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Microscopic dynamics of a large-scale pedestrian evacuation model

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Numerous models have been developed for estimating the time required to evacuate from a variety of places under various conditions. For high-traffic places, such as commercial and industrial buildings, it is vital to be able to accurately calculate the evacuation time required in order to ensure the safety of the occupants. To this end, various models of pedestrian dynamics have been proposed, either as a whole system or focusing only on the psychological interaction between pedestrians. However, most of these published studies do not take into account the pedestrian's ability to select the exit route in their models. To resolve these issues, we have developed a model to simulate evacuations from a hall using the social force model that incorporated with the degree of pedestrians' impatience along with the distance to exits and the density of the crowd, in determining pedestrians' selection of evacuation routes. For validation, the results obtained with the proposed model are compared with published data. Finally, the model is applied to predict a specific adjustment to the hallway that would improve the output of the system. Simulations show that reasonable improvement is achieved, with an additional 14.2% pedestrians being evacuated within a 12 min interval.

Key words: Social force model, exit route selection, crowd dynamic, density effect, intelligent agent.

INTRODUCTION

Pedestrian modeling is among the most interesting areas in transportation science. Understanding the dynamics of pedestrian flow is key to the design of public places that aims to reduce the loss of life and property in the event of disasters. However, pedestrian evacuation is a complex process complicated by human behaviors and emotions such as panic. It is difficult to capture scenes of pedestrian flows during evacuation for research purposes, and it is nearly impossible to simulate real-life evacuations.

Researchers, therefore, rely on simulation models to study pedestrian behavior during evacuation. Various useful models have been put forward, among which are models based on particle flow (Helbing, 1992; Helbing et al., 2003; Sakai et al., 2006; Raja and Pugazhenthi, 2011), social force (Helbing et al., 2000; Helbing and Molnar, 1995; Helbing et al., 2001; Okazaki and Matsushita, 1993; Teknomo et al., 2001) and cellular automata (CA) (Siamak et al., 2011; Alizadeh, 2011; Kirchner et al., 2004; Burstedde et al., 2001; Varas et al., 2007; Fang et al., 2010; Liu et al., 2009; Lim, 2011). Particle flow and social force models use physical models

to simulate movement. CA models divide the environment into cells between which pedestrians move. Each model has strengths and weaknesses. For example, the social force model (SFM) produces smoother movements than the CA model owing to its continuous nature.

Kosinski and Grabowski (2010) introduced an intelligent agent system using the Langevin equation with an additional SFM term to represent the level of panic during evacuation that takes the speed of movement in the exit area as the decisive factor. However, the model produces less realistic pedestrian movements as compared to the social force model. Parisi and Dorso (2005) devised a model to study the level of panic with different exit widths. However, their model is limited to crowds of about 200 pedestrians and focused on the effect of exit width on panic levels during evacuation. Durupinar et al. (2011) introduced an impatience rule for multi-agents systems for route selection. Similar to Parisi and Dorso (2005) work, the model is the only application for simulating limited number of pedestrian in the system due to it heavy computational algorithm. Frank and Dorso

(2011) investigation of the impact of human behavior during evacuation shows clogging and cluster forming during escape from a room with a single exit and a fixed obstacle. They found that the distance of the obstacle from the exit influenced the level of panic as well as the flow of evacuation. Following this work, Zheng et al. (2011) also suggested that psychological term such as impulsive or impatience behavior should be considered for improving evacuation model in order to make the model more accessible to reality. Ding et al. (2011) proposed a psychological force into SFM which is mainly applied for pedestrian avoidance in rail transit lane area, but not for improving pedestrian intelligent in selection less congested route for exiting the area.

