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Single-Electron Transistors (SET): Literature Review

AMIZA RASMI AND UDA HASHIM
Microfabrication Cleanroom
School of Microelectronic Engineering,
Kolej Universiti Kejuruteraan Utara Malaysia,
Jejawi, 02600 Arau, Perlis.
E-mail:aemmiey_zs@yahoo.com Tel: 04-9798342

ABSTRACT

Single-electron transistor (SET) is a key element in our research field where device operation is based on one-by-one electron through the channel utilizing the Coulomb blockade effect. The SET are often discussed as elements of nanometer scale because SET can be made very small and can detect the motion of individual electrons. However, SET has low voltage gain, high input impedances, and sensitive to random background charges. This makes it unlikely that SET would ever replace field-effect transistor (FET) in applications where large voltage gain or low output impedance is necessary. In this paper, we provide an overview of research developments of SET. The theoretical study of single electronics include orthodox theory, coulomb blockade, tunneling effects, and Kondo effect are discussed. On the other hand, the methods for modeling and simulation single-electron circuit are reviewed.

Keyword: single-electron transistor (SET), orthodox theory, Coulomb blockade, tunneling effects, quantum dot, Kondo effect, modeling, and simulation.

INTRODUCTION

Single-electron devices (SED) [1], which literally have potential to manipulate electrons on the level of elementary charge, are thus considered to be the devices that will allow such a charge. In addition to their low-power nature, SED have a rather simple operation principle, and because of this, operation basically guaranteed even when device size is reduced to the molecular level. On the other hand, their performance improves as they become smaller. These properties are quite beneficial for large-scale integration (LSI).

The most fundamental three-terminal single-electron devices (SED) are called the single-electron transistors (SET) [2-4]. The schematic structure of SET is shown in Figure 1. As shown in the figure, the structure of SET is almost the same as that of MOSFET. The SET also has one coulomb blockade (CB) island but it has another tunnel capacitor and voltage source. The electrode with the normal capacitor is the gate and the other two electrodes are the source and drain.

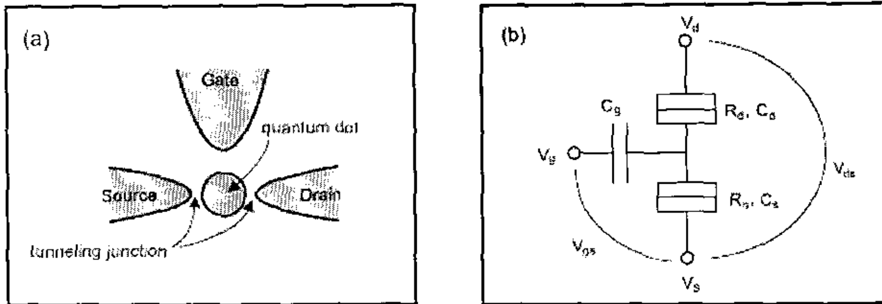

Figure 1: (a) Schematic structure of single-electron transistors.
(b) Equivalent circuit of single-electron transistors.

The operation principle of a SET is show in Figure 2. A SET is a device whose operation relies on single electron tunneling through a nanoscale junction. The electrons tunnels are transferred one-by-one through the channel (in contrast with conventional MOSFET) due to its particular architecture that includes two tunneling junctions and *one conductive island* [5, for a detailed description see ref.1].

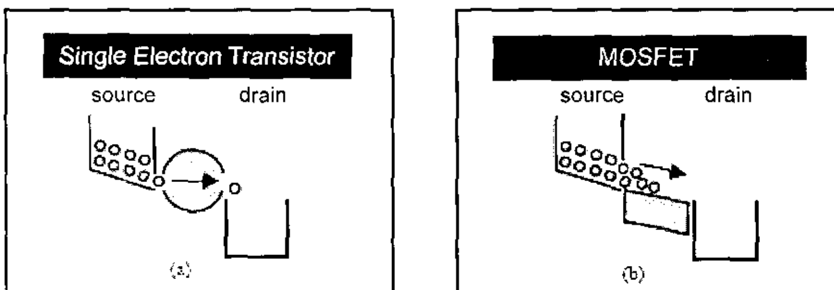

Figure 2: Transfer of electrons is (a) one-by-one in Single Electron Transistor (SET), which is in contrast with (b) conventional MOSFET where many electrons *simultaneously participate to the drain current.*

A SET can be considered as field effect transistor (FET) whose channel consists of two tunneling junctions forming a quantum dot (QD) island. The quantum dot island is channeled to the source *RTD1* and drain *RTD2* so current can flow under the influence of the source bias voltage, *Vs* and drain bias voltages, *Vd* respectively. The island is capacitive coupled to a gate voltage, *Vg*. Beside that, the charging energy of the *system depends on the number of electron, n* on the quantum dot island and the gate voltage, *Vg*. An equivalent circuit representation of the SET is as shown in Figure 3.

