Affinity Laws: Interpretations and Applications for Centrifugal Pumps

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1.0 ABSTRACT

Operation of rotating equipment at its best efficiency point (BEP) or close to it is every operator's objective as this leads to less wastage, higher cost savings, less wear in the machine and a bundle of other benefits. However, in common practice, most process data acquired during front-end engineering design (FEED) or conceptual study phases do not accurately tally with actual process conditions during actual plant startup. This of course is due to many uncontrollable or unpredictable conditions and as a result, in most cases, pumps do not operate at rated points. A correction method should be implemented to correct this deviation or to vary pump performance based on process conditions that change over time, permitting peak performance throughout the pumps life cycle. One such method presented here is Affinity Laws, where performance improvements are achieved by toggling with the impeller diameter or rotational speed. This article presents a detailed interpretation of Affinity Laws and their usage for centrifugal pumps, alongside correction methods to mitigate the large inaccuracies that have overshadowed the widespread usage of Affinity Laws. These methods have been evaluated with vendor pump curves with the results presented and discussed.

2.0 INTRODUCTION

Affinity Laws, also known as 'Fan Laws', derive their name from their initial usage to re-rate fans. These laws have proven successful in their application for fans generally in industrial plants, where the fluid medium being dealt with was air (light gas) with a relatively low head rise and volume ratios that are commonly negligible. Their applications in other rotating equipment soon became apparent, as centrifugal pumps and compressors are both ideally governed by similar velocity triangles as fans. Affinity Laws for pumps have often been described in terms of dimensionless numbers, used to predict flow, head and power changes with diameter or speed variation under similar aerodynamic, flow, and geometrical conditions, with a caveat that errors can be in the order of 20% [(1)].

The objective of this article is to bring awareness and put forward a comprehensive reference to the usage of Affinity Laws for application in re-rating pumps, whether due to changes in process requirements over time or unexpected flow conditions at commissioning. This article discusses methods generally overlooked by the general rotating equipment engineering community. It is worth to note that proper application of this law could provide a scenario allowing the pump to operate closer or at its BEP, providing more efficient operation and in turn cost savings. However, it is outlined without going in-depth into the definition of Euler's rotor dynamics equation (interested readers are directed to (2); (3)).

The problem was tackled by references to multiple literature and analysis of vendor pump curves to validate literature claims along with field experience by industrial experts.

The discussion would also be limited to the following conditions, viz.:

- Pump has no entrained solid or gas [(2)].
- Scope does not include application to hydromagnetic pumps [(2)].
- Information presented generally cover medium range specific speed pumps.
- Simultaneous variation of diameter and rotational speed at once is not included in the scope of this article and not recommended [(4)].
- Viscosity and fluid vaporization are negligible [(2)].

3.0 BACKGROUND

3.1 Velocity Triangles

Affinity Laws for head relies on Euler's rotor dynamics equation (Equation 1) which is derived from velocity triangles. Noticeable from these triangles as depicted in Figure 1(a) is that as periphery speed, U (also known as tip speed) varies, the absolute fluid exit velocity, V, follows suit linearly. The same observation is noticed for variation in the relative fluid exit velocity, W[1], as illustrated in Figure 1(b).

$$\Delta H = U_2 \cdot C_{u_2} - U_1 \cdot C_{u_1} \qquad \textit{Equation 1}$$

Where;

H = Specific Head, kJ/kg *C_u = Meridional velocity, m/s*
U = Periphery velocity, m/s

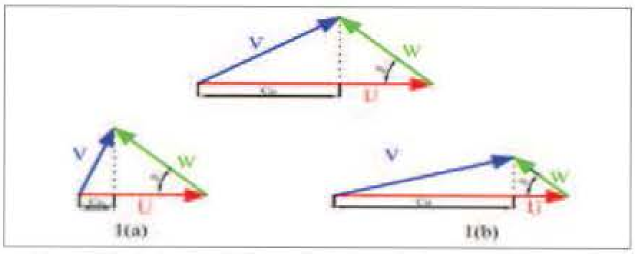

Figure 1: Velocity triangles (a) variation in periphery speed (b) variation relative fluid exit velocity

3.2 Affinity Laws

Affinity Laws are derived from the basis that head and flow rate are both dependent on velocity triangles for geometrically similar impellers, and under ideal conditions (as presented in Figure 1), they react linearly to change either individually or simultaneously.

