SHEAR DESIGN OF WIDE BEAM RIBBED SLABS

(Date received:16.2.09/Date accepted:1.3.11)

Lau Teck Leong[1] and L A Clark OBE[2]
[1]Assistant Professor, Department of Civil Engineering, University of Nottingham
43500 Jalan Broga, Semenyih, Selangor
[2]Pro-Vice-Chancellor, The University of Birmingham,
E-mail: [1]teckleong.Lau@nottingham.edu.my

ABSTRACT

A method for the shear design of wide beam ribbed slabs is proposed. The method modifies the current UK code design method for solid slabs by applying a shear area factor which reduces the area of the code critical shear perimeter to take account of the loss of shear area from that of a solid slab. The proposed method gives good agreement with test data for internal column situations, and underestimates the strength at edge columns. The conservativeness in relation to edge columns arises because of an empirical assumption made in the basic code method for solid slabs and is not due to the modification that it is proposed for wide beam ribbed slabs.

Keywords: *Punching Shear, Ribbed Slab, Shear Transfer Mechanism*

1.0 INTRODUCTION

Reinforced concrete flat slabs have been widely used in building construction since the early twentieth century. However, in recent years, wide beam ribbed slabs have become increasingly popular owing to their economic benefits. As can be seen from Figure 1, a wide beam ribbed slab consists of major wide beams that are much wider than the supporting columns, spanning in the two orthogonal directions, and ribs spanning between the beams in only one direction.

A flat slab can develop a type of local shear failure at the column or under a concentrated load, which is known as a "punching shear failure" (see Figure 2). At failure, a solid revolution of concrete (marked as 'I'), which is the portion of concrete surrounded by the inclined shear cracks, separates normally from the slab leaving the rest of the slab (marked as 'II') remaining rigid. The punching resistance of the slab is the sum of all the shear strength on the shear failure surface.

Since the introduction of flat slab structures, an extensive amount of research has been undertaken to aid the understanding of punching at columns [1]. However despite the increasing popularity of wide beam ribbed slabs, the current understanding about their punching behaviour derives from that of solid flat slabs. Only a small amount of work has been carried out on waffle slabs [2, 3,] and the only advice on the shear design of wide beam ribbed slabs is that of Simpson [4].