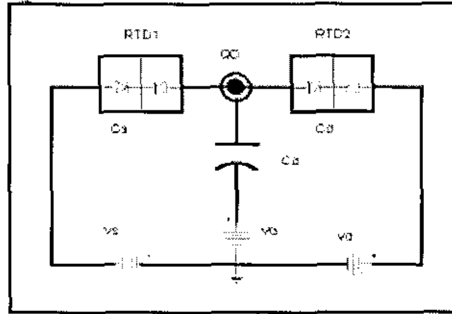

Figure 3: SET circuit.

This present paper is organized as follows. We start with the theoretical study of SET such as orthodox theory, coulomb blockade, and tunneling effects in Theory of Single Electron. In Modelling and Simulation of Single-Electron Transistor we discussed the methods of modeling and simulation of single-electron circuits. Finally, the conclusion of our discussion is presented.

THEORY OF SINGLE ELECTRONICS

Recent progress in physics and fabrication technology of nanostructures ushered in a new field of electronics is single electronics [7]. We talk about single electronics whenever it is possible to control the movement and position of a single or small number of electrons. In this section, we discussed about theoretical of the single electronics including a basic theory of orthodox, coulomb blockade and tunneling effects.

Orthodox Theory

K. Likharev [1] developed the 'orthodox' theory of single-electron tunneling, which quantitatively describes an important charging effect such as Coulomb blockade and Coulomb oscillation. On the other hand, they also describe an electron transport in an arbitrary single-electron circuit consisting of tunnel junctions, capacitors, and voltage sources, as a sequence of jumps of single electrons [8]. The orthodox theory makes the following approximations [1]:

1. The electron energy quantization inside the conductors is ignored, i.e. the electron energy spectrum is treated as continuous.
2. The time T_t of electron tunneling through the barrier is assumed to be negligibly small in comparison with other time scales (including the interval between neighboring tunneling events). This assumption is valid for tunnel barriers used in single-electron devices of practical interest, where $T_t \sim 10^{-15}$ s.
3. Coherent quantum processes consisting of several simultaneous tunneling events ("*cotunneling*") are ignored. This assumption is valid if the resistance R of the tunnel barriers of the system is much higher than the quantum unit of resistance R_Q:

$$R >> R_Q, R_Q = h/e^2 \approx 25.8 \text{ k}\Omega. \quad (1)$$

This condition ensures that the electrons are traversing the insulating gap one at a time. The tunnel resistance should not be too big also because tunneling cannot to be occurred. In terms of circuit design, this high tunnel resistance will have an immediate consequence for the maximum speed operation.

The tunneling resistance is a phenomenological quantity that is defined in the situation where a fixed voltage difference V is imposed to the two electrodes on either side of the barrier. The tunneling rate Γ of an electron through the barrier is then proportional to V [8]:

$$\Gamma = \frac{V}{eR_T} \quad (2)$$

At the junction, the charge e will be incremented continuously by a current source and drained discretely by the tunneling of electrons.

Despite the limitations list above, the orthodox theory is in quantitative agreement with virtually all the experimental data for systems with metallic conductors (with their small values of the electron wavelength on the Fermi surface, λ_F) and gives at least a qualitative description of most results for most semiconductor structures (where the quantization effects are more noticeable, due to larger λ_F). The main result of the orthodox theory is that the rate of a tunnel event strongly depends on the change in free energy the event causes [9].

Coulomb Blockade Effects

The SET is known as a highly sensitive electrometer based on the Coulomb blockade phenomena. The Coulomb blockade or single-electron charging effect, which allows for the precise control of small numbers of electrons, provides an alternative operating principle for nanometer-scale devices. In addition, the reduction in the number of electrons in a switching transition greatly reduces circuit power dissipation, raising the possibility of even higher levels of circuit integration [10].

The SET uses this effect to control the charging of a small 'island' electrode by electrons one at a time [10]. Figure 4 shows a basic SET, where an island electrode is isolated from source and drain electrodes by tunneling potential barriers with capacitance C_1 and C_2. A gate electrode couples to the island electrostatically via the gate capacitance C_G and C_G is used to modulate the generated current.

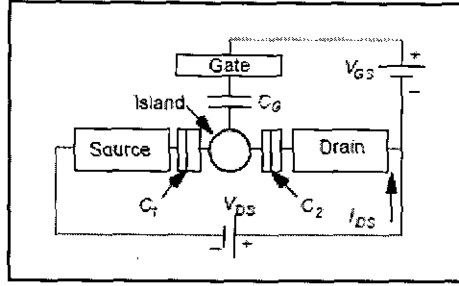

Figure 4: Circuit diagram of single-island double tunnel junction SET.