In situations of panic, the crowd tends to jostle to try and escape in the shortest time. Congestion close to the exit may be avoided by placing an obstacle to disperse the crowd (Helbing et al., 2005; Kirchner et al., 2003). The size of the obstacle, as well as its distance to the exit, ought to be considered carefully to attain optimal evacuation time. Otherwise, the exit area may become overcrowded, or congestion may grow to a critical level outside this area. Properly done, an obstacle may improve pedestrian flow up to 30% or twice the flow that may occur without the obstacle (Helbing et al., 2005; Escobar and Rosa, 2003). The reduction in evacuation time is achieved when pedestrians are required to walk a longer path in order to avoid the obstacle. Detrimental results may occur if the obstacle is placed too near the exit. To achieve optimal evacuation time, the obstacle should be moved slightly away from the exit and placed in the center of the area next to the exit (Frank and Dorso, 2011; Yanagisawa et al., 2009). Helbing et al. (2000) proposed placing a column as obstacle near the exit area. Understanding evacuation dynamics would therefore allow safer designs of public facilities.

The objective of the present paper is to study the microscopic mechanism involved in the hallway flow during evacuation from a hall. In our study, we use a social force model to examine the psychological factor of impatience in pedestrians in the hallway area during mass evacuation from a hall. The social force model (Helbing and Molnar, 1995) produces realistic movements in simulating the evacuation process by taking into consideration discrete characteristics of pedestrian flow, thus allowing individuals' physical variables to be set, such as mass, shoulder width, desired speed and target destination. The forces of interaction that may cause high pressure capable of bringing down a brick wall or causing suffocation can thus be determined. These continuous flow characteristics cannot be simulated with CA models.

The efficiency of an obstacle in relieving congestion is affected by pedestrians' behavior during the evacuation process. Most studies assume a fixed route taken by pedestrians without considering the degree of impatience, which may arise in such situations (Kirchner et al., 2003; Escobar and Rosa, 2003; Yanagisawa et al., 2009). A more realistic scene would be that some

impatient pedestrians will alter their initial preferred route and head for the nearest exit available, pushing through other pedestrians in the process. Subsequently, we introduce a rule for exit route selection influenced by the degree of impatience. Next, the social force model of Helbing et al. (2000) is discussed, then we present and discuss the results of simulations performed with our model. Finally, we conclude the paper with suggestions for future research.

RULE FOR EXIT ROUTE SELECTION

During an evacuation, an impatient pedestrian will tend to show characteristics, such as walking faster than their normal speed, pushing nearby pedestrians and rushing toward the nearest exit available.

An impatient pedestrian's action can be expressed mainly in terms of changes in their speed. A pedestrian's degree of impatience, $n_i(t)$, at time t can be expressed as:

$$n_i(t) = \frac{v^{(i)}(t) - v^{(i)}(0)}{v_{\text{max}}^{(i)} - v^{(i)}(0)}$$
(1)

where $v^{(i)}(t)$ is the speed of pedestrian i at time t, $v^{(i)}(0)$ is the

initial speed of pedestrian i and $v_{\max}^{(i)}$ is the maximum speed desired by pedestrian i.

Research has shown that pedestrians' emotions, such as impatience, affect their choice of escape route (Helbing et al., 2000; Okazaki and Matsushita, 1993; Fang et al., 2010). Impatient pedestrians would behave in such a way that causes an unstable flow, leading to delays or congestion during evacuation.

Exit selection is mainly based on distance. To select a route, a pedestrian would consider two factors: (1) their distance from an exit and (2) the presence of people flocking to that exit. Assuming there are m(x = 1, 2, 3, ..., k, ..., m) evacuation exits, the probability of pedestrian i selecting exit k as the evacuation route can be defined as:

$$P_k^{(i)} = (1 - n_i)P_1 + n_i P_2 (2)$$

where P_1 is the probability of reaching the nearest exit, P_2 is the probability of people flocking to that exit and n_i is the degree of impatience of pedestrian i. When n_i approximates 0, pedestrian i is in normal mood. However, when n_i approximates 1, pedestrian i is in an extremely impatient mood, rushing to get out as fast as possible.

The probability of reaching the nearest exit, P_1 , can be defined as:

$$P_1 = \frac{d_{(i)}}{d_{\text{max}}} \tag{3}$$

where $d_{(i)}$ is the distance of pedestrian i to exit k, and d_{\max} is the maximum distance measured from all pedestrians to exit k. Equation 3 indicates that the shorter the distance of pedestrian i to exit k, the higher the probability of pedestrian i selecting exit k as the evacuation route. Conversely, if the distance is longer, the probability of selecting exit k decreases.