Fluid flow rate defined by general textbooks suggest the following:

$$Q = UA\begin{cases} A \propto r_2^2 \propto D_2^2 \\ U \propto N \cdot r_2 \propto N \cdot D_2 \end{cases}$$

Equation 2a

$$\therefore Q \propto N \cdot D_2^3$$

Equation 2b

However, empirical data shows that flow rate to be directly proportional to diameter and speed change. The following was proposed by A.J. Stepanoff and suggest (5) its validity for a considerable range of specific speeds[2] .

$$Q \propto N \cdot D_2$$

Equation 2c

Where;

Q= Flow Rate, kg/s A=Impeller exit area, m^2	
N = Rotational Speed, rpm	D = Impeller diameter, m
r = Impeller radius, m	

Head being a function of two velocities as indicated in Equation 1 with velocity being a product of rotational speed and impeller diameter as indicated in Equation 2a then suggest;

$$\Delta H = \Delta(U \cdot C_u)$$

Equation 3a

$$\Delta H \propto (N \cdot D)^2$$

Equation 3b

Power, P being defined as a product of flow and head is then represented by referring to Equations 2c and 3b;

$$P = Q \cdot \Delta H$$

Equation 4a

$$P \propto (N \cdot D_2)^3$$

Equation 4b

Equations 2c, 3b and 4b all represent the general Affinity Laws applicable for pumps[3]. It should be noted that other forms of these laws exist by either holding one of the variables on the right side of these equations constant or implementing correction constants, which would be elaborated further in Section 4 of this article.

4.0 EVALUATION OF FAN LAWS

4.1 Effect of Scaling / Similitude

In order to accurately apply Affinity Laws, Gullich suggested that geometrical and dynamic similarity be maintained; to be more specific, all wetted parts are sealed in the same ratio whilst ensuring constant Euler, Froude and Reynolds number [(6)]. The effects of Froude[4] and Reynolds[5] number for pumps are however, generally negligible unless detailed loss investigation is required. Jacques and Florjancic both added to these recommendations by implying that Affinity Laws can be applied accurately by maintaining pumps efficiency, specific and suction specific speed [(4), (3)]. Jacques further suggested that only the same impeller and not one geometrically similar be used for scaling as manufacturing variation on wetted parts surface properties will affect the accuracy of Affinity Laws [(4)] (refer Section 4.1.1 for further information).

By paraphrase, conservation of fluid impeller exit angle before and after modifications would ensure the accuracy of Affinity Laws. The requirements outlined are to ensure that flow kinematics are maintained which prevents variation in slip. As illustrated beforehand in Section 3.2, observation suggests that variation in impeller outer diameter and rotational speed are major factors which govern corresponding change in the pumps performance. The following sub-sections will elaborate the effect of variation in these parameters on pump performance.

4.1.1 Impeller Diameter Variation

It is common practice with most pump vendors to construct impellers to be slightly larger than theoretically computed, as rated flow within 80% – 110% of BEP is generally acceptable [(7)]. If required pump performance modifications are carried out by trimming the impeller or throttling the flow. This practice provides a buffer for the minor effects neglected during detailed engineering calculations and also safeguard against under tolerance in contractual terms. Developing an impeller to its exact theoretical calculated size might prove to be detrimental commercially and to the delivery schedule, if the flow properties end up not meeting process requirements and remanufacturing is required.

However, one should take note that throttling the flow would mean the pump is over-sized, leading to detrimental effects, and to name a few:

- Higher manufacturing and operation cost
- Higher energy consumption as the pump would be running on part load (due to reduced pump efficiency)
- Higher maintenance cost and frequency (due to premature wear)
- Possibility of pulsation (if inducers or suction impellers are used to reduce the higher NPSHr of a larger pump)
- Higher possibility of seal failure
- Higher vibrations and noise levels (due to part load operation) [(3)].

Reducing impeller diameter on the other hand does not significantly cost any of the before mentioned side effects besides the higher capital cost; if performed properly and within the allowable range. Most literatures would suggest the upper limit of the allowable range to be 20% of maximum diameter, whereas some propose it to be up to 30% of maximum impeller diameter. However, in practice it is often trimmed to about 3% – 5%[6] only of maximum impeller diameter. Further impeller trim beyond the allowable range might cause considerable efficiency drop and unstable pump performance due to increase in energy loss to turbulent flow. In addition, if the impeller to casing ratio exceeds the pump design limit, an excessive increment of specific vane loading would occur, resulting in re-circulation flow distribution pattern across the impeller exit to become highly unstable increasing the tendency of back flow in the pump especially in high energy and double suction flow pumps [(1)].