The charging energy of the system is defined as [10]:

$$E_C = \frac{e^2}{2C_\Sigma} \tag{3}$$

where e is the charge of an electron and C_Σ is the capacitance of this system ($C_\Sigma = C_1 + C_2 + C_G$), associated with a single electron prevents sequential tunneling through the island at below a threshold voltage, V_t which can be controlled by applying a voltage V_g to the gate. The threshold voltage, V_{th} is the minimum value required for tunneling as shown below [8]:

$$V_{th} = \frac{E_C}{e} = \frac{e}{2C_\Sigma} \tag{4}$$

E_C is the charging energy of the system. As long as the threshold voltage is not reached the junction is blocked.

In order to observe coulomb blockade effects, there are two necessary conditions. One condition is that the charging energy E_C of single excess electron on a quantum dot is much greater than the thermal energy is described by [9, 11]

$$E_C = \frac{e^2}{2C_\Sigma} \rangle\rangle k_B T \tag{5}$$

In the above expressions, k_B is Boltzmann's constant and T is the temperature of the system. From this equation, we can write

$$T \langle \frac{E_C}{k_B} \tag{6}$$

Here, the Coulomb blockade is effective at this temperature. The other condition is that the tunneling resistance, R_T of the tunneling junction must be larger than quantum resistance is given by [9]

$$R_T \rangle\rangle R_Q = \frac{h}{e^2} \approx 25.8k\Omega \tag{7}$$

where h is Planck's constant and R_Q is the quantum resistance.

Based on the Coulomb blockade effects, many interesting devices are possible such as precise current standards, very sensitive electrometers, logic gates and memories with ultra low power consumption, down scalability to atomic dimensions, and high speed operation [8]. Therefore, these effects are experimentally verifiable only for very small high-resistance tunnel junctions, meaning small particles with small capacitances and/or very low temperatures.

Tunneling Effects

In 1928, the main features of electron emission from cold metals by high external electric fields on the basis of tunneling through a triangular potential barrier were explained. Conclusive experimental evidence for tunneling was found by L.Esaki in 1957 [12] and by I.Giaever in 1960 [13]. Esaki's [12] tunnel diode had a large impact on the physics of semiconductors, leading to important developments such as the tunneling spectroscopy, and to increased understanding of tunneling phenomena in solids. Beside that, R.Davis and H.Hosack [9,14] first introduced the concept of resonant tunneling in double barriers.

In this part, we discussed two other important tunneling effects that are transmission probability and tunnel rate.

Transmission Probability

A wave packet with probability amplitude B_1 hits a potential barrier. Part of the wave will be reflected and the rest will be transmitted through the barrier. The probability amplitude of the transmitted part is denoted by B_3. The transmission probability $|T|^2$ is defined as the square of the ratio between transmitted and incoming probability amplitudes [9]

$$|T|^2 = \frac{|B_3|^2}{|B_1|^2} \tag{8}$$

The transmission probability can be derived from Schrödinger's equation is

$$\mathrm{H}\ \Psi = \mathrm{E}\ \Psi$$

For a rectangular potential barrier with width d and height E_0, the transmission probability $|T|^2$ is defined as [11]

$$|T|^2 = \frac{|B3|^2}{|B1|^2} = \left[\frac{1}{2} - \frac{1}{8}\left(\frac{k_1}{k_2} - \frac{k_2}{k_1}\right)^2 + \frac{1}{8}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)^2 \cosh\left(2k_2 d\right)\right]^{-1} \quad (9)$$

with $k_1 = \frac{1}{\hbar}\sqrt{2m^* E}$ and $k_2 = \frac{1}{\hbar}\sqrt{2m^*\left(E_0 - E\right)}$

The strength parameter $d\sqrt{2m * E_0 / \hbar}$ allows a classification of tunnel barriers according to their opaqueness. For atypical strength parameter of 30 the transmission probability is shown in Figure 5.

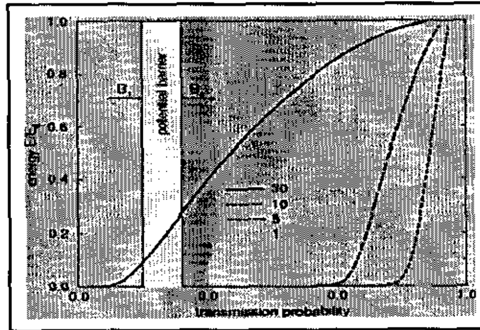

Figure 5: Tranmission probability for a barrier with a strength parameter of 30, 10, 5 and 1.

A strength parameter of 1 is typical of the barriers present in most ohmic contacts to semiconductors, a value of 5 is typical for the ionization of an atom by an electric field and a value of 30 is characteristic for the α-decay from a radioactive nucleus [9,15].