The probability of the flocking phenomenon occurring in the exit area is defined as:

$$P_{2} = 1 - \frac{N_{(k)}}{\sum_{\phi=1}^{m} N_{(\phi)}}$$
 (4)

where $N_{(k)}$ represents the number of pedestrians that select exit k as the evacuation route, while $\sum_{i=1}^m N_{(\phi)}$ is the total number of

pedestrians that select exit ϕ as the evacuation route. The degree of impatience of pedestrian i in the system, n_i can be expressed as:

$$n_{i} = \left| \frac{v^{(i)}(t) - v^{(i)}(0)}{v_{\text{max}}^{(i)} - v^{(i)}(0)} \right|.$$
 (5)

According to Helbing (1992), Helbing et al. (2000) and Helbing and Molnar (1995), the maximum desired speed $v_{\rm max}^{(i)}$ that can be achieved by a pedestrian is 3 m/s. When an unexpected situation occurs, such as flocking, congestion or panic, pedestrian speed $v^{(i)}(t)$ drops to less than 1.5 m/s. The velocities of a pedestrian are uniformly distributed in the range of 0.5 to 1.5 m/s in simulations, while the initial speed $v^{(i)}(0)$ of a pedestrian is approximately 1.0 m/s. Hence, the decision of pedestrian i to select exit x as the evacuation route is determined by comparing probability $P_x^{(i)}$ (x = 1, 2, 3, ..., k, ... m) against the highest value of $P_x^{(i)}$ as the criterion of selection.

THE SOCIAL FORCE MODEL

According to the social force model, pedestrians' movement is determined by their desire to arrive at the destination, as well as the effects of the surroundings on them (Helbing et al., 2000; Helbing and Molnar, 1995). Recent social force models include the social force and granular force, while earlier models are based on one force known as the desire force.

Suppose that a pedestrian is moving at a desired speed of v_d in a given direction \vec{e}_d . In actual situations, pedestrians always walk a bit out of the actual desired path and they never walk exactly at the desired speed v_d . A pedestrian's actual speed v(t) is influenced mainly by environmental factors (e.g., obstacles, exit size). Hence, pedestrians have to increase or decrease their speed with the intention of reaching the destination at the desired speed v_d . This acceleration or deceleration corresponds to the desire force, as it is dictated by their will and motivation. Therefore, the desire force of pedestrian i can be defined in mathematical terms as:

$$f_d^{(i)}(t) = \frac{v_d^{(i)}(t)\vec{e}_d^{(i)}(t) - v_i(t)}{\tau} \tag{6}$$

where all parameters are assumed to be functions of time, and τ represents the relaxation time required to achieve the desired speed. The value of τ is determined by experiment.

The pedestrian's reactions to environmental stimuli are represented by social forces. Although, there exist stimuli such as family members or friends that generate attraction, they are not included

in our model. However, the basic rule that pedestrians tend to preserve their private space between other pedestrians still applies (Helbing and Molnar, 1995). When people get closer to one another, the repulsive force will become stronger. In other words, the repulsive force is mainly dependent on the inter-pedestrian distance d, which can be modeled as an exponentially decaying function defined as:

$$f_s^{(ij)} = A_i n_{ij} e^{(r_{ij} - d_{ij})/B_i}$$
(7)

where i and j correspond to any two pedestrians, d_{ij} is the distance between the centers of mass of the two pedestrians, $n_{ij} = \left(n_{ij}^{(1)}, n_{ij}^{(2)}\right)$ represents the unit vector in direction $j\vec{i}$ and $r_{ij} = r_i + r_j$ is the sum of pedestrian radii for pedestrians i and j. Parameters A_i and B_i are determined by experiment (Helbing et al., 2000).

