Static and Dynamic Analysis Methods

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This paper introduces a quasi-static method of analysis which circumvents issues generated by uncertainties in the natural period properties of real building structures. Decisions leading to the proposal are explained and terminologies clarified.

KEYWORDS

National Annex to Eurocode 8, Seismic Design Actions, quasi-static method of analysis.

INTRODUCTION

A quasi-static method of analysis, which is essentially the "Code Lateral Force Method", offers an alternative to the conventional procedure to circumvent issues generated by uncertainties in the natural period properties of real buildings. Eurocode 8 (EC8) (CEN, 2004) makes reference to the lateral force method of analysis and the dynamic modal response spectrum method of analysis. The lateral force method is essentially a static analysis method based on a pre-determined lateral force which is representative of the design seismic actions. The dynamic analysis method is particularly encouraged in EC8 and is regarded as the "Reference Method" in view of the availability of commercial packages possessing dynamic analysis capability in most structural design offices in Europe and other advanced economies in other parts of the globe.

Static analysis is still permitted by EC8 but stringent pre-requisites apply as summarised in the following:

i. The fundamental natural period of vibration (T_1) of the building does not exceed $4T_c$ or 2s whichever is the less, where T_c (the corner period of the response spectrum) is 0.3s on rock or stiff soil sites.
ii. Criteria for verticality of the building in elevation, or vertical regularity must be satisfied.

The first criterion is controlled by the $4T_c$ (or 1.2s) threshold on rock or stiff soil sites. Most buildings of up to 50m in height (16 storeys) comply with this requirement. In view of most structures having some form of irregularity to fulfil architectural and functional requirements, the second criterion can be described as very stringent and this may preclude the majority of building structures from design by static analysis only.

Although most design offices possess software having dynamic analysis capability, most undergraduate degree programs in civil/structural engineering do not have substantial coverage on this topic in the core curriculum. The average engineering graduate may not have adequate knowledge and training to review dynamic analysis results generated by the computer and have them incorporated in the calculation of design actions (namely bending moment and shear force) at the member level. Enforcing dynamic analysis on structures can be counter-productive when the underlying principles are not well understood. A static analysis, despite its short comings of not allowing for higher mode effects in a dynamic response, has the merit of being easy to comprehend by the average structural engineering designer.

The vertical regularity prerequisite in EC8 should be relaxed in view of recent findings from the literature that buildings with $T_1 < 1.5$s (which is fulfilled by most buildings with height of up to 50m, or 16 storeys) are unlikely to experience any significant higher mode effects in their dynamic response to earthquake ground shaking. Analyses that have been reported to support this proposition include buildings possessing mass and stiffness irregularity in the elevation of the building (Su *et al.*, 2011; Fardipour *et al.*, 2011; Zhu *et al.*, 2007). In Australia (AS 1170.4, 2007; AEES, 2009), dynamic analysis is only required for buildings exceeding 50m (16 storeys) which are found on rock or stiff soil. In Singapore (NA to SS EC8, 2013; BC3, 2013), only one of the two prerequisites listed in the above need to be fulfilled.

In view of findings reported from the literature and prerequisites imposed by codes of practices in other areas of low to moderate seismicity, it is recommended that buildings of up to 25m in height be subjected to lateral force analysis method, irrespective of its regularity conditions in elevation.

LATERAL FORCE METHOD OF ANALYSIS

The lateral force method of analysis, as stipulated in EC8, entails the determination of the natural period of vibration, T_1, using equation (1a), the determination of the design base shear, F_b, using equation (1b) and the determination of lateral forces, F_i, applied to individual floor levels in the building using Eq. (1c).

$T_1 = 0.05H^{0.75}$ where H is the building height. (1a)

$F_b = S_d(T_1) / \lambda m$ (1b)

where $S_d(T_1)$ is the design response spectral acceleration at period T_1, and λm is the effective mass of the building and λ can be taken as 85% of the total mass (Clause 4.3.3.2.2(1)P).

$$F_i = F_b \frac{\delta_i m_i}{\sum_i \delta_i m_i} \quad (1c)$$

where δ_i is the deflection at floor level i of the building when subject to the lateral force and m_i is the floor mass.

