most obvious methods of encoding positive and negative bit sequences. This situation is generated as below in
example. Example using 4 bits-lengths with 16th of sequences:
Binary=[1111]; [111-1]; [11-11]; [11-1-1]; [1-111]; [1-11-1]; [1-1-11]; [1-1-1-1]; [-1111]; [-111-1]; [ 11-11]; [-11-1-1]; [-1-111]; [-1-11-1]; [-1-1-11]; [-1-1-1-1]

### 2.2 Synthesis Using Cyclic and Bit Addition

### 2.2.1 Cyclic Shift (Mutation Method)

Mutation is one of the operators that randomly changes by flip method. The purpose of the mutation operator is to check whether the element changes better than itself. The mutation is carried out according to the mutation probability. Here is the mutation methods are implemented for binary genes only:

## Flip bit

This mutation method simply changes (flips) a randomly selected bit one until the end of their tap of length:

Before mutation: $\left[\begin{array}{lllllll}-1 & 1 & -1 & 1 & 1 & 1 & 1\end{array}\right.$
$\left.\begin{array}{llllll}1 & -1 & -1 & 1 & 1 & -1\end{array}\right]$
After mutation: $\left[\begin{array}{lllllll}-1 & -1 & 1 & -1 & 1 & 1 & 1\end{array}\right.$
$\left.\begin{array}{llllll}1 & 1 & -1 & -1 & 1 & 1\end{array}\right]$
The effectiveness of this method is capable to solve this problem. Combining both of them is aimed to seek out the new multilevel sequences and will be used as an experimental. This method must generate iteratively by repeating the process of inversion more than 10 times to get the samples. From that observation, the inverse filter method must be clipped and normalized to obtain SLE, ER and PSL which would utilize for review. These parameters are very important to fulfil the characteristics of good ACF of a sequence. Responding from that selection, the sequences should be getting through with a mutation procedure to mutate all the elements one by one to scan the best sequences. The flow chart of new methods for multilevel sequence set design is shown in Figure 4.

## Bit Addition

The binary addition algorithm operates on two bit patterns and resulted in a bit pattern. Usually all two patterns are the different size or exceed from the first operand whereby all three represent unsigned integers or signed integers. The leftmost bit of the one-bit result is used to carry into the next column. For example, here is the sum of two thirteen-bit integers, shown in Figure 3:

| -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 |  |  |  |  |  |  | 0 | 0 | 0 | 0 |  |
| 1 | first operand |  |  |  |  |  |  |  |  |  |  |  |
| -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |

Fig. 3. Example of the sum of the bit integers
From Figure 4 using cyclic shift and bit addition, shows the visually presented flow of data through information of these methods.

The operations performed within the procedure and the starting sequence was considered as the binary sequences with good correlation properties (ex: Barker Code).


Using the binary sequences ex: Barker Code/sequence that have good ACF


Fig. 4. Cyclic Shift and Bit Addition flowchart
Computers (usually) add two N-bit integers together to produce a N -bit result and a carry-out of the leftmost column. Every bit of the result must have a value. The following shows a 15 -bit addition from the carry column that has 13-length.

## 3 Result and Discussion

Based on Figure 1 using the element complementation, the lengths of the 13 until 21 sequences are generated to acquire all of the possibility sequences that are the lowest SLE. The possibility sequences of length thirteen are listed in Table 2 and here it is able to detect that indices 203 are the Barker sequence. Additional
information for the indices of 2657, 5536 and 7990 are the same contributions in SLE, ER and PSL parameter.

Table 2. Listing of the possibility sequences of length=13

| Length-13; $2^{13}=8192$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Indices | Sequences | SLE | ER | PSL |
| 203 | $\begin{aligned} & {\left[\begin{array}{llllllllllllllllllllllllllllll} 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & 1 \end{array}\right]} \\ & \text { Barker Sequences } \end{aligned}$ | 0.0710 | 13 | 0.0769 |
| 2657 |  | 0.0710 | 13 | 0.0769 |
| 5536 | $\left.\begin{array}{llllllllll} \hline-1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ -1 & -1 & -1 & -1 \end{array}\right]$ | 0.0710 | 13 | 0.0769 |
| 7990 | $\begin{aligned} & {\left[\begin{array}{llllllll} -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \end{array}\right.} \\ & 1 \end{aligned}$ | 0.0710 | 13 | 0.0769 |

From the total 32768 indices only eight sequences are detected with the lowest SLE. As shown in Table 3, 0.1333 is the lowest value in SLE for each indices below.