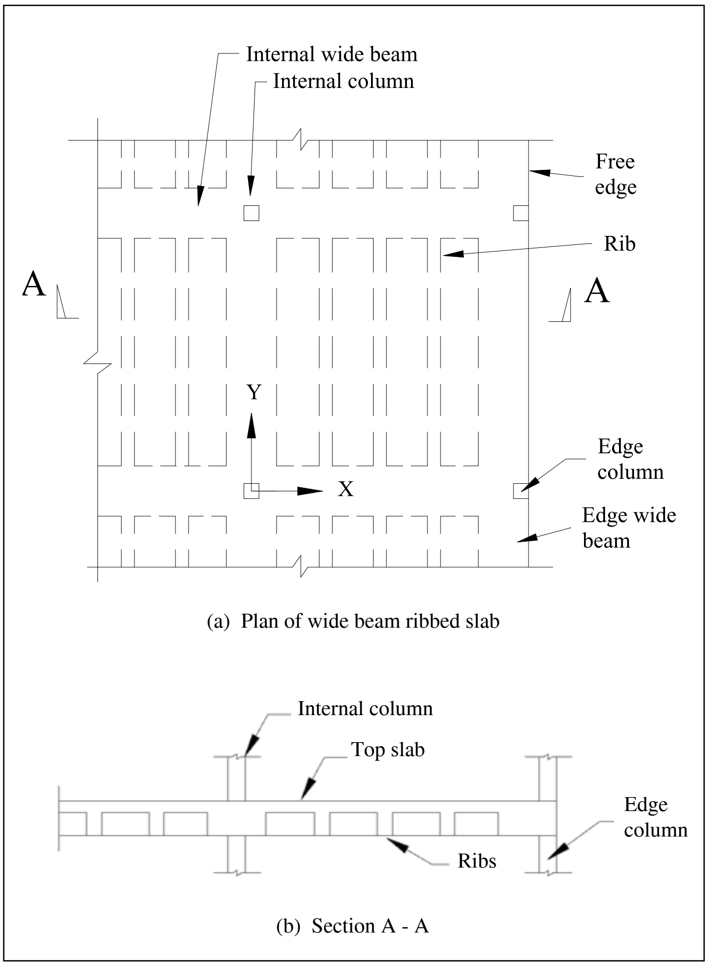


Figure 1: Wide beam ribbed slab

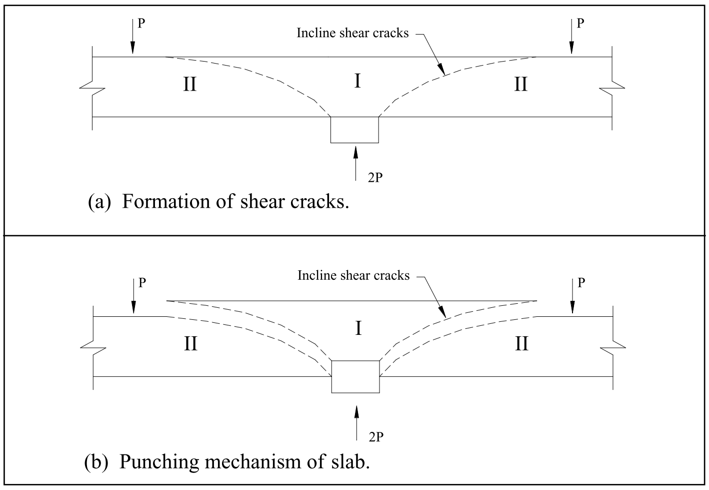


Figure 2: Punching shear mechanism

It is apparent that the shear design procedures for wide beam ribbed slabs are not covered adequately in the current UK design code, BS8110[5]. In particular, it is not clear how to apply the code clauses for solid slabs to wide beam ribbed slabs, because, when the wide beams are very wide, the punching failure surface could form within the full depth section [see Figure 3(a)], but if the beams are narrower, the punching failure surface could pass through the reduced depth section [see Figure 3(b)]. As a result, a smaller shear failure surface could be mobilised, which, consequently, would lead to a lower punching shear capacity.

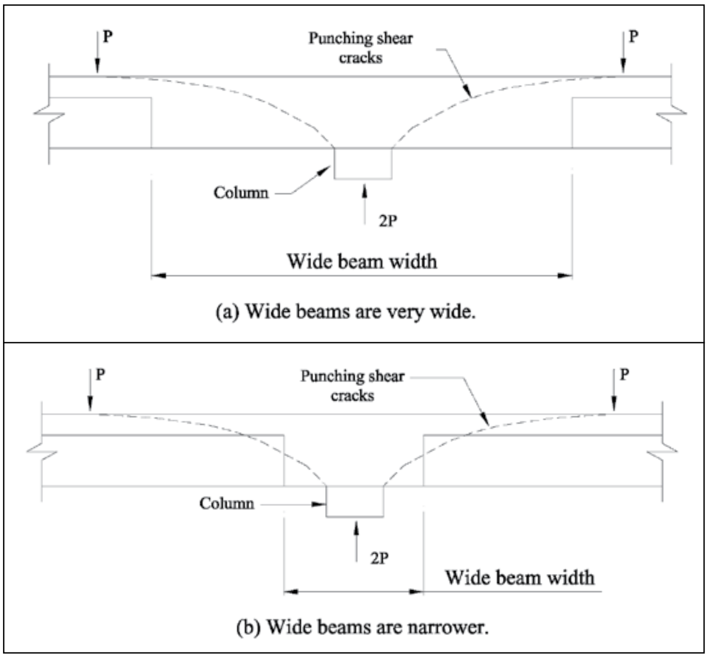


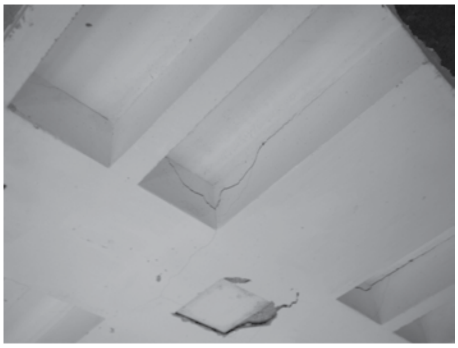
Figure 3: Punching shear failure mechanism of wide beam ribbed slab.

In previous publications [6-8], the Authors presented the results of tests on micro-concrete models of wide beam ribbed slabs with either an internal or an edge column. They also presented theoretical models for the shear capacity of wide beam ribbed slabs at both internal and edge columns based on plasticity theory and achieved good agreement with the test data. In the current paper, design models based on the current UK code, BS8110[5] are presented.

Test Results

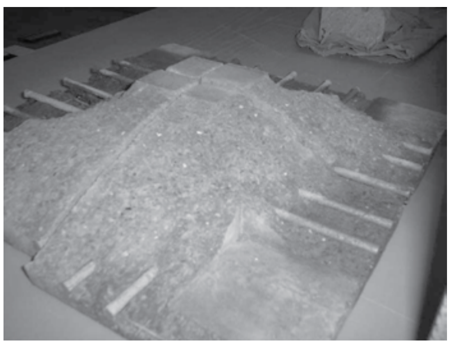
The tests were conducted in three series: IRS with an internal column and four equal point loads on the four beams framing into the column; ISS with an internal column and different loads on the four beams framing into the column; and ERS with an edge column. Details of the test specimens and procedures are given in References [6-8].

Each of the slabs in Series IRS and ERS failed suddenly by punching shear in a mode very similar to the punching mechanism observed in solid slabs with the failure surface inclined at about 22° to the horizontal and intersecting the top surface at about 2.5 times the overall slab thickness from the column. However, unlike a solid slab, when the wide beam width was less than five times the overall slab depth, the failure surface was an incomplete surface of revolution because some of the potential failure surface was lost when it entered the reduced section, as shown in Figures 4 and 5 for an internal and edge column, respectively.