While the prescriptive based lateral force method, as summarised above, appears straight forward, the estimated lateral actions on the building may be significantly higher than the actual values, mainly because of uncertainties in the natural period properties of the building concerned. The conservatism stems from inconsistencies in the natural period value calculated by equation (1a) and that reported by the computer analysis of the structural frame model of the building. This problem can be circumvented by introducing the capacity spectrum method (in a linear elastic analysis setting) which makes use of the calculated static deflection of the building to infer on an improved estimate of the fundamental natural period of vibration of the building. The revised lateral forces and the corresponding deflection can be significantly lower than that estimated by equations (1a) to (1c). Only static analyses are involved and these are easy for the average structural engineer to understand.

ILLUSTRATION BY A WORKED EXAMPLE

The lateral force method and the capacity spectrum method (which is referred herein collectively as the quasi-static method of analysis) are illustrated in the following eight-storey building under Malaysian seismic actions. The reinforced concrete hospital building (Figure 1), corresponding to Class IV importance level and situated on a flexible soil site (site period T_s = 0.5 s), measures 31.2m × 93.8m on plan and stands at a height of 25.6m above ground. The lateral force resisting system is contributed by wall-frame interaction. The typical storey height is 3.2m, typical span is 7.8m with 600mm × 600mm secondary beams separating the 150mm-thick slabs into one-way action. The main beams are sized at 800mm × 600mm. The wall thickness is 250mm, dimension of major columns is 850mm × 800mm, except for the 450mm × 450mm corner columns at the two wings. For gravity load, a superimposed dead load of 5.2 kPa is estimated for partitions, finishes and ceilings, and an average live load of 5 kPa is adopted (Looi *et al.,* 2015).

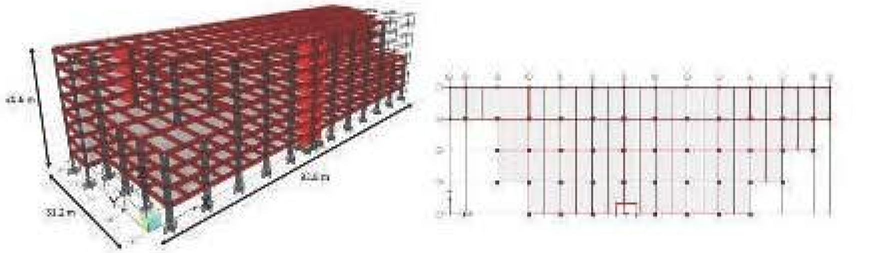

Figure 1: Eight-storey RC hospital building

LATERAL FORCE METHOD

The Lateral Force method of analysis, as stipulated in EC8, entails the determination of the natural period of vibration, T_1, using equation (1a), the determination of the design base shear, F_b, using equation (1b) and the determination of lateral forces, F_i, applied to individual floor levels in the building using equation (1c).

Step One: Identifying building height (H), calculating codified natural period of vibration (T_1) using equation (1a) and calculating response spectral acceleration (Figure 2).

H = 25.6 m

$T_1 = 0.05\,(25.6)^{0.75} = 0.57$ s

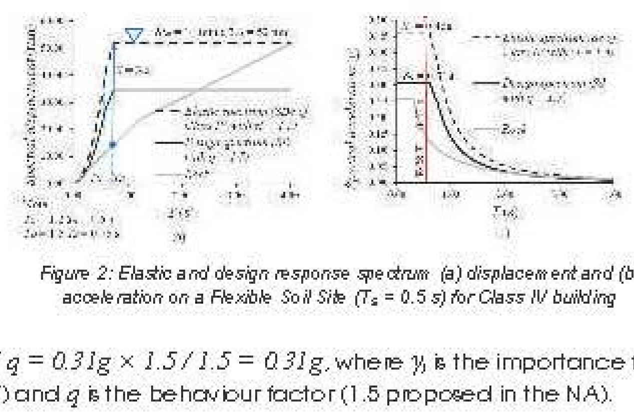

Figure 2: Elastic and design response spectrum (a) displacement and (b) acceleration on a Flexible Soil Site (T_s = 0.5 s) for Class IV building

$S_d = S_e\,\gamma_I / q = 0.31g \times 1.5 / 1.5 = 0.31g$, where γ_I is the importance factor (1.5 for Class IV) and q is the behaviour factor (1.5 proposed in the NA).

Step Two: Finding base shear F_B using equation (1b)

Mass, m = 76,862 ton

F_b = 0.31g (0.85)(76,862) = 198,683 kN

Step Three: Distributing the base shear into equivalent static force at each storey using equation (1c) by replacing lateral displacement (δ) with heights (z) of the masses, assuming fundamental mode shape is approximated by δ increasing with z (see Table 1). The static load should be applied to two orthogonal directions on plan. The lateral force method as required by EC8 is completed at this point. Analysis may continue with the quasi-static method for obtaining improved estimates.