Also Affinity Laws can be easily applied to estimate the amount of trim required for a desired flow property with a caveat that the results by using this Law does not accurately reflect actual physical data. Reasons for this occurrence are breached to the geometrical similitude, before and after the impeller trim. Factors which contribute to this include failure of geometrical impeller trim to vary proportionally with:

- Impeller surface roughness (commonly negligible with the exception to application of Affinity Law in high head or flow operations (3))
- Impeller width and internal leakage clearances
- Impeller to casing ratio

These mentioned factors cause increase in slip, which evidently changes the angle of relative fluid exit velocity. Due to this, impeller diameters are often trimmed in phases of small increments and re-tested until desired flow conditions are achieved. Each time the impeller is trimmed it would require to be re-balanced which could results in costly modifications.

A silver lining to the setbacks introduced by the usage of Affinity Laws was proposed by Bloch and Budris as illustrated by Figure 2; a correction chart for impeller diameter trim to compensate for hydraulic mismatches and mechanical imperfections [(8);(9)]. Validity of this chart has been tested with two separate pump vendor performance curves and initial observation shows that improvement of accuracy ranging between 0.2 to approximately 10%. More importantly, usage of the correction factor as indicated in Figure 3 and Figure 4 prevents impeller over cut, which is not correctable unless by change of impeller or variation of rotational speed. The question that lies here is what if the driver is not variable speed, which is the common case in most scenarios where impeller trimming is practiced. Do note however, even with the usage of the correction factor the exact required trim is not indicated with negligible effects, this is especially the case when referring to Figure 3. This is highly likely owed to the high flow of this pump in which the usage of these laws is highly not recommended as addressed later. Note

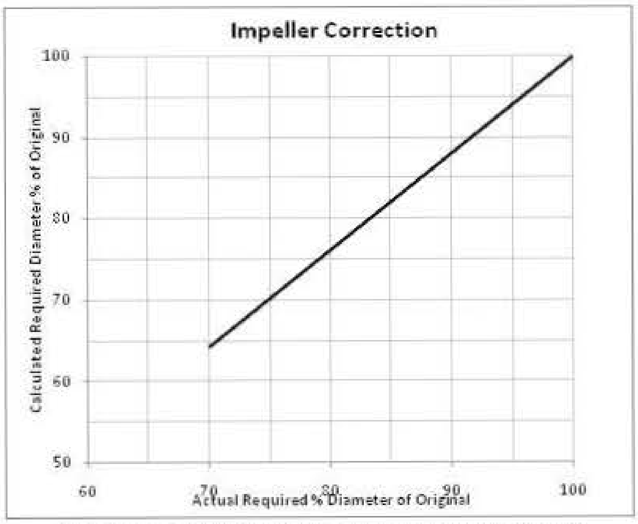

Figure 2: Impeller trim correction [Image adopted from (9); (8)].

however, the mere 0.01% deviation for an approximately 20% maximum impeller diameter trim for pump vendor B as demonstrated on Figure 4, which is likely owed to its moderate flow and head (medium range specific speed). Other methods that may also be applied to make impeller trimming more favourable is trimming of vanes in an oblique cut instead of the entire shroud (for closed impeller designs). This reduces the effect of balancing issues and provides a more uniform exit flow at the impeller exit. However, care also should be taken when performing this to ensure that shroud mechanical strength is not compromised.

It is safe to perform interpolation of actual flow data being utilised (vendor to test and validate impeller with similar specific speed at varying sizes), and extrapolation under any circumstances avoided. If a new impeller is to be purchased to increase head or flow conditions, a complete hydraulic analysis is recommended. The failure of the correction factor when extrapolated is indicated on Figure 3, showing an undersized impeller, which would ultimately lead to dissatisfactory pump performance. Pump underperformance or over sizing is to be expected if the

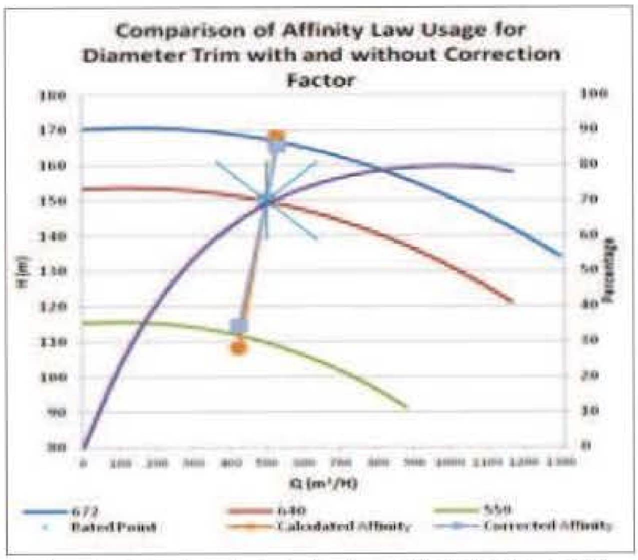

Figure 3: Comparison with and without correction factor proposed on general vendor A pump curve (SI Specific Speed: 762)