(a) Loss of failure surface

(b) Punching failure surface



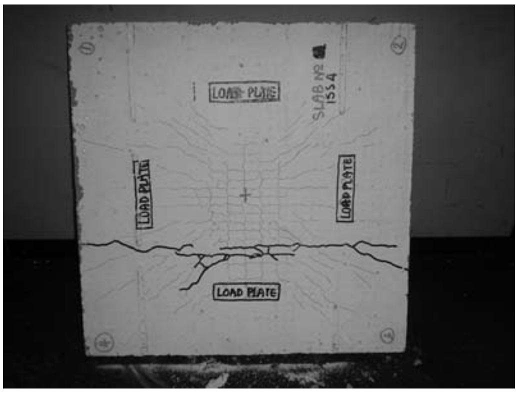
(c) Punched out piece

Figure 4: Internal column punching failure mechanism

(a) Inclination of internal cracks *(b) Loss of failure surface*

Figure 5: Edge column punching failure mechanism

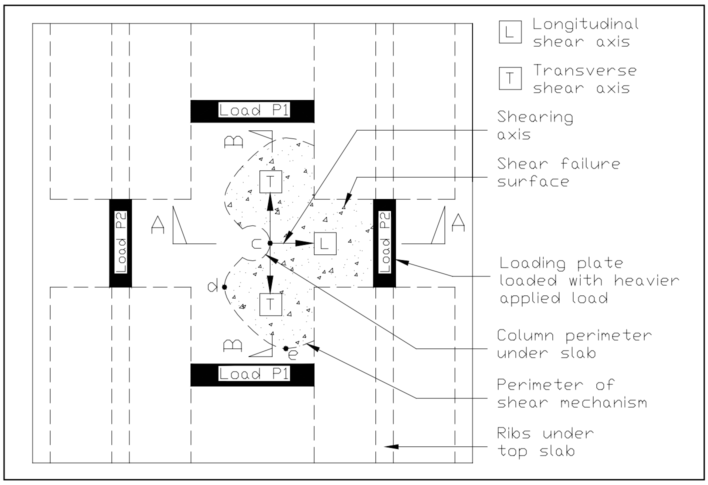
Each of the slabs in Series ISS failed by wide beam shear as shown in Figure 6 and diagrammatically in Figure 7. The longitudinal internal cracks within the column width propagated from the column vicinity through the slab thickness at about 22º inclination towards the load. Beyond the column width (moving away from the column in the transverse direction) these longitudinal shear cracks lost their longitudinal restraint from the column and extended behind the column. In the transverse direction, at the column side vicinities, shear cracks also formed to enable a collapse mechanism to form. These cracks occurred because the column was located at the centre of a solid concrete region that was much wider than the column. In general these shear cracks were found to propagate away from the column in the transverse direction and reached peak height, point 'd' in Figure 7(c), at the edges of the longitudinal wide-beam, which coincided with its peak length, point 'd' in Figure 7(a), in the longitudinal direction. These cracks then gradually reduced in height and intersected the bottom surface, point 'e' in Figure 7(c), at a distance (in the transverse direction) of about 2.5 times the overall slab depth away from the column side faces.



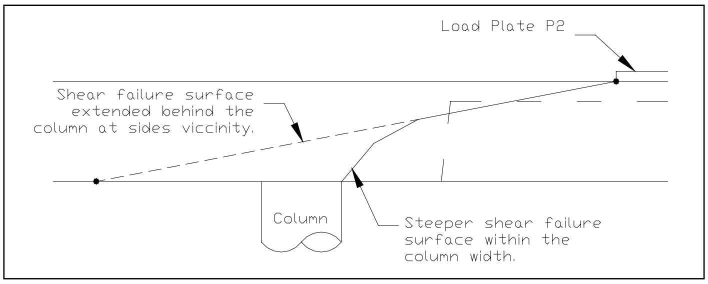
(a) Top view

(b) Bottom view

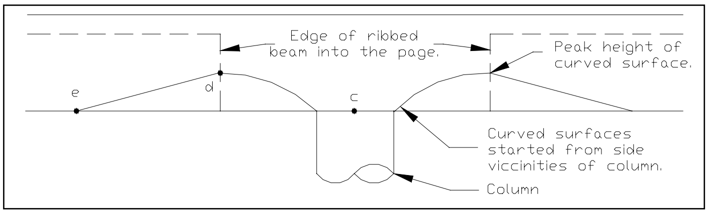
Figure 6: Wide beam shear failure mechanism



(a) Plan view



(b) Section A-A



(c) Section B-B

Figure 7: Wide beam shear failure

The failure loads (V_{test}) are given in Tables 1 to 3 for Series IRS, ISS and ERS, respectively. These results are discussed fully in references [6-8].