Table 3. Listing of the possibility sequences of length=15

| Length-15; ${ }^{15}=32768$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Indices | Sequences | SLE | ER | PSL |
| 822 | $\begin{aligned} & {\left[\begin{array}{lllllllllll} 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & - \\ 1 & 1 & -1 & 1 & -1 \end{array}\right]} \end{aligned}$ | 0.1333 | 15 | 0.2000 |
| 3822 |  | 0.1333 | 15 | 0.2000 |
| 9288 |  | 0.1333 | 15 | 0.2000 |
| 10656 |  | 0.1333 | 15 | 0.2000 |
| 22113 |  | 0.1333 | 15 | 0.2000 |
| 23481 | $\begin{aligned} & {\left[\begin{array}{llllllllll} -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & - \\ 1 & -1 & -1 & 1 & 1 & 1 \end{array}\right]} \end{aligned}$ | 0.1333 | 15 | 0.2000 |
| 28947 | $\begin{aligned} & {\left[\begin{array}{lllllllllll} -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & - \\ 1 & 1 & 1 & -1 & 1 \end{array}\right]} \\ & \hline \end{aligned}$ | 0.1333 | 15 | 0.2000 |
| 31947 |  | 0.1333 | 15 | 0.2000 |

The result of the next length are shown in Table 4. It shows that this is the best sequence (binary sequence) respectively and has the distinctive characteristic. Nevertheless, there are risks when the lengths are increase and it caused the stack of data overflow. It has become the out-of-memory error as a warning while the program is running.

Table 4. The possibility sequences of length-17, 18 and 19

| Length | Sequences | SLE | ER | PSL |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline 12^{17} \\ & (131072) \end{aligned}$ | $\begin{array}{lllllllllll} {\left[\begin{array}{lllllllllllll} 1 & 1 & 1 & - & -1 & 1 & 1 \\ -1 & 1 & 1 & - & 1 & -1 & 1 & -1 \\ -1 \end{array}\right]} & & & & \\ \hline \end{array}$ | 0.2215 | 17 | 0.1176 |
| $\begin{aligned} & \hline 12^{19} \\ & (524288) \end{aligned}$ | $\begin{array}{llllllllll} {\left[\begin{array}{lllllllll} 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & -1 & - \\ 1 & 1 & -1 & -1 \end{array}\right]} & & & \\ \hline \end{array}$ | 0.1607 | 19 | 0.1579 |
| $\begin{aligned} & \hline 12^{21} \\ & (2097152) \end{aligned}$ | $\begin{aligned} & {\left[\begin{array}{lllllll} 1 & 1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 \end{array}\right]} \end{aligned}$ | 0.1179 | 21 | 0.1429 |

Figure 2 shows the relation between SLE and length of sequences using the collected results from Table 2, Table 3 and Table 4. The figure clearly shows that for
the lowest value in binary sequences is in their length. Through this synthesizing process by using the element of complementation method, is able to generate the best binary sequences and can be created for any desired length. In sum, the results from this method commensurate with any other methods applied.


Fig. 2. Side lobe Energy versus Length of Sequences for 13 until 21 lengths using the Synthesis of Element Complementation.

Table 6 is then determined by the cyclic shift by simply changing the sign and unsign numbers which result in flipping are particularly prized. A dimensional cyclic shift of a matrix column is obtained from mutating the sequences by shifting it to the left. The $n-1$ samples are available where n is the length of the starting sequence when calculated by counting the number shifted from this procedure.

To avoid the same sequences from repeating, the types of sign or unsigned element detection of the sequences are placed in flow procedure. In this context, the gain time is highly emphasized and that particular sequence will be stacked. The addition of bit integer $k$ is for complementing the $2^{\mathrm{k}}$ to create the new sequences by adding the starting sequence or the shifted sequences. Table 5 shows the result of the bit addition. The Barker sequence of length-13 added with $2^{\mathrm{k}}$ where k is equal to 2. Therefore, the total element of the sequence is length15. Here, all of sequences are normalized before the SLE, ER and PSL are calculated.
In the final algorithm, the results from the sequences in Table 7 uses the cyclic shifted to track the lowest SLE. Figure 5 shows the results. Referring from that figure, the lowest value traces is 0.2044 in sm2 and sm3, and has three sets of sequences as shown in Table 7.
$\mathrm{sm} 1=[0.3111,0.3822,0.5956,0.5600,0.7022,0.4533$, $0.5244,0.5956,0.7022,0.8800,0.5244,0.3111,0.3822$, $0.4533,0.3822 ;$ ]
$\mathrm{sm} 2=[0.2756,0.3822,0.5600,0.7733,0.5956,0.3111$, $0.3467,0.3111,0.5956,0.4533,0.6311,0.5956,0.6311$, 0.4533, 0.2044;]
$\mathrm{sm} 3=[0.2756,0.2044,0.3111,0.3111,0.2044,0.2756$, $0.5244,0.5600,0.6311,0.4178,0.3822,0.2756,0.5244$, 0.2756, 0.3822;]
$\operatorname{sm} 4=[0.3550,0.3550,0.3787,0.3550,0.3314$, $0.4970,0.8047,0.4497,0.3550,0.2367,0.4970,0.5207$, $0.3550,0.2840,0.3822 ;$ ]