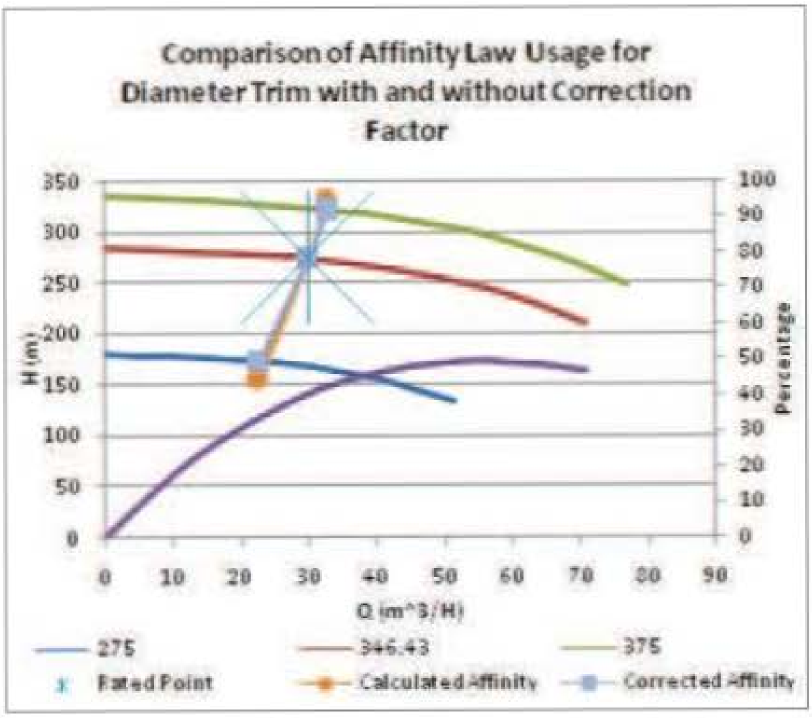

Figure 4: Comparison with and without correction factor proposed on general vendor B pump curve (SI Specific Speed: 287)

process conditions are extrapolated beyond a certain flow threshold.

Another important limitation to consider is the applicable specific speed range which Affinity Laws govern. Pumps with low specific speed are designed to maintain the impeller clearance width for a considerable distance away from the outer diameter; this disrupts the width to diameter ratio which maintains a near constant change as seen in middle range specific speed pumps. High specific speed pumps on the other hand have very short blades. Hence, they are very sensitive to diameter change[5] due to possibility of inadequate vane overlap causing hydraulic problems to arise[9]. Consequently, both these specific speed ranges do not correspond well with Affinity Laws. Mentioned in Section 4.1 that Affinity Laws rely on the pumps efficiency to remain constant before and after the impeller trim, and pump efficiency for system curves with high static head tend to deviate a significant amount for small changes in duty point rendering Affinity Laws highly unreliable in these cases[4].

4.1.2 Impeller Rotational Speed Variation

An efficient way to control pump performance especially in frequently varying process conditions is by changing the impeller rotational speed with variable speed drive system (VSD). Use of VSDs with electric motor drive include variable frequency drive (VFD), hydraulic coupling or 2-step gear box. Where electric motor drive is not used the VSD drive can be variable speed steam turbine, gas turbine, reciprocating engine or even hydraulic power recovery turbine. Note all of these VSD systems have different limitations on allowable speed range which must be taken into account.