Design Models

General

The proposed design models are based on the current UK code, BS8110[5]. The shear strength used in these models is that in BS8110[5]. However, the partial safety factor is set to unity, and the concrete cube strength is not restricted to be below 40 N/mm^2 as in BS 8110[5]. In fact, Gardner[9] and Concrete Society Technical Report R49[10] indicate that the equation can be used for cube strengths of up to 70 N/mm^2 for slabs. The shear strength (v_c) is therefore defined as:

$$v_c = 0.79 \left(\frac{100A_s}{bd}\right)^{\frac{1}{3}} \left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}} \left(\frac{400}{d}\right)^{\frac{1}{4}} \qquad (1)$$

Where A_s = area of longitudinal steel crossing shear plane
b = width of shear plane
d = effective depth
f_{cu} = concrete compressive cube strength

In developing the plasticity theory prediction models presented [6-8] it was found necessary to include a shear retention factor, α, when calculating the strength of the micro-concrete models to allow for the reduced post-peak shear resistance which is apparent when a small aggregate size such as the 2.36 mm used in the micro-concrete model tests referred to in this paper. The value of a of 0.7 adopted for micro-concrete is based on the test data of Boswell and Wong[11-12] and its application to the wide beam ribbed slab tests is discussed fully in [6-8]. It is emphasised that the shear strength reduction factor of 0.7 has to be applied only for comparison with the micro-concrete model test data and does not have to be applied when designing full-size slabs.

Internal Punching Mechanism

The proposed design model takes the section variations of a ribbed slab into account while predicting the punching capacity. The model adopts the shear perimeter as that in BS8110[5], at 1.5d from the column faces. Depending on the wide beams' width and the top slab's thickness the shear surface area to form a punching mechanism for a ribbed slab could be less than that in a flat slab.

An effective shear area factor, γ, is therefore introduced to simulate the loss of shear area and which is the ratio of the projected section area A_{DEF} to A_{ABC} as indicated in the following equations (see Figure 8):

$$\gamma_X = 1 - \frac{a_{X2}\, d_2}{a_X\, d} \qquad (2)$$

$$\gamma_Y = 1 - \frac{a_{Y2}\, d_2}{a_Y\, d} \qquad (3)$$

where, a_x = shear span in x direction which is $\leq 2.6d$.
a_y = shear span in y direction which is $\leq 2.6d$.
a_{x2}, a_{y2}, d_2 = as defined in Figure 8.
d = overall slab section effective depth.

The critical shear area within the column width is not to be altered by this factor. The reason for the limit of 2.6d on the shear spans is that, for an inclined shear surface, the optimum angle of inclination for the plasticity theory model[7, 8] against which the modified BS8110 design model was calibrated is 1 in 2.6. The implication is that if the shear span between the column and load is less than 2.6 times the slab thickness the shear crack extends to the load, but if the shear span is more than 2.6 times the slab thickness the shear crack intersects the slab top surface at 2.6 times the slab thickness from the column face. However, since the BS8110 shear perimeter is expressed in terms of effective depth rather than overall depth, the limit on shear span has also been expressed in term of effective depth.

The effective perimeter of the critical shear area for punching at an internal column is:

$$u = 2(c_X + 3d\,\gamma_X) + 2\,(c_Y + 3d\,\gamma_Y) \qquad (4)$$

where, c_x, c_y = column sizes.
γ_y, γ_y = effective shear area factor in x and y direction.
d = overall slab section effective depth.

The internal column punching capacity of a wide beam ribbed slab is predicted using the following equation.

$$V = \alpha\, v_c\, u\, d \qquad (5)$$

where, α = concrete shear factor, 0.70 for micro concrete and 1.0 for normal concrete.
v_c = concrete shear strength as in Equation 1.
d = overall slab section effective depth.

The predicted punching capacity from this equation decreases as the effective shear area decreases. However, if the wide beams' width or the top slab's thickness were sufficient, the effective shear area factor will increase to unity and this model would become identical to the model for a flat slab in BS 8110[5].

Edge Punching Mechanism

The proposed design model also adopts the shear perimeter as that in BS8110[5]. The effective shear area factors are also calculated [Equations 2 and 3], but using the various dimensions from Figure 9.