Quasi-static method analysis

Step Four: *Structural analysis to obtain the force at each floor* (F_i), displacement at each floor (δ_i) (see Table 1) and effective displacement value (δ_{eff}) are calculated using equation (2).

Table 3: Force and displacement at individual floors in lateral Y direction.

Flr. no.	Mass m_i (ton)	z_i cumulative (m)	$m_i z_i$	F_i (kN)	δ_i (mm)	δ_i^2 (mm2)	$m_i \delta_i$	$m_i \delta_i^2$
8	8700	25.6	222,720	41,523	72.8	5295.0	633,070	46,066,436
7	8864	22.4	198,554	37,017	67.0	4494.3	594,237	39,837,308
6	8864	19.2	170,189	31,729	59.5	3535.8	527,073	31,340,917
5	8864	16	141,824	26,441	50.3	2528.6	445,726	22,413,344
4	10,370	12.8	132,736	24,747	39.7	1577.1	411,814	16,354,018
3	10,400	9.6	99,840	18,614	29.1	846.5	302,580	8,803,314
2	10,400	6.4	66,560	12,409	18.2	331.6	189,391	3,448,948
1	10,400	3.2	33,280	6,205	7.8	61.5	81,590	640,090
SUM	76,862		1,065,702	198,683			3,185,482	168,904,374

$$\delta_{eff} = \left(\frac{\sum m_i \delta_i^2}{\sum m_i \delta_i}\right) = \frac{168{,}904{,}374}{3{,}185{,}482} \approx 53mm \qquad (2)$$

Step Five: Calculating effective mass (m_{eff}) and improved estimate of response spectral acceleration (S_a) from equation (1b)

$$m_{eff} = \left(\frac{\left(\sum m_i \delta_i\right)^2}{\sum m_i \delta_i^2}\right) = \frac{(3{,}185{,}482)^2}{168{,}904{,}374} \approx 60{,}077 \text{ tons} \qquad (3)$$

$$S_a = \frac{F_b}{\lambda m} = \frac{F_b}{m_{eff}} = \frac{198{,}683}{60{,}077} \Big/ 9.81 \approx 0.34g$$

Step Six: Calculating effective stiffness (k_{eff}), natural period of vibration (T_{eff}) and drawing acceleration-displacement diagram for the building structure.

$$k_{eff} = 198{,}683/0.053 = 3{,}747{,}103 \text{ kN/m}$$

$$T_{eff} = 2p\sqrt{\frac{m_{eff}}{k_{eff}}} = 2p\sqrt{\frac{60{,}077}{3{,}747{,}103}} = 0.8\,s$$

Compared to the results obtained from ETABS (CSI, 2003) simulation, the first mode shape period is 0.81 s in the X direction and second mode shape period is 0.79 s in the Y direction (Figure 3).

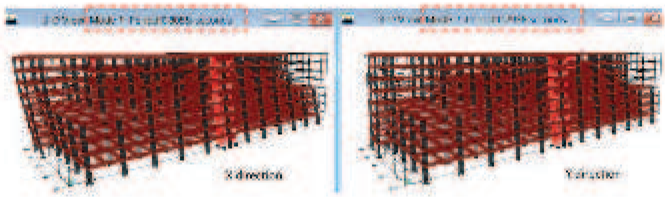

Figure 3: ETABS simulation results

Step Seven: Calculate seismic demand and superpose demand diagram on the acceleration-displacement diagram for the building (Figure. 4).

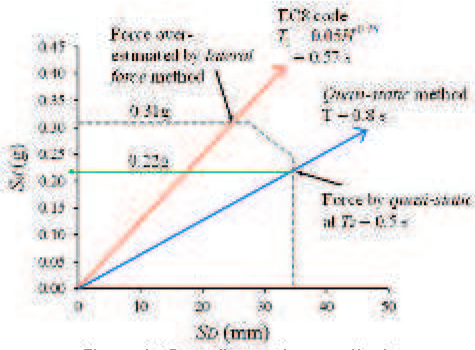

Figure 4: Capacity spectrum method

Step Eight: Repeat Step Three with the improved accuracy of demand.

F_b = 0.22g (0.8)(76,862) = 132,707 kN

Table 4: Improved force estimation and displacement at individual floors in lateral Y direction.