Tunnel Rate

The tunnel rate from an initial state i to a final state f, considering the change in free energy and using Fermi's rule is expressed as

$$\Gamma_{i\to j}\left(\Delta F\right) = \frac{2\pi}{\hbar}\left|T_{ki,kf}\right|^2 \delta\left(E_i - E_f - \Delta F\right) \quad (10)$$

where $\Delta F=F_f-F_i$ is the difference in final and initial free energy. Hence a transition to a state with lower free energy produces a negative change in free energy. The difference of initial energy E_i and final energy E_f of the tunneling electron has to account for the change in free energy its transition causes. The total tunnel rate from occupied states on one side of the barrier to unoccupied states on the other side of the barrier is given by the expression [11]

$$\Gamma(\Delta F)=\frac{2\pi}{\hbar}\sum_i\sum_j\left|T_{ki,kf}\right|2\left(1-f\left(E_f\right)\right)\delta\left(E_i-E_f-\Delta F\right) \qquad (11)$$

Here, $f(E)$ is the Fermi-Dirac distribution or Fermi function which gives the occupation probability of energy levels. $(1-f(E))$ is therefore the probability of finding an empty state.

$$f(E)=\frac{1}{1+e^{\frac{E-E_f}{k_BT}}} \quad \text{and} \quad 1-f(E)=f(-E)=\frac{1}{1+e^{\frac{-E+E_f}{k_BT}}} \qquad (12)$$

For a typical metal tunnel junction, the barrier consists of a thin oxide with a relatively high barrier height (see Figure 6).

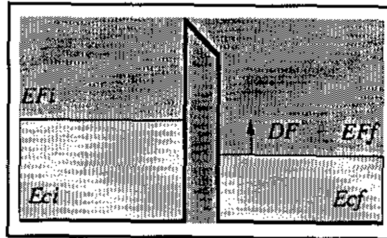

Figure 6: Tunnel barriers.

Kondo Effect

In 1964, Kondo [16] made a startling discovery when considering the scattering from a magnetic ion that interacts with the spins of the conducting electrons. He found that the resistance of a metal increases logarithmically when the temperature is lowered. Hence the name is 'Kondo effect'. The behaviour of the resistance of a metal and a 'quantum dot' are remarkably different, as shown in Figure 7.

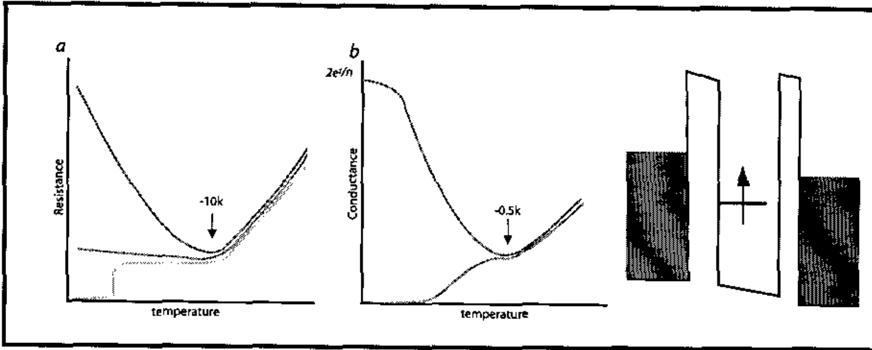

Figure 7: Kondo effect in (a) A metal, and (b) A quantum dot.

Kondo's theory correctly describes the increase in resistance at low temperatures. However, it also makes the unphysical prediction that the resistance will be infinite at even lower temperatures. It turns out that Kondo's result is correct only above a certain temperature, which became known as the Kondo temperature, T_K [16].

The Kondo effect occurs when an impurity atom with an unpaired electron is placed in a metal, and the energy of the unpaired electron is far below the Fermi energy [17,18]. At low temperatures a spin singlet state is formed between the unpaired localized electron and delocalized electrons at the Fermi energy as shown in Figure 8.

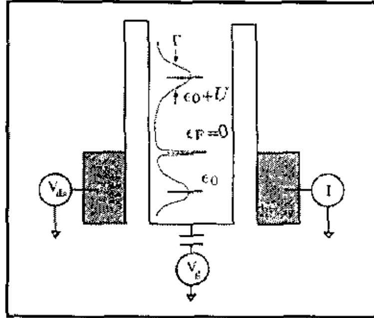

Figure 8: Schematic energy level diagram for SET.

As can been seen in Figure 8, the states below the Fermi energy E_F in the drain and source are filled. A singly occupied level at ε_0 gives rise to the sharp Kondo resonance in the density of states at E_F. The doubly occupied level is higher than ε_0 by U. the tunneling between the localized states and the leads, gives the density of states resulting from each level a width Γ [19].

The confined droplet of electrons interacting with the leads of a single electron transistor (SET) is closely analogous to an impurity atom interacting with the delocalized electrons in a metal [20]. Goldhaber-Gordon et al [18] reported the measurements on a new generation of SET that displays all the aspects of the Kondo problems:

1. The spin singlet forms and causes an enhancement of the zero-bias

conductance when the number of electrons on the artificial atom is odd but not when it is even.
2. The singlet is altered by applying a voltage or magnetic field or by increasing the temperature, all in ways that agree with predictions [21].