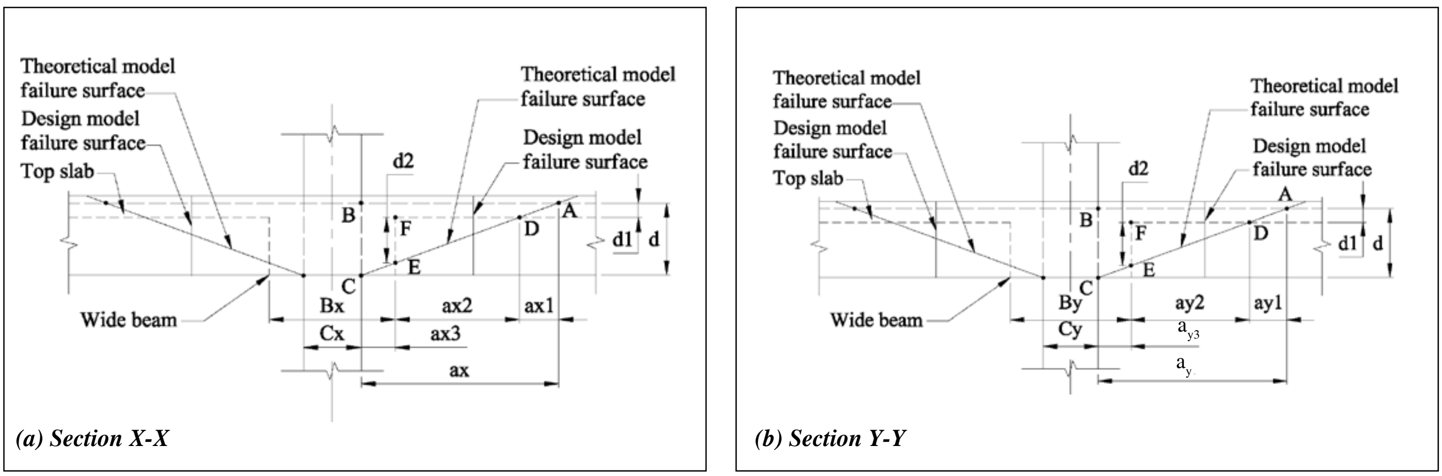
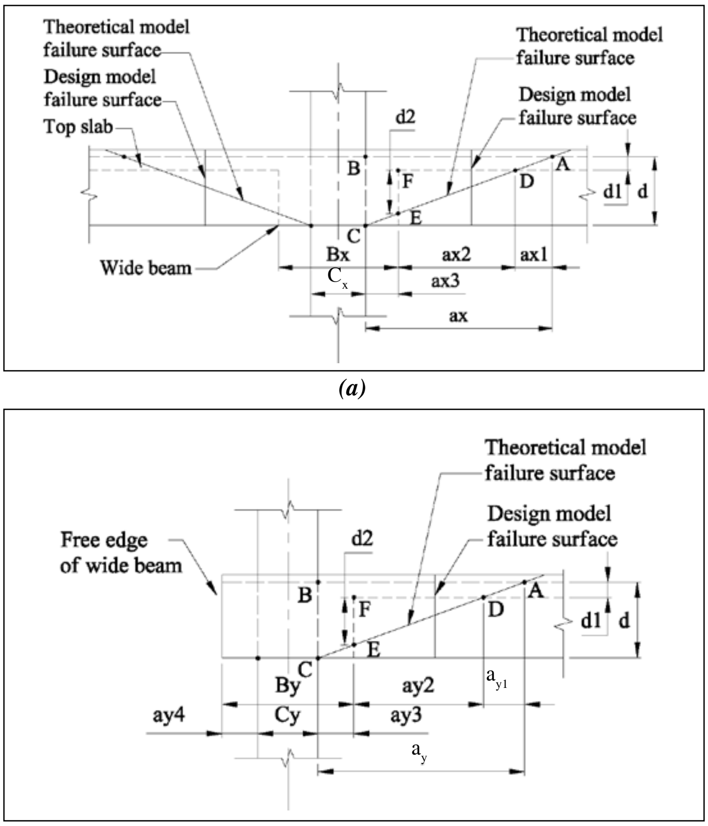


Figure 8: Design model for internal column punching shear area



(a)

(b)

Figure 9: Design model for edge column punching shear area

It is assumed that the critical shear area within the column width is not to be altered by this factor. The effective perimeter of the critical shear area for punching at an edge column is:

$$u\,d = (c_X + 3d\gamma_X)\,d + 2\,(c_Y + 1.5d\gamma_Y + a_{Y4})d \quad (6)$$

where, c_x, c_y = column sizes.
a_{Y4} = edge beam width as defined in Figure 9.
γ_X, γ_y = effective shear area factor in x and y direction.
d = overall slab section effective depth.

The edge column punching capacity of a wide beam ribbed slab is predicted using the following equation.

$$V = \frac{1}{1.25}\,\alpha\, v_c\, u\, d \quad (7)$$

where, α = concrete shear factor, 0.70 for micro concrete and 1.0 for normal concrete.
v_c = concrete shear strength as in Equation 1.
d = overall slab section effective depth.

The reason for the inclusion of the factor 1.25 is that, to allow for the effect of moment transfer perpendicular to the slab edge, BS8110[5] effectively reduces the shear capacity by a factor of 1.25.

In general, the predicted punching capacity from this model decreases as the effective shear area decreases. However, if the wide beams' width or the top slab's thickness were sufficient, the effective shear area factor will increases to unity and model would become identical to the model for flat slab in BS 8110.

Wide Beam Mechanism

The failure mechanism of the proposed model is shown in Figure 10. The shear perimeter of the model is as shown in the figure from A to D, and its critical shear area is defined in the following equation.

$$u\,d = 2\,d\,\sqrt{(a^2_{XI} + b^2_{XI})} + d\,c_X \quad (8)$$

where, a_{x1} = shear span.
b_{x1} = width as defined in Figure 10.
c_x = column width.
d = section effective depth

The shear capacity is obtained from Equation 5.