Equation 7 is also applicable to environmental factors (e.g., obstacles). Pedestrians will tend to maintain a distance from obstacles to avoid being injured. Hence, r_{ij} and d_{ij} in Equation 7 are substituted by r_i and d_i , corresponding to the pedestrian's radius and their distance to the obstacle, respectively.

The final term in the social force model, which expresses the sliding friction that appears between pedestrians in contact with each other and with walls, is known as the granular force. By assuming the pedestrian's relative velocity as a linear function, the granular force can be expressed as:

$$f_g^{(ij)} = \kappa g(r_{ij} - d_{ij}) \Delta v_{ij} \cdot t_{ij}$$
(8)

where $\Delta v_{ij} = v_j - v_i$ is the speed difference between pedestrian i and pedestrian j. If pedestrian i comes to a wall, then v_j becomes

zero in Equation 8. The function
$$t_{ij} = \left(-n_{ij}^{(2)}, n_{ij}^{(1)}\right)$$
 is the unit

tangential vector orthogonal to n_{ij} , and K is an experimental parameter. The g(.) function is set to zero when the argument value is negative (that is, $r_{ij} < d_{ij}$), and it is equal to the argument value for any other cases.

Body compression may occur in extremely crowded situations (Helbing et al., 2000). However, as reported by Parisi and Dorso (2005), body compression forces play no significant role during the evacuation process. Hence, it is not taken into consideration in the proposed model. A more detailed explanation of $f_s(t)$ and $f_g(t)$ can be found in the literature (Ding et al., 2011; Escobar and Rosa, 2003; Kosinski and Grabowski, 2010; Lim, 2011). Table 1 lists typical values for the experimental parameters in Equations 6 to 8. Consequently, both the desire and granular forces control pedestrians' dynamical characteristics by changing their speed. The movement of pedestrian i can be expressed as:

$$\frac{dv_i}{dt}(t) = f_d^{(i)}(t) + \frac{1}{m_i} \left[\sum_{i \neq j} f_s^{(ij)}(t) + \sum_{i \neq j} f_g^{(ij)}(t) \right]$$
(9)

where m_i is the mass of pedestrian i. The subscript j represents all other pedestrians except pedestrian i and environmental factors.

The magnitude of the desired speed v_d in Equation 6 corresponds to the pedestrian's movement at free-flow speed. Additionally, the moving direction \vec{e}_d sets the level of anxiety for the pedestrian, eager to reach a particular exit. An impatient pedestrian will tend to change their initial desired direction for the nearest exit available (Escobar and Rosa, 2003).

Parameter	Symbol	Value	Units
Force at $d_{ij} = r_{ij}$	A_i	2000	N
Characteristic length	B_i	0.08	m
Pedestrian mass	m_i	70	kg
Contact distance	r _{ij}	0.5 ± 0.2	m
Acceleration time	au	0.5	S
Friction coefficient	K	2.4×10^{5}	kg m ⁻¹ s ⁻¹

Table 1. Variables for evacuation simulations.

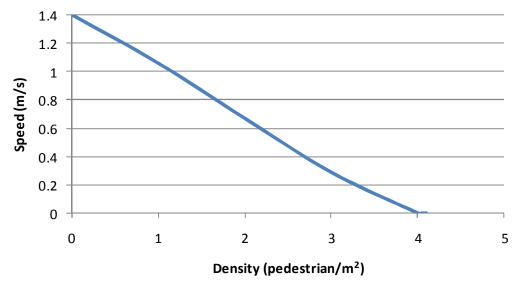


Figure 1. Relationship of pedestrian speed and density.

Effect of pedestrian density on speed

The higher the pedestrian density in an area, the slower pedestrians move as compared to their desired free-flow speed in order to avoid collision and injury. In the proposed model, pedestrians are moving at maximum speeds in the hallway area, but they slow down when there are obstacles or other pedestrians nearby. Fruin (1971), studying the effect of density on speed, concluded that, as pedestrian density increases, the speed of movement drops correspondingly. According to Fruin's study, pedestrian speed approximates zero when density approaches 4 pedestrians/m² (Figure 1).