When the channel of a transistor is made very small and is isolated from its leads by tunnel barriers, it behaves in an unusual way. A transistor can be thought of as an electronic switch that is on when it conducts current, and off when it does not. Whereas a conventional field-effect transistor (FET), such as one in a computer memory, turns on only once when electrons are added to it, the small transistor turns on and off again every time a single electron is added to it [22,23]. This increased functionality may eventually make these single electron transistors (SET) technologically important. The unusual behavior of SET is a manifestation of the quantization of charge and energy caused by the confinement of the droplet of electrons in the small channel. Since similar quantization occurs when electrons are confined in an atom, the small droplet of electrons is often called an artificial atom [24,25]. Indeed, the confined droplet of electrons interacting with the leads of an SET is closely analogous to an impurity atom interacting with the delocalized electrons in a metal [20].

Several theoretical papers [17-19] have predicted that a Kondo singlet could form in an SET, which would make it possible to study aspects of the Kondo phenomenon inaccessible in conventional systems:

1. With an SET the number of electrons on the artificial atom can be changed from odd to even.
2. The difference in energy between the localized state and the Fermi energy can be tuned.
3. The coupling to the leads can be adjusted.
4. The voltage differences can be applied revealing non-equilibrium Kondo phenomena [21]; and
5. A single localized state can be studied rather than a statistical distribution of many impurity states.

However, for SET fabricated previously, the binding energy of the spin singlet has been too small to observe Kondo phenomena.

MODELING AND SIMULATION OF SINGLE-ELECTRON TRANSISTOR

SET has been considered as one of the candidates for future high-density, high speed and low power circuit applications [26]. Due to the few electrons transport nature and completely different electrical characteristics of SET, device modeling becomes quite important and necessary for understanding their characteristics and practical applications [26]. A conventional SET circuit symbol is given in Figure 9, which reflects the general structure of a SET, i.e. a small island is sandwiched between two tunneling junctions.

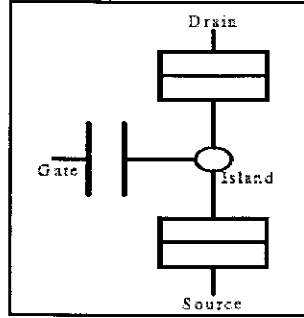

Figure 9: The conventional circuit symbol for SET.

Three different approaches have been used for SET modeling are Monte-Carlo method, macro modeling, and analytical modeling [26]. Although it is recognized that Monte-Carlo method can give the most accurate results in SET characteristics, it is time-consuming and not suitable for mixed circuits (i.e. circuits composed of both conventional MOSFET and SET) applications. On the other hand, less computation time is needed for both the macro-models and analytical models. While analytical model gives a direct insight into the tunneling probability of single electron transistors, macro model is compact and friendly to those circuit designers who are not familiar with device physics of single electron transistors or quantum physics. In this section, we discussed a macro modeling, analytical modeling, Monte Carlo method, and SIMON for a single-electron transistor (SET) circuits modeling and simulation.

The Macro Model

As in the case of the conventional CMOS circuit design; the modeling of devices and the simulation of the circuit would be a key step to design the SET circuits. In the case of the conventional circuit, the compact simulators such as SPICE are used to simulate the characteristics of the given circuit topology.

In these simulators, two assumptions are implicitly used to build the model [27]. The first assumption is that once the parameter of the isolated transistor is determined from the device simulator or other modeling tools, it can be used in the whole circuit. When the model parameters are determined from the device simulator, the current–voltage (I-V) characteristic of the device is only a function of the device width. The second assumption is that the I-V characteristics of the device are affected by neighbouring transistors only through the changes of the terminal voltages of those transistors. The interaction between adjacent devices is usually neglected. In the case of the circuits with SET, it is known that the second assumption may not be valid. The terminal currents of the SET are determined from the average charge state of the Coulomb island of the SET. When several SET is connected, the charge state of the Coulomb island of one SET is strongly affected by the charge states of neighboring islands of other SET. Therefore, the terminal currents of the SET in the circuit may be different from those of the isolated SET even at the same bias condition.

For the application of full conventional simulation techniques to single-electron circuits, parameter-based compact modeling is essential for the characteristics of isolated SET rather than the characteristics obtained from the Monte Carlo method. In this section, a macro modeling for SET, which is fully compatible to SPICE, is introduced by Yu et al [27-28] as shown in Figure 10.