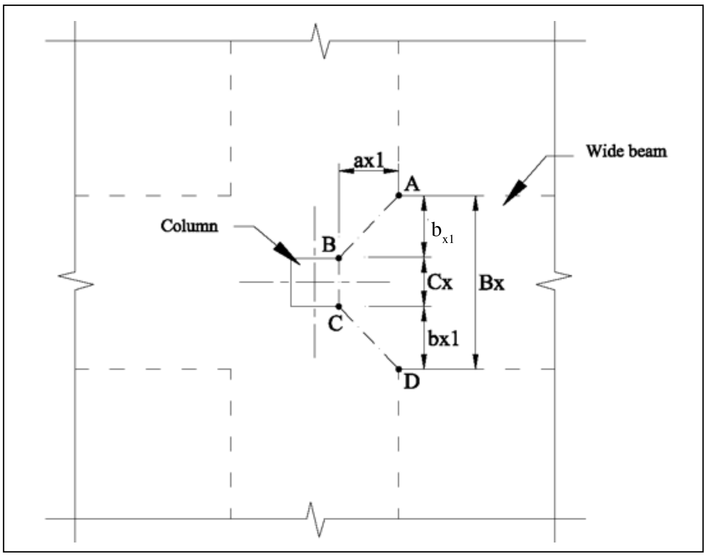


Figure 10: Design model for wide beam shear area

Comparisons with test results

Comparisons of the shear capacities predicted by the modified BS8110 design method (V_{BS}) with the test results (V_t) are presented in Tables 1 to 3 for Series IRS, ISS and ERS, respectively. The design method used is that for internal punching, wide beam and edge punching, respectively. For completeness the strengths (V_p) predicted by the plastic theories of [7,8] and the ratios of the test results to the strengths predicted by the plastic theories are also shown. The mean ratios of test shear strength to plastic theory prediction were 1.01, 1.13 and 0.99 for Series IRS, ISS and ERS, respectively. In contrast the mean ratios of test shear strength to the modified BS8110 design method prediction are 1.24, 1.23 and 1.86, respectively.

Table 1: Internal punching shear predictions for Series IRS specimens

Slab	V_t (kN)	V_p (kN)	V_{BS} (kN)	$1.25V_{BS}$ (kN)	V_t / V_p	V_t / V_{BS}	$V_t / 1.25V_{BS}$
IRS 1A	48	49	42.20	52.75	0.96	1.14	0.91
IRS 2A	64	61	47.83	59.79	1.05	1.34	1.07
IRS 3A	52	52	44.15	55.19	0.98	1.18	0.94
IRS 4A	64	58	46.79	58.48	1.10	1.37	1.09
IRS 5A	56	62	49.53	61.91	0.90	1.13	0.91
IRS 6A	72	71	52.92	66.16	1.01	1.36	1.09
IRS 7A	44	48	41.47	51.83	0.92	1.06	0.85
IRS 8A	58	57	46.75	58.43	1.02	1.24	0.99
IRS 1B	62	62	52.56	65.70	1.00	1.18	0.94
IRS 2A	74	73	57.53	71.91	1.14	1.29	1.03
IRS 1C	64	63	55.42	69.28	1.02	1.55	0.92
IRS 2C	76	74	60.88	76.11	1.03	1.25	0.99
				Mean	1.01	1.24	0.99
				Standard Deviation	0.07	0.11	0.09

Table 2: Wide beam shear predictions for Series ISS specimens

Slab	V_t (kN)	V_p (kN)	V_{BS} (kN)	$1.25V_{BS}$ (kN)	V_t / V_p	V_t / V_{BS}	$V_t / 1.25V_{BS}$
ISS 1	15.00	14.29	12.88	16.10	1.05	1.17	0.93
ISS 2	16.00	15.73	13.42	16.78	1.02	1.19	0.95
ISS 3	12.00	9.88	11.38	14.23	1.04	1.06	0.84
ISS 4	14.50	14.12	12.74	15.93	1.03	1.14	0.91
ISS 5	15.00	10.33	12.77	15.96	1.25	1.18	0.94
ISS 6	14.50	11.64	12.44	15.55	1.10	1.17	0.93
ISS 7	15.00	9.64	10.59	13.24	1.33	1.42	1.13
ISS 8	19.00	15.41	12.57	15.71	1.23	1.51	1.21
				Mean	1.13	1.23	0.98
				Standard Deviation	0.12	0.15	0.13

Table 3: Edge punching shear predictions for Series ERS specimens

Slab	V_t (kN)	V_p (kN)	V_{BS} (kN)	$1.25V_{BS}$ (kN)	V_t / V_p	V_t / V_{BS}	$V_t / 1.25V_{BS}$
ERS 1A	48.55	49.94	29.30	36.63	0.97	1.70	1.36
ERS 2A	60.69	60.60	29.84	37.30	1.00	2.09	1.67
ERS 3A	69.36	70.10	31.15	38.93	0.99	2.29	1.83
ERS 1B	34.68	34.67	27.74	34.67	1.00	1.28	1.03
ERS 2B	48.55	48.22	29.98	37.48	1.01	1.66	1.33
ERS 3B	64.16	66.12	31.25	39.06	0.97	2.11	1.69
				Mean	0.99	1.86	1.48
				Standard Deviation	0.02	0.37	0.30