Based on the findings of Fruin (1971), we introduce the density effect in Equation 10 as defined by Siamak et al. (2011), for the purpose of evaluation. The path is defined as the movable neighborhood of the desired route. The three regions of the path located near the exits are all fixed at equal sizes for evaluation purposes.

Density Effect =
$$\begin{cases} \frac{\beta_{path}}{\beta_0}, \ \beta_{path} < \beta_0 \\ 1, \text{ otherwise} \end{cases}$$
 (10)

In Equation 10, eta_{path} is the path area density, while the margin area density represented by eta_0 is equal to the size of the path area.

SIMULATION RESULTS

From the simulation results of the proposed model, the speed-density relation is determined and then compared to results reported by Schadschneider et al. (2008). In addition, simulation snapshots were compared with actual photographs taken by a surveillance camera installed at the corner of the scene. The relationship between pedestrian flow rates and evacuation time is then examined. Finally, the model is applied to predict whether placing an obstacle in the center of the hallway area would significantly enhance the overall system output.

Speed-density graph comparison

As suggested by Siamak et al. (2011), we use the speed-density graph as shown in Figure 1 earlier to compare our results with those of other studies. We use two graphs found in Schadschneider et al. (2008), from Fruin's and Predtechenskii-Milinskii's (PM) studies. Fruin's and PM's graphs produce the highest (1.4 m/s) and the lowest (1.0 m/s) desired speeds, respectively. Fruin's results indicate that the crowd will stop moving

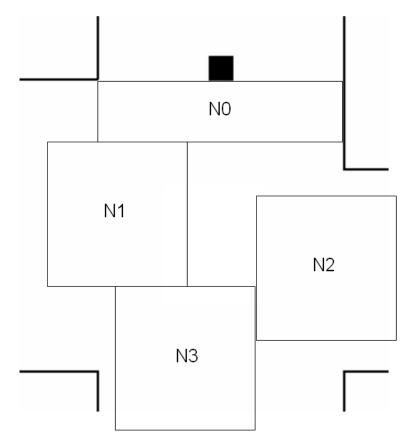


Figure 2. Sections of the hallway area for speed-density evaluation.

when density increases to 4 pedestrians/m², while PM's findings show that pedestrians will continue moving at a speed below 0.2 m/s even when density reaches 8 pedestrians/m². High densities exceeding 4 pedestrian/m² can sometimes be observed in the hallway area. Therefore, we infer that the speed-density relation for the hallway area is similar to that described in PM's graph.

According to Siamak et al. (2011), every evacuation area is made up of sections with different properties. To determine the speed-density relation for each section of the evacuation area in our study, we adopted the guidelines of Siamak et al. (2011). We divided the hallway area into sections as shown in Figure 2. Section N0 is the most crowded area, where pedestrians stream out of the hall's two adjacent doorways and move in different directions toward the exits in N1, N2 and N3 sections. Slowdown in N0 is caused partly by pedestrians in N1 heading for N2 or N3. Slower movement is also observed in spots where pedestrians are close to the walls in N2 or heading for N1. Pedestrians crossing over to different sections causes further delay. Since N1. N2 and N3 exhibit none of the extreme characteristics of N0, they are suitable for evaluating the efficiency of our model in simulating evacuation under different conditions (Figure 3).

Figure 4 compares the speed-density curves for the N1, N2 and N3 sections in our model against Fruin's and PM's graphs. We can see that the N1, N2 and N3 curves behave similarly to PM's curve, with an initial desired speed of around 1.2 m/s. We can thus infer that our model is comparable to PM's. The maximum density in our model is 4.25 pedestrians/m². Even with extremely high demand levels, densities higher than 5 pedestrians/ m² cannot be achieved in the N1, N2 and N3 section.

Visual comparison

It was mentioned earlier that the hallway area has certain characteristics in different sections that lead to congestion in the N0 section. We performed evacuation simulations that were based mainly on videos of movements in the hallway area when participants leave a hall after an event. Most people would move toward one of the exits as fast as possible. The crowd in N0 creates resistance to pedestrian flow behind this area. This resistance leads to serious congestion in N0, whereas a less congested scene is observed in other sections.