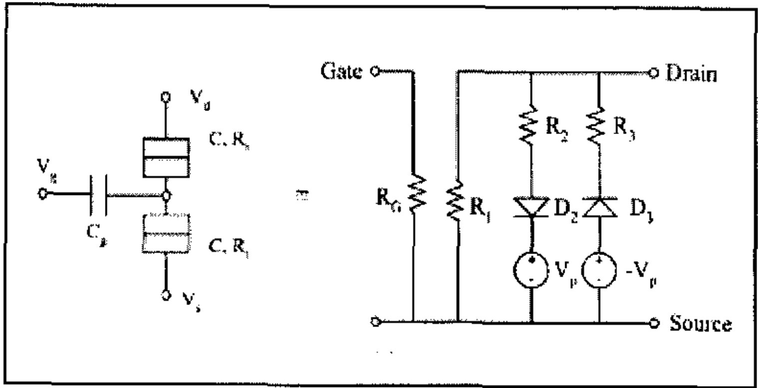

Figure 10: The proposed macro modeling of SET circuit by Yu et. al.

Figure 10 shows the macro modeling of SET circuit. Symmetric features of the drain-source current–voltage (I–V) characteristics are incorporated with two branches consisting of the combinations of resistors, diodes, and voltage sources. R2/D2/V2 and R3/D3/V3 denotes them, respectively. The directions of D2 and V2 are opposite with those of D3 and V3 to have adequate current flow in both positive and negative drain-source bias. The charging energy, periodically changing as a function of the gate bias, is included in R1, R2, and R3 where the cosine of the gate bias is used. They are expressed as follows [27-28] :

$$R_1(V_G) = CR1 + CR2\cos(CF1 \cdot V_G) \tag{13}$$

and

$$R2(V_g, V_{ds}) = R3(Vgs, Vds) = \frac{CVp}{CI2 - 2CVp / R1(Vgs, Vds)} \tag{14}$$

The parameters, *CF1, CVp,CI2, CR1* and *CR2* are used to fit the characteristics at various gate biases.

However, in this model that proposed by Yu et al [27-28], the base line of I_{ds} increases with the gate-source voltage V_{gs} in the I_{ds}-V_{gs} characteristics due to the leakage current flows between the gate and the source of the SET. Besides, the parametric equations given in Yu's model cannot describe accurately the exponential nature in Coulomb blockade region in I_{ds}-V_{ds} characteristic. Therefore, Wu and Lin [26] proposed an improved compact macro model for SET as is depicted in Figure 11.

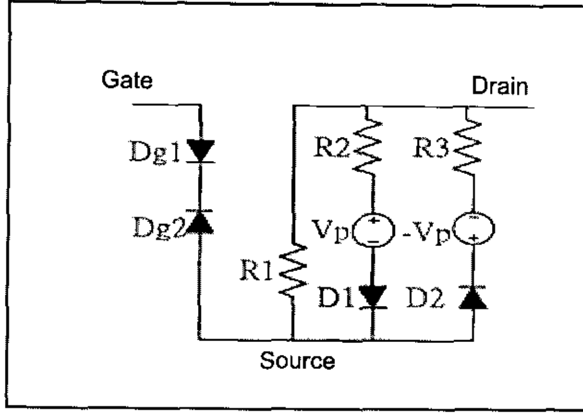

Figure 11: The proposed macro-model for SET circuit by Wu and Lin.

As can be seen in Figure 11, two face-to-face ideal diodes, D_{g1} and D_{g2}, rather than a large resistor are used in the proposed model to block all possible current flows from the gate to the source caused by the application of gate-source voltage V_{gs}. Two branches of combinations of resistors, diodes and voltage-sources are included in the model, in which R1 is the primary resistor in the Coulomb blockade region, R2 and R3 are resistors of the SET in the non-Coulomb blockade region when the drain-source voltage V_{ds} exceeds a certain value in the positive and negative directions, respectively. The voltage source V_p connected in each branch of R2 and R3 defines the boundary between the Coulomb blockade and non-Coulomb blockade region. They also modified the parametric equations as given below to comply with the exponential I_{ds} behavior in Coulomb blockade region in I_{ds}-V_{ds} characteristic [26] :

$$R1(V_g, V_{ds}) = CR1 + CR2[\cos(\pi \times (CF1 \times V_{gs} + CF2 \times V_{ds}) + 1] \times 2^{|CV_p - V_{ds}|x} \tag{15}$$

and

$$R2(V_g, V_{ds}) = R3(Vgs, Vds) = \frac{CVp}{CI2 - 2CVp / R1(Vgs, Vds)} \tag{16}$$

where $CF1 = \frac{C_g}{e}$ and $CF2 = \frac{C_d}{e}$

Here, *CF1* and *CF2* is the parameter that determines the frequency of Coulomb oscillation in I_{ds}-V_{gs} characteristics and equals to $2C_g/e$ and $2C_d/e$; C_g is the normal capacitance of the gate, C_d is the normal capacitance of the drain, and *e* is the electronic charge. The parameter *CR2* gives the amplitude and the parameter *CR1* gives the base line of the Coulomb oscillation. The parameter V_p is used to define the range of Coulomb blockade region.

The Analytical Model

Various analytical models have been proposed for SET [29-30]. Among them, the one used the concept of free energy of the tunneling junctions was first proposed by Wasshuber [9] and involves the most physical meanings of SET. However, in his model electron tunneling across only one tunneling junction is considered at a time, which does not reflect the real operating situation of SET.