Similar to diameter trimming, speed modification does have its limitations too. For one, the larger number of equipment required directly increases capital and maintenance cost, and skid foot print. Limitations concerning to accuracy of Affinity Laws with speed change also exist, as this impacts the fluid velocity, which in turn varies the friction losses due to surface roughness not varying proportionally with speed change. Nevertheless, this impact is commonly negligible with exception given to high head or high flow operations, as friction losses become more predominant in these scenarios[3]. Also as addressed in Section 4.1, for Affinity Laws to be used

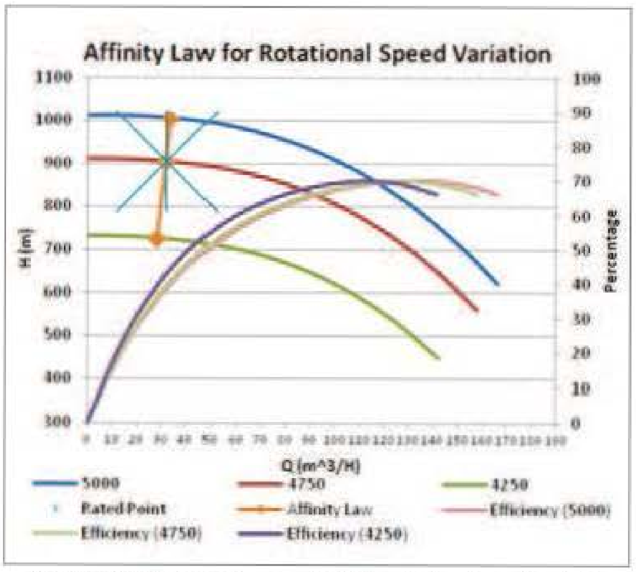

Figure 5: Affinity Laws (Pump vendor A) for rotational speed variation (SI Specific Speed: 178)

accurately, constant pump efficiency is required. This however is not seen in practice, being supported from a large study of commercial pumps by H.H. Anderson suggesting that centrifugal pump efficiency as a function of multiple variables, one of which is rotational speed[2]. Though for small speed changes the effects that cause inaccuracy to Affinity Laws are more than often negligible as illustrated on Figure 5. A pump manufacturer had introduced a method shown in Equation 5 to estimate the new pump efficiency after speed variation [10].

$$\eta_{new} = 1 - (1 - \eta_{old}) \cdot \left(\frac{n_{old}}{n_{new}}\right)^{0.1}$$

Equation 5

Beside accuracy limitations, when Affinity Laws are applied to vary rotational speed, one should be cautious and take mechanical limitations of the pump into consideration, as increasing the rotational speed without a proper rotor dynamic study may cause the shaft to operate dangerously close to its critical speed resulting in excessive noise and vibration, with pump part failures following suit. Increasing the shaft size to counter this effect can be considered with a detailed engineering study, as the shaft size directly affects the rotor dynamics, performance and efficiency of the pump [9]. Prudence should also be taken when the NPSH margin of the pump in operation is very small, as common pump NPSHr curves would show increment of NPSHr as speed is increased[11].

5.0 CONCLUSION

Affinity laws have not always been used in practice owing to the common fallacy of low accuracy predictions and unreliability. This is compounded with competition from present day high speed computational hardware and advancing CFD software with the capability to accurately map new flow conditions for variation of rotational speed and impeller diameter. These factors are main causes as to why not much attention has been paid to Affinity Laws. However, this article brings to light the possibility of these laws being used reliably to estimate expected flow conditions with tolerable accuracy during feasibility stage or even during trouble-shooting, if applied with proper care and the methods outlined. Although further thorough research is strongly suggested before field application of these laws is put into practice. Future works to be considered should include establishing a proven quantitative range for maximum impeller trim, rotational speed variation, applicable specific speeds as well as applicable head and flow. Nevertheless, caution should always be taken especially when trimming impeller diameter, as flow kinematics that prevail at certain flow coefficients are greatly influenced by geometrical properties. ■

APPENDIX A: BIBLIOGRAPHY

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[11] **Brennen, Christopher E.** *Hydrodynamics of Pumps.* Caltech, Pasadena, California : Oxford University Press, 1994.

[1]Relative exit fluid velocity is a function of the fluid flowrate, Q.

[2]Note that Affinity Laws are not applicable for very low and very high specific speeds as will be discussed further in section 4.
A number of other literatures also support this claim i.e. [9];[10];[4].

[3]These equations are also applicable for compressors and fans with caveat that the operating fluid may undergo substantial compression that has to be taken into consideration; as opposed to liquid compression in pumps that is generally negligible (and such will be treated in this article).

[4]Effect of flow resistance does not exceed a threshold that can be considered to cause major influence to flow kinematics.

[5]Flow is generally in turbulent phase along most points in the allowable operating range, flow does not cross Reynolds number threshold to cause significant effect to flow kinematics.

[6]Bloch's observation suggest for up to 5% speed change or impeller diameter trim efficiency remains almost constant, for mid range specific speed pumps.

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