Figure 5 shows the snapshots of simulations at medium, high and very high flow rates, along with actual

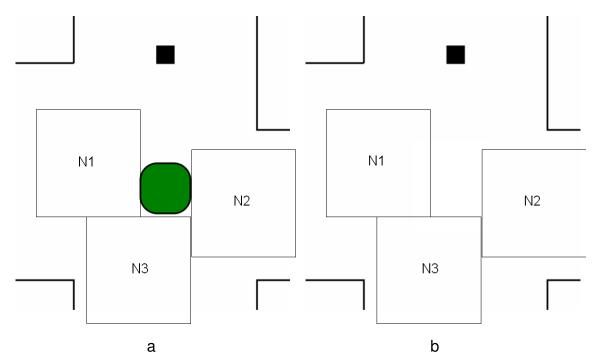


Figure 3. Speed-density evaluation of the hallway area (a) with and (b) without an obstacle.

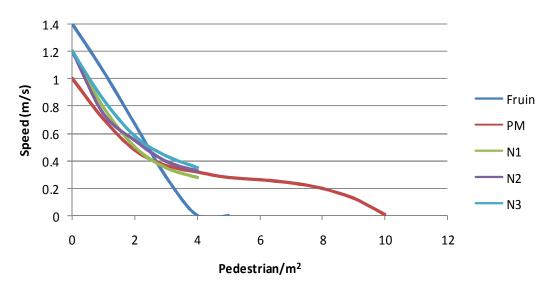


Figure 4. Speed-density relation in the N1, N2, and N3 sections.

scenes taken from surveillance photographs showing the flow of people leaving a hall. The area just outside the hall is most congested, corresponding to the situation in N0 in our simulation.

Evacuation time and flow rates

As the number of people in the hallway area builds up, movement slows down. To see the effect of this build-up

on evaluation time, we ran the simulation for five different flow rates, which are 25, 75, 175, 225 and 275 pedestrians/min, corresponding to low, medium, high, very high and extremely high flow rates. Figure 6 shows the results of these simulations. As the flow rate increases, evacuation time decreases.

Additionally, as the flow rate increases up to 75 pedestrians/min, evacuation time drops significantly. The point before which a sharp drop in evacuation time occurs around 50 pedestrians/min. Simulations also show

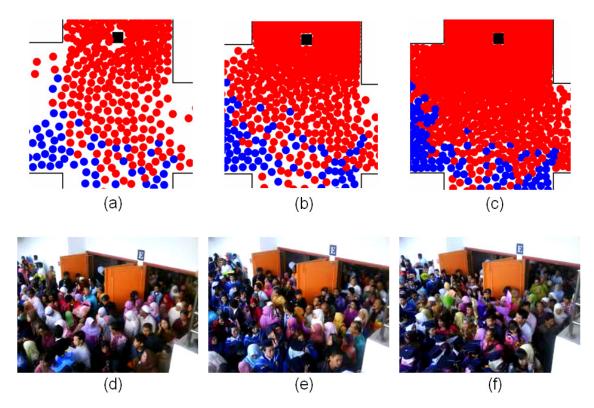


Figure 5. Simulation snapshots of (a) medium, (b) high and (c) very high pedestrian flow rates. Surveillance photographs of real scenes (d to f).

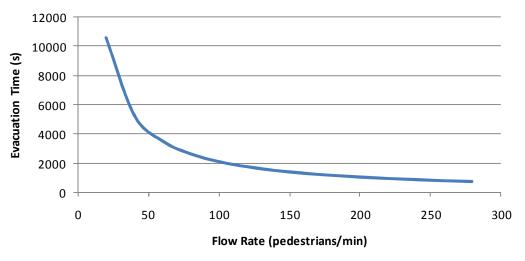


Figure 6. Evacuation time for different pedestrian flow rates.

that congestion quickly builds up in N0 once the flow rate reaches over 75 pedestrians/min. Congestion also begins to spread to N1 and N2, which delays evacuation. Such a level of congestion could develop into a hazardous situation as impatient or panicked pedestrians start to push one another.