Therefore, Uchida et al [30] derive an analytical model of SET, which have two tunneling junctions and an additional capacitor connected to a conductive center island as shown in Figure 12.

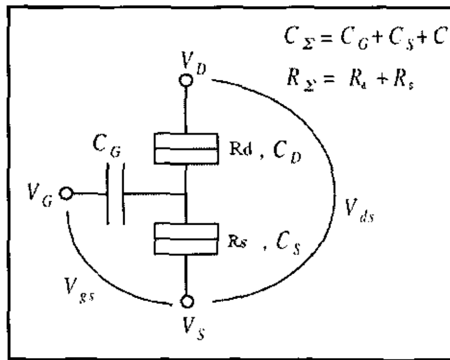

Figure 12: *Schematic of the single-electron transistor. The tunneling resistance of the source is assumed to be same as that of the drain.*

For deriving the analytical SET model, they make three assumptions are as follows. Firstly, all the source and drain terminals of SET are connected to the capacitors whose capacitance is much larger than the total capacitance of the SET island or biased by constant-voltage sources. This assures that the SET characteristics are affected by neighboring circuit components only through the node voltages of SET terminals [30]. Secondly, the source and drain resistances are assumed to be the same. Although this assumption puts a small restriction on the SET structures, it not only greatly reduces the calculation task but also renders the final formula simple and concise. Because only a few circuits utilizing a resistance mismatch have been proposed so far, it is considered that this restriction will not affect the usefulness of this model. Thirdly, at each given gate voltages, the two most probable numbers of the electrons in the SET island are taken into account. For example, when the gate voltage is 0 (e/C_G) and the drain voltage is positive, they consider that the number of electrons in the SET island is 0 or -1 (1 or 0). Besides these assumptions, the tunneling resistance is supposed to be much larger than the quantum of resistance, $h/e^2 \sim 25.8$ kΩ, in order to suppress tunneling by quantum fluctuation and to maintain the accuracy of the 'orthodox' theory [1].

On the other hand, they derive the analytical model of a double-junction SET on the basis of the 'orthodox' theory and the steady-state master equation method. Based on the assumptions described above, the steady-state master equations can be accurately calculated, and thus, the I-V characteristics of SET having n or n+1 electrons in its island is given by [30]

$$I_n = \frac{e}{2R_\Sigma C_\Sigma} \frac{(V_{gs,n}{}^2 - V_{ds,n}{}^2)\sinh(V_{ds}/T)}{V_{gs,n}\sinh(V_{gs,n}/T) - V_{ds,n}\sinh(V_{ds}/T)} \quad (17)$$

where

$$V_{gs,n} = \frac{2C_G V_{gs}}{e} - \frac{(C_G + C_S - C_D)V_{ds}}{e} - 1 - 2n \quad (18)$$

$$Vds = \frac{C_\Sigma V_{ds}}{e} \quad (19)$$

$$T = \frac{2k_B T C_\Sigma}{e^2} \quad (20)$$

and $R_\Sigma = R_d + R_s$. It should be noted that the dependence of I_n on $V_{gs,n}$ takes a symmetrical hump shape having a maximum at $V_{gs,n} = 0$. It is, therefore, reasonable to consider that I_n corresponds to one period of Coulomb oscillations. Consequently, by incorporating $V_{gs,n} = 0$ into equation (2), the gate voltage giving the peak of Coulomb oscillations as

$$V_{gs} = \frac{e}{2C_G} + \frac{ne}{C_G} + \frac{(C_G - C_S - C_D)V_{ds}}{2C_G} \quad (21)$$

Considering the period of Coulomb oscillations, e/C_G, the gate voltage range, for which the model current I_n holds, as

$$\frac{ne}{C_G} + \frac{(C_G - C_S - C_D)V_{ds}}{2C_G} \langle V_{gs} \langle \frac{(n+1)e}{C_G} + \frac{(C_G + C_S - C_D)V_{ds}}{2C_G} \quad (22)$$

which is equivalent to $-1 < V_{gs,n} < 1$.

In addition, the present SET model can be easily implemented in conventional circuit simulators, such as the simulation program with integrated circuit emphasis (SPICE), as a small circuit comprising voltage-controlled current-sources and capacitors. This implementation allows us to simulate circuits including SET using a conventional circuit simulator alone. However, it should be noted that the circuits should satisfy the condition that all the source and drain terminals of SET are connected to large capacitors whose capacitance is much larger than the total capacitance of the SET island.