Table 5. The Generated Sequences for Bit Addition Method with Complement Number of Two

| Sequences | SLE | ER | PSL |
| :---: | :---: | :---: | :---: |
|  | 0.0710 | 13 | 0.0769 |
| $\left.\right]$ | 0.3822 | 15 | 0.2000 |
| $$ | 0.2044 | 15 | 0.1333 |
| $\operatorname{sm15(3)}=\left[\begin{array}{llllll}1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & 1\end{array}\right.$    -1 <br> 1 -1 1  $]$ | 0.3822 | 15 | 0.2000 |
| $\left.\right]$ | 0.3822 | 15 | 0.2000 |



Fig. 5. Side Lobe Energy of the Generated Sequences (Table 5) when applying the Mutation Method

Table 6. The Generated Sequences for Cyclic Shift Method

| Sequences | SLE | ER | PSL |
| :---: | :---: | :---: | :---: |
| Barker=[11 $11 \begin{array}{lllll}1 & 1 & 1 & 1\end{array}$ | 0.0710 | 13 | 0.0769 |
| $\begin{array}{cccccc} -1 & -1 & 1 & 1 & -1 & 1 \\ -1 & 11 & 1 \end{array}$ |  |  |  |
| Fm(1) | 0.1657 | 13 | 0.1538 |
| $\operatorname{Fm}(2)$ | 0.4024 | 13 | 0.2308 |
| Fm(3) | 0.2308 | 13 | 0.4024 |
| Fm(4) | 0.1657 | 13 | 0.1538 |
| Fm(5) | 0.2604 | 13 | 0.2308 |
| Fm(6) | 0.2308 | 13 | 0.3550 |
| $\operatorname{Fm}(7)$ | 0.4024 | 13 | 0.2308 |
| Fm(8) | 0.3550 | 13 | 0.2308 |
| Fm(9 | 0.2604 | 13 | 0.2308 |
| Fm(10) | 0.1657 | 13 | 0.1538 |
| Fm(11) | 0.4497 | 13 | 0.2308 |
| Fm(12) | 0.3550 | 13 | 0.2308 |

Table 7. THREE Set of Sequences Obtained using Mutated Process

|  | Sequences | SLE | ER | PSL |
| :--- | :--- | :--- | :--- | :--- |
| sm2(15) | $[11111-1-$ <br> $111-11-$ <br> $111-1]$ | 0.2044 | 15 | 0.1333 |
| $\operatorname{sm3(2)}$ | $[-1111111-$ <br> $1-111-11-$ <br> $11]$ | 0.2044 | 15 | 0.2000 |
| $\operatorname{sm3(5)}$ | $[1-11-$ <br> $111111-$ <br> $1-111-1]$ | 0.2044 | 15 | 0.2000 |
|  |  |  |  |  |

## 4 Conclusion

In this chapter, the pulsed compression theory has been introduced to get a high range resolution as well as a good detection probability. One of the basic types of pulse compression is a binary phase coding which encodes the transmitted pulse with the information that is compressed in the receiver radar.

The desired properties and general types of methods for finding and generating such waveform are defined in this chapter. Variety of methods are applied to keep an overview for introducing the existing methods to be done as a routine. The table and the histogram of complete results are presented to elaborate them visually and to predict the future. It has been proved by $[14,15]$ to compare with this method designed.

In conclusion, the binary code pulse trains design can be as the effective way to improve the overall performance of desired properties. It can enhance the radar performance by using the element complementation, cyclic shift and bit addition methods. It has the flexibility to dispose the noise ratio to improve the ER in that pulse trains. This performance result was evaluated via computer simulations and mathematical equation.

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[^0]:    * Corresponding author: sitijulia@unimap.edu.my