It is noticeable that the two ratios for the internal column situation are very close together and are conservative (1.24 and 1.23). It is believed that this discrepancy arose because the depth factor in BS8110 (see the last term of Equation 1) gives a relatively lower enhancement to the shear strength in comparison to the value of the theoretical models[6-8]. In Figure 11 the two depth factors are compared as a function of overall depth. Since the BS8110 depth factor is a function of effective depth (see the last term of Equation 1), it has been assumed that, for comparison purpose, the effective depth is 90% of the overall depth. According to Figure 11, for a 50mm thick slab, as was used in the test specimens under consideration, the difference is as much as 25%. As a result, a 25% increase was introduced to the modified BS8110 predictions for the small scale tests, and subsequently, the mean ratios of test shear strength to the modified BS8110 design method prediction reduced to 0.99, 0.98 and 1.48, respectively. It is emphasised that the 25% enhancement should not be used in practice when designing a full size slab for which the BS8110 depth factor and that adopted in the plastic theory are very similar (refer to Figure 11).

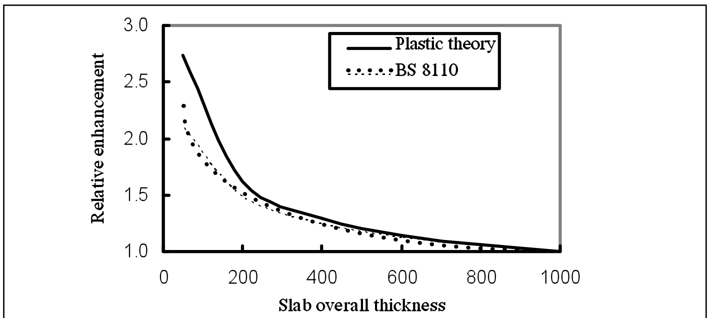


Figure 11: Comparison of depth factors

When the 25 % enhancement factor is applied the agreement between the test results and the modified BS8110 predictions are excellent for the internal column situation with mean ratios of 0.99 and 0.98, and the standard deviations are very similar to those for the plasticity theory models. However, the mean ratio of 1.48 is still very conservative for the edge column situation. It is believed that the reason for this is the manner in which BS8110 takes account of the effect of moment transfer perpendicular to the slab edge by designing for an enhanced shear force of V_{eff} = 1.25 Vt. It is emphasised that this factor of 1.25 is not the same as the 25% enhancement referred to previously in connection with the discussion on depth factors. The factor of 1.25 applied to V_t to give an enhanced design shear force is an approximation for the effect of moment transfer suggested by Regan[13]. However, Regan[13] indicated that the ratio of test to predicted strength when using this approximation is in the range of 1.0 to 1.5. Therefore, it is not surprising to find that the proposed method for an edge column, which is based on BS8110, can be conservative. Another point to note is that for the test data analysed by Regan[13], the ratios of test to predicted strengths were greater for structurally indeterminate specimens, compared to those obtained from isolated single column specimens. This is consistent with the fact that, as pointed out by Long[14], double edge column specimens are stronger than single edge column specimens, primarily due to the fact that single edge column specimens are not capable of permitting the redistribution of moments which can occur in a double edge column specimen. The specimens tested in the current research were similar to double edge column specimen, because the slabs were clamped at the column supported edges[6-8] and hence a conservative prediction is expected for a method based on BS8110. It is again emphasised that the conservatism is inherent in the basic BS8110 approach for solid slabs rather than being a result of the proposed modification for wide beam ribbed slabs.

Comparisons with Theoretical Model and BS 8110

In addition to comparisons with the test results, the modified BS8110 method was also compared with the basic BS 8110 method with no allowance for the material lost from the failure perimeter in a wide beam ribbed slab and with the plasticity theory models over a range of wide beams' width and top slab's thickness. Full comparisons [6], and examples of the comparisons for the internal column punching method are given in Figures 12 and 13 for ranges of ratios of top slab to overall slab thickness and for ranges of ratios of wide beam width to column width, respectively. Figure 12 is for slabs having ratios of wide beam

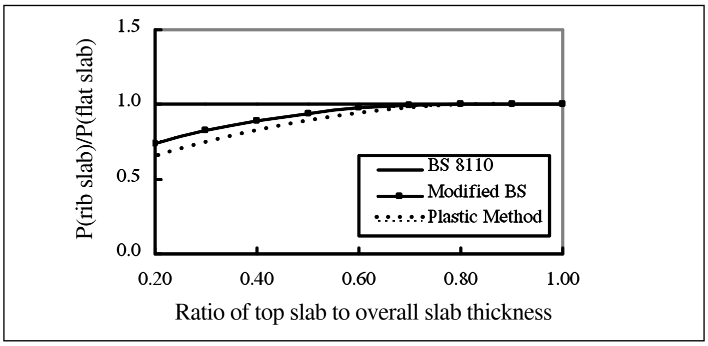


Figure 12: Comparison of prediction methods as top slab thickness varies

to column width, effective depth to overall depth and shear span to effective depth of 1.6, 0.75 and 2.6, respectively. Figure 13 is for slabs having ratios of top slab to overall thickness, effective depth to overall depth and shear span to effective depth of 0.2, 0.75 and 2.6, respectively. For each prediction method the relative strength is presented as the ratio of predicted strength for a wide beam ribbed slab to that for a solid slab. It is apparent that the strength reduction predicted by the proposed modified BS8110 method gives good agreement with that for the plasticity theory model, and that, without the proposed modification, BS8110 would overestimate the capacity significantly for some geometries of slab.