Flr. no.	Mass m_i (ton)	δ_i (mm)	$m_i \delta_i$	Improved F_i (kN)	δ_{ei} (mm)
8	8700	72.8	633,070	28,022	51.6
7	8864	67.0	594,237	26,303	47.6
6	8864	59.5	527,073	23,330	42.3
5	8864	50.3	445,726	19,729	35.8
4	10,370	39.7	411,814	18,228	28.3
3	10,400	29.1	302,580	13,393	20.7
2	10,400	18.2	189,391	8383	13.0
1	10,400	7.8	81,590	3611	5.6
SUM	76,862		3,185,482	141,001	

Table 5. Interstorey drift ratio check in the Y-direction

Flr. no.	Floor height z (mm)	$d_s = q_d d_e$ (mm)	Design interstorey drift d_r (mm)	Reduced interstorey drift $v d_r$ (mm)	Reduced interstorey drift ratio (%)	EC8 Cl. 4.4.3.2 (1) a)(%)
8	3200	77.4	6.0	3.0	0.093	0.5
7	3200	71.4	7.9	4.0	0.124	0.5
6	3200	63.5	9.7	4.9	0.152	0.5
5	3200	53.8	11.3	5.6	0.176	0.5
4	3200	42.5	11.4	5.7	0.178	0.5
3	3200	31.1	11.7	5.8	0.182	0.5
2	3200	19.4	11.1	5.5	0.173	0.5
1	3200	8.4	8.4	4.2	0.131	0.5

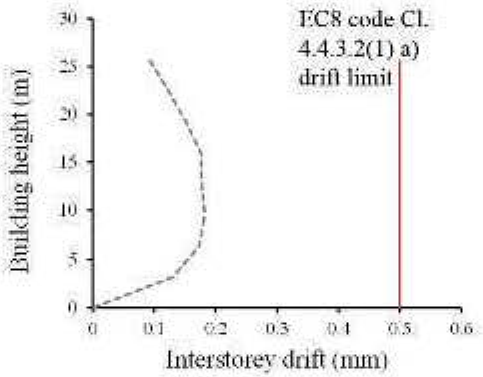

Figure 5: Interstorey drift in the Y-direction

Subsequent rigorous design based on acceptance criteria for ultimate strength and damage limitation (serviceability) drift check of structural members should be carried out accordingly. An example of damage limitation check in the Y-direction for the hospital (Class IV building) is shown in Table 5 and Fig. 5. The displacement behaviour factor (q_d) is assumed equal to q as 1.5, the reduction factor (ν) is taken as 0.5 (Lam *et al.*, 2016) and the limitation for interstorey drift ratio for "buildings having non-structural elements of brittle materials attached to the structure" is assumed as 0.5%.

CONCLUSIONS

1. Lateral Force method of analysis is allowed for buildings of up to 50m (16 storeys) in height and is recommended for buildings of up to 25m (8 storeys), irrespective of the regularity conditions in elevation.
2. Estimates by the Lateral Force method may be overly conservative because of uncertainties in the natural period properties of the building concerned.
3. The predicted lateral forces and the corresponding deflections may be revised to lower values by applying the capacity spectrum method (in a linear elastic analysis setting) which makes use of the calculated static deflection of the building to infer an improved estimate of the fundamental natural period of vibration of the building.
4. The Lateral Force method and the Capacity Spectrum method are collectively described as the quasi-static method of analysis, which is illustrated by an example eight-storey building.
5. It is shown by example that the lateral force and deflection demand have been overstated by the Lateral Force method by a factor of 1.4 approximately.

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Notations

F_b	design base shear
F_i	lateral force at floor level i
H	height of building
$S_D(T)$	design displacement response spectrum
$S_{De}(T)$	elastic displacement response spectrum
$S_d(T)$	design response spectral acceleration
$S_d(T_1)$	design response spectral acceleration at period T_1
$S_e(T)$	elastic horizontal ground acceleration response
T_1	fundamental natural period of vibration
T_C	first corner period
T_D	second corner period

T_s	initial low-amplitude site natural period (note: this symbol is different from EC8 where T_S is referred as the duration of the stationary part of the seismic motion)
T_{eff}	effective natural period of vibration
d_e	displacement of the same point of the structural system, as determined by a linear analysis based on the design response spectrum
d_r	design interstorey drift
d_s	displacement of a point of the structural system induced by the design seismic action
k_{eff}	effective stiffness
m_{eff}	effective mass
m_i	floor mass at floor level i
q	behaviour factor
z_i	floor height at floor level i
δ_i	deflection at floor level i
γ_I	importance factor
λ	mass correction factor
v	reduction factor for interstorey drift limit associated with the damage limitation requirement. ■

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