The average pedestrian speed and evacuation time in the simulations match data obtained through the surveillance camera for the real situations. However, it is suggested that more data should be obtained to allow the parameters to be calibrated.

Application of the model

We then applied the model to evaluate the extent of

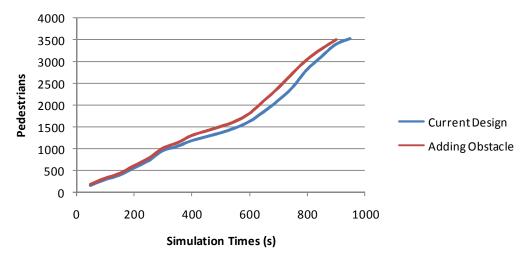


Figure 7. Comparison of simulations with the current design of the hallway and a what-if scenario where an obstacle is added.

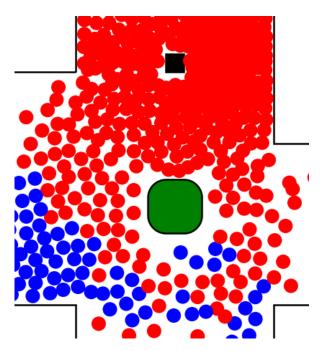


Figure 8. Simulation snapshot of the what-if scenario.

improvement in throughput with the implementation of changes designed to improve flow. In an experiment with a what-if scenario, an obstacle was placed in the center of the hallway area. Figure 7 shows that this improves the speed of evacuation. The reason is that the obstacle splits up pedestrian flow and reduces collisions between pedestrians from N0 and those crossing from N1 to N2. It also creates a smoother flow for pedestrians moving from N0 to N2 and to N1.

In the absence of the obstacle, pedestrians heading for N3 would walk past the central area in collision with

others. With the obstacle in the center, the number of pedestrians crossing over to N1 and N2 is reduced. Most of the pedestrians going from N1 to N2 would take the route behind the obstacle to avoid the crowd in N0. The outcome is smoother flows and less congestion in the center of the hallway area. Overall, congestion in the hallway area and evacuation time are reduced. Figure 8 shows a simulation snapshot of the what-if scenario. In such a scenario, at a flow rate of 175 pedestrians/min, the simulation yields 318 pedestrians, or 14.2% additional output, within a 12 min interval.

Conclusion

The findings show that our social force model is suitable for simulating complex and highly crowded situations. We incorporated into the model the elements of impatience and route selection ability for more realistic simulations. We performed simulations of evacuation from a hall to the hallway area to investigate pedestrian behavior during the evacuation process. This model was applied to a what-if situation to predict whether changes to the hallway area would produce a better outcome as compared to the current design. For future research, pedestrian dynamics during evacuation that involve grouping will be explored, and additional factors such as age and other psychological characteristics of individuals will be included, with the aim of reproducing more realistic simulations for analysis and study.

REFERENCES

Alizadeh R (2011). A dynamic cellular automaton model for evacuation process with obstacles, Safety Sci., 49: 315-323.

Burstedde C, Klauck K, Schadschneider A, Zittartz J (2001). Simulation of pedestrian dynamics using a two-dimensional cellular automaton,