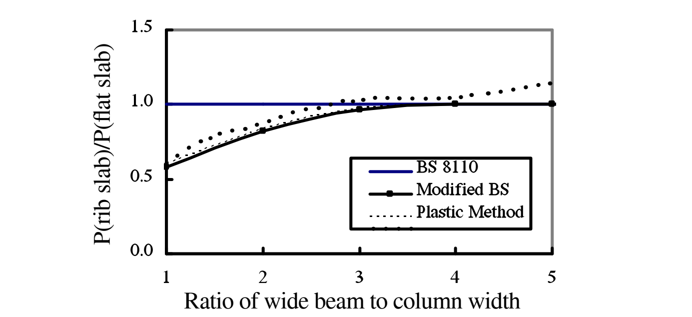


Figure 13: Comparison of prediction methods as wide beam width varies

2.0 CONCLUSIONS

- Shear failure of a wide beam ribbed slab can occur either by punching shear or wide beam shear.
- Although a punching shear failure of a wide beam ribbed slab is very similar to that of a solid slab, the shear capacity is reduced relatively, because some of the potential shear failure surface is lost when it enters the ribbed section.
- The existing empirical model in BS 8110 can be extended to predict the punching and wide beam shear strengths of wide beam ribbed slabs by making an allowance for the “lost” area of the shear failure surface which occurs because of the non-uniform overall slab thickness.

ACKNOWLEDGEMENTS

The first Author would like to express his thanks to the Committee of Vice-Chancellors and Principals of the United Kingdom (CVCP) for the award of an Overseas Research Scholarship, and the University of Birmingham for their financial support. ■

NOTATION

a_x, a_y	shear span in x and y direction
A_s	area of longitudinal reinforcement crossing shear plane
b	width of shear plane
d	effective depth of slab sections
c_x, c_y	column sizes in x and y direction
f_{cu}	concrete cube compressive strength
u	effective perimeter of critical shear area
v_c	shear strength
V_{ef}	enhanced effective shear force at edge column
V_p	theoretical predicted shear capacity using plasticity theory
V_t	test shear capacity
α	shear retention factor
γ_x, γ_y	effective shear area factor in x and y direction

REFERENCES

[1] Federation Internationale du Beton. Punching of structural concrete slabs. fib Bulletin No 12, 2000pp.314.

[2] Xiang. X. Z. Punching Shear strength of waffle slabs at internal columns, PhD thesis, The University of Leeds, 1993.

[3] Hussein A. F. Punching shear strength at edge columns in ribbed flat slabs, PhD thesis, The University of Leeds, 1994.

[4] Simpson D., Designing out the difficulties: Wide flat beam strips, Concrete, 37/3, 2003, p 39.

[5] British Standard Institution, BS 8110-1: 1997, The structural use of concrete – Part 1: Code of Practice for Design and Construction, 1997.

[6] Lau, T L. Shear transfer mechanism between reinforced concrete ribbed slabs and columns. PhD Thesis, University of Birmingham, November 2004, pp 314.

[7] Lau, T L and Clark, L A. Shear transfer between ribbed slab and internal column. Magazine of Concrete Research, Vol 59, No 7, September 2007, pp 507-516.

[8] Lau, T L and Clark, L A., Shear transfer between ribbed slab and edge column. Magazine of Concrete Research, Vol 60, No 6, August 2008, pp 411-419.

[9] Gardner, N. J., and Shao, X. Y., Punching Shear of continuous flat reinforced concrete slabs, ACI Structural Journal, 93/2, 1996, p. 219-228.

[10] Concrete Society. Design guidance for high strength concrete, Concrete Society Technical Report No. 49, 1998, pp 168.

[11] Boswell L.F. and Wong S.S., The shear behaviour of micro-concrete and its inclusion in a proposed yield criterion, Proceedings of Cement and Concrete Association Research Seminar, 1981, pp 217-227.

[12] Wong S.S., Collapse behaviour of micro-concrete box girders bridges, PhD thesis, The City University, London, 1998.

[13] Regan P. E., Behaviour of reinforced concrete flat slabs, CIRIA Report 89, Construction Industry Research and Information Association, London 1981.

[14] Long, A. E., and Gilbert, S. G., Behaviour of flat slab / edge column joints, Proceedings of the International Conference on Concrete Slabs, Dundee University, 1979, pp 185.

PROFILE

IR. DR LAU TECK LEONG
is an Assistant Professor in DEpartment of Civvil Engineering, The University of Nottingham, Malaysia. He is a graduate member of BEM since 2006. He obtained his Ph.D from university of Birmingham, Uniteed Kingdom in 2005 specialising in concrete slab punchingh mechanism.