Monte Carlo Method

A Monte Carlo has widely been used for single-electron circuit simulation. It simulates the stochastic nature of charge tunneling utilizing random numbers. The Monte Carlo approach starts with all possible tunnel events, calculates their probabilities, and chooses one of the possible events randomly, weighted according to their probabilities [31]. This is done many times to simulate the transport of electrons through the network. Tunnel events are considered to be independent and exponentially distributed. It is handy for transient simulation and crude dc simulation [32]. The simulation procedure is as follows. Given a tunneling rate Γ for a tunneling event, the probability that the tunneling will occur in t is

$$R(t) = 1 - \exp(-\Gamma t) \tag{23}$$

Therefore, time to the tunneling event is determined as

$$\Delta t = -\frac{1}{\Gamma}\ln(1-r) \tag{24}$$

where Γ is the tunnel rate and r is a random number uniformly distributed over the interval $0 \le r < 1$. Δt is calculated for each tunneling event starting from the present state. The tunneling event with the shortest Δt of all (shorter than the simulation time step t_{step}) is chosen as the one actually occurs. This procedure is followed repeatedly.

In the Monte Carlo simulation, it is assumed that tunneling occurs instantaneously, which means that tunneling current cannot be defined in transient simulation [32]. Additionally, a Monte Carlo method is superior to other approaches because of the following advantages [31]:

- It gives better transient and dynamic characteristics of SET circuits because it models the underlying microscopic physics in a very direct manner.
- It is not required to find the relevant states before one can start with the actual simulation.
- It is easy to trade accuracy with simulation time, and therefore one can quickly achieve approximate results of very large circuits.

Nevertheless, there is one major disadvantage of the Monte Carlo method. When it comes to simulating co-tunneling, a plain Monte Carlo approach has its limitations. Co-tunneling is a very rare process that is difficult to resolve by a Monte Carlo method and it demands very long simulation times [31]. Another drawback of the Monte Carlo method is the slow convergence of the averaging calculation. I-V curves obtained by the Monte Carlo method are in general not smooth [32]. Therefore, the Monte Carlo method is not the choice if accuracy is required.

Simulation of Nanostructures Method (SIMON)

The single-electron circuit simulators such as SIMON have procedures to calculate the charge states of all the Coulomb islands altogether to take into account of the interaction between neighboring Coulomb islands [27]. These procedures are usually based on the Monte Carlo technique and require a huge amount of computation time because the Monte Carlo method requires the calculation of the average charge states in each step. It features the arbitrary connection of tunnel junctions, capacitors, constant voltage sources, piece-wise-linear time-dependent voltage sources and voltage controlled voltage sources. Furthermore SIMON features two simulation modes are a transient mode and a quasi-stationary mode [33].

Wasshuber et al [33] make two assumptions of SIMON are as follows. The first significant assumption is that voltage sources are considered to have no internal resistance. Consequently, charging and discharging of capacitances occurs instantaneously. The second assumption is that the simulator treats electrons as point charges that hope from island to island via tunnel junctions. In other words, the electron states must be localized. This requires all tunnel resistances R_T to be much larger than the quantum resistance R_q is given by

$$R_T \rangle\rangle R_q = \frac{h}{q_e^2} \approx 25.8k\Omega \tag{25}$$

Here h is Planck's constant and q_e is the elementary charge.

The Coulomb blockade is a manifestation of the tunnel rate dependence on the change in free energy. Free energy is the difference of electrostatic energy U stored in the network and work done by voltage sources W [31,33].

$$F = U - W \tag{26}$$

Every time an electron tunnels, the state of the circuit changes. The state of the circuit is determined by the set of all node charges and node voltages. A change in state causes a change in free energy. Since the circuit will tend to a lower energy state, events that reduce the free energy will be more likely. Once the change in free energy for all possible tunnel events is determined, the tunnel rates can be calculated by [32]

$$\Gamma = \frac{\Delta F}{q_e^2 R_T \left(1 - \exp\left(-\frac{\Delta F}{k_B T}\right)\right)} \tag{27}$$

With this information the Monte Carlo part of the simulator is entered. Here ΔF is the change in free energy, k_B is the Boltzmann constant, T is the absolute temperature, R_T is the tunnel resistance and q_e the elementary charge. Tunneling is modeled as a

Poisson process [33]. From the probability distribution of the Poisson process one can deduce the following formula [33]

$$\Delta t = -\frac{\ln r}{\Gamma(T, R_T, \Delta F)} \tag{28}$$

Here r is an evenly distributed random number in the interval [0; 1]; $\Gamma(T; R_T; \Delta F)$ is the tunnel rate depending on temperature T , the tunnel resistance is R_T and the *change in free energy* is ΔF. *The winner is the tunnel process with the shortest time* Δt. This calculation of tunnel rates, time intervals and winners is done many times to either simulates the transient behavior of the network or to determine quasi-stationary characteristics by averaging over many events.

CONCLUSION

A SET is a three-terminal device whereby electrons are transferred one-by-one using Coulomb blockade effects. Modeling and simulation of SET are very important to understand behavior and characteristic of electron before start designing and fabricating the devices. Thus, in this research the macro model, analytical model, and Monte Carlo method are reviewed for further investigation.

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