- Phys. A. 295: 507-525.
- Durupinar F, Pelechano N, Allbeck JM, Gudukbay U, Badler NI (2011). How the ocean personality model affects the perception of crowds, IEEE Computer Graphics Appl., 11: 22-31.
- Ding QY, Wang XF, Shan QC, Zhang XY (2011). Modeling and simulation of rail transit pedestrian flow, J. Trans. Systems Eng. Inform. Technol., 11(5): 99-106.
- Escobar R, Rosa ADL (2003). Architectural design for survival optimization of panicking fleeing victims, Springer-Verlag, Berlin, pp. 97-106.
- Fang ZM, Song WG, Zhang J, Wu H (2010). Experiment and modelling of exit-selecting behavior during a building evacuation, Phys. A. 389: 815-824.
- Frank GA, Dorso CO (2011). Room evacuation in the presence of an obstacle, Phys. A, 390: 2135-2145.
- Fruin JJ (1971). Pedestrian planning and design, Metropolitan Association of Urban Designers and Environmental Planners, New York, pp. 39-46.
- Helbing D (1992). A fluid-dynamic model for the movement of pedestrians, Complex Systems. 6: 391-415.
- Helbing D, Isobe M, Nagatani T, Takimoto K (2003). Lattice gas simulation of experimentally studied evacuation dynamics, Physical Review. E 67: 067101.
- Helbing D, Farakas I, Vicsek T (2000). Simulating dynamical features of escape panic, Nature. 407: 487-490.
- Helbing D, Molnar P (1995). A social force model for pedestrian dynamics, Phys. Rev., E51: 4282-4286.
- Helbing D, Molnar P, Farkas J (2001). Self-organizing pedestrian movement, Environment and Planning B: Planning Design, 28: 361-383.
- Helbing D, Buzna L, Johansson A, Werner T (2005). Self-organized pedestrian crowd dynamics: experiments, simulations, and design solutions, Transportation Sci., 39: 1-24.
- Kirchner A, Klupfel H, Nishinari K, Schadschneider A, Schreckenberg M (2004). Discretization effects and the influence of walking speed in cellular automata models for pedestrian dynamics, J. Statistical Mech., 10: 10011-10032.
- Kirchner A, Nishinari K, Schadschneider A (2003). Friction effects and clogging in a cellular automaton model for pedestrian dynamics, Phys. Rev., E67: 056122.
- Kosinski RA, Grabowski A (2010). Intelligent agents system for evacuation processes modeling, Proceeding of IEEE/WIC/ACM International Conference, Web Intell. Intell. Agent Technol., pp. 557-560.
- Lim EA (2011). Exit-selection behaviors during a classroom evacuation, Int. J. Phys. Sci., 6(13): 3218-3231.
- Liu SB, Yang LZ, Fang TY, Li J (2009). Evacuation from a classroom considering the occupant density around exits, Physical, A 388: 1921-1928.

- Okazaki S, Matsushita S (1993). A study of simulation model for pedestrian movement with evacuation and queuing, Proceedings Int. Confer. Eng. Crowd Safety. pp. 271-280.
- Parisi DR, Dorso CO (2005). Microscopic dynamics of pedestrian evacuation, Physical. A 354: 606-618.
- Raja P, Pugazhenthi S (2011). Path planning for a mobile robot in dynamic environments, Int. J. Phys. Sci., 6(20): 4721-4731.
- Sakai S, Nishinari K, Lida S (2006). A new Stochastic cellular automaton model on traffic flow and its jamming phase transition, J. Phys. Math. General, 39: 15327-15339.
- Schadschneider A, Klingsch W, Klupfel H (2008). Modeling and applications, Meyer, B. (Editor), Encyclopedia Complexity System Sci. Springer, Berlin, p. 3142.
- Siamak S, Fazilah H, Abdullah ZT (2011). A cellular automata model for circular movements of pedestrians during Tawaf, Simulation Modeling Practice Theory, 19: 969-985.
- Teknomo K, Takeyama Y, Inamura H (2001). Microscopic pedestrian simulation model to evaluate lane-like segregation of pedestrian crossing, Proceeding of Infrastructure Planning Conference, Japan, pp. 208-218.
- Varas A, Cornejo MD, Mainemer D, Toledo B, Rogan J, Munoz V, Valdivia JA (2007). Cellular automaton model for evacuation process with obstacles, Physica. A 382: 631-642.
- Yanagisawa D, Kimura A, Tomoeda A, Nishi R, Suma Y, Ohtsuka K, Nishinari K (2009). Introduction of frictional and turning function for pedestrian outflow with obstacle, Physical Review. E 80: 036110.
- Zheng Y, Jia B, Li XG, Zhu N (2011). Evacuation dynamics with fire spreading based on cellular automaton, Physical A. 390: 3147-3156.