Analysis and Design of Trapezoid Web Steel Section

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1.0 INTRODUCTION
1.1 General

A trapezoid web steel section is built up by welding flanges and a web of trapezoidally corrugated profile (Figure 1). The main purpose in particular is to increase the out-of-plane stiffness and shear buckling strength without the use of vertical stiffeners. It allows the use of thin plate webs without the need of stiffeners, thus considerably reducing the cost of fabrication. Since there is no standard design method for the new section, this research has been carried out to develop a complete design guide based on analytical and experimental study. The design of beam and column is based on the British Standard BS 5950 (Part 1:2000).
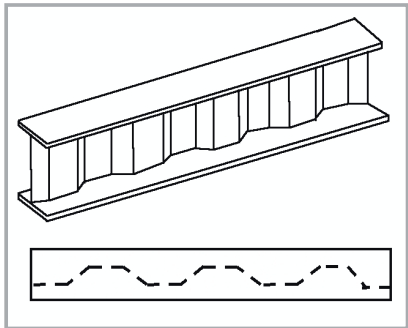

Figure 1: Trapezoid web section as compared with the conventional steel web

The design concept of TWP sections is not much different from the design of conventional welded steel sections. In almost all cases, the webs for both sections is classified as slender while the flanges are semi-compact section.

The derivation of the design shear capacity of TWP sections is based on the results of experimental and theoretical study, and is the major part of the research program.

1.2 Design of bending members

Bending members are checked for shear and bending capacities. The shear capacity is limited by the failure in shear buckling which is governed by the slenderness and the depth of the web. Only the shear resistance of web is considered, while the shear resistance of the flange is neglected. The design checking for shear is given by:

$$V \ < \ V_{cr} \qquad (1)$$

where ,
V is the applied shear force
V_{cr} is the shear buckling capacity

The bending moment applied on the section is resisted by the flange only. The web resistance is neglected due to its high slenderness.

The design checking for bending is

$$\frac{M}{M_{cx}} + \frac{VC_o}{M_{sec}} \leq 1.0 \qquad (2)$$

where,
M is the applied bending moment
M_{cx} is the section bending capacity
M_{sec} is the secondary bending moment capacity
C_o is the coefficient of secondary bending moment

For an unrestrained beam, it must be checked for lateral torsional buckling, i.e.:

$$mM_A \ < \ M_b \qquad (3)$$

where,
M_A is the maximum moment in the member under consideration,
M_b is the buckling moment resistance
m is an equivalent uniform moment factor determined from BS5950

1.3 Design of compression members

Compression members are checked for local capacity at the points of greatest bending moment and axial load. The member should also be checked for overall buckling.

The local capacity check is by ensuring that :

$$\frac{F}{P_z} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} + \frac{VC_o}{M_{sec}} \leq 1.0 \qquad (4)$$

where,
F is the applied axial load
P_z is the load capacity of the section

The overall buckling check is given by:

$$\frac{F}{P_c} + \frac{M_x}{M_b} + \frac{M_y}{M_{cy}} \leq 1.0 \qquad (5)$$

where,
P_c is the critical buckling strength of the member.

2.0 SHEAR BUCKLING CAPACITY

Previously, engineers use the shear capacity formula for trapezoid web section as:

$$V_{cr} \ = \ 0.65 \, \tau_y \qquad (6)$$

$$\tau_y = \frac{P_y}{\sqrt{3}} \qquad (7)$$

However, the formula is too conservative for a thick web, while it may become unsafe for slender web. Experimental and analytical studies have been carried out to develop a better equation.

2.1 Experimental Study

Predominantly shear loads were applied on a number of full-scale beam specimens ranging from 300 to 1600 mm in depth and spanning 1.5 to 5 m. A typical loading and support condition in
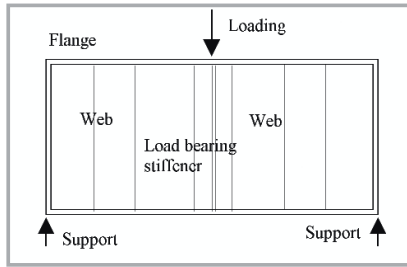
Figure 2 : Loading and support condition in the shear test

Figure 3: Photograph of the shear test

the experimental setup is shown in Figure 2. Figure 3 shows a photograph of a 1.0 m deep beam being tested in the lab.

Typical shear failure modes are shown in Figure 4 in which the web buckles in the direction of tensile membrane action across the web.

The ultimate shear capacity of a trapezoid web section is higher than the elastic design capacity, which only accounts for the web elastic shear buckling limit. However, it has low post buckling strength. It is because the inclined panel of the corrugated web is too thin to act as a normal vertical stiffener to the web. Each sub-panel of the trapezoid web acts as flat plate and also as partial stiffener that mutually

Figure 4: Failure of web in shear buckling

anchors each other. In a normal section with a flat web, the web is anchored to rigid stiffeners and when the shear load applied to the web reaches its elastic buckling capacity, the stiffeners give additional post buckling capacity to the web. In a trapezoid web section, the inclined web has very limited post buckling strength.

2.2 Theoretical study

Finite element analysis was used to study the critical shear by using eigenvalue-buckling analysis. A total of 80 finite element models of shear web panels with various parameters were analysed. A typical mode of shear failure shown in Figure 5.
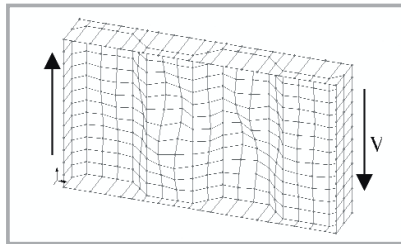

Figure 5: A typical shear mode failure

To study the influence of the corrugation pattern to the critical shear buckling of the trapezoid web panel, two key parameters, i.e. the web slenderness ratio d/t and the sub-panel aspect ratio b/d were varied. It was assumed that the web panels are simply supported to both top and bottom flanges. Based on the analytical study, a theoretical buckling curve has been developed.

2.3 Empirical formula of shear buckling capacity

Based on the theoretical buckling curve and experimental data, plus the test data from the collaborative team from the University of Braunshweig, Germany, a shear buckling formula for various sections has been developed.

The buckling formula proposed is given in the function of d/t and b/d and is shown typically in Figure 6. The elastic critical shear buckling is derived as follows:

$$\tau_{cre} = k \frac{1.6\pi^2 E}{12(1-v^2)(d/t)^2} \tag{8}$$

$$k = 1.8/(b/d)^2 - (b/d)^3 + 8*(b/d) + 9 \tag{9}$$

The shear capacity is given by:

$$V_{cr} = \tau_{cr} dt, \ \ \tau_{cr} = \tau_{cre} \tag{10}$$

In the case where $\tau_{cre} > 0.8\,\tau_y$, inelastic buckling will occur and the critical buckling is taken as equal to the inelastic buckling stress τ_{cri} calculated from:

$$\tau_{cri} = \sqrt{0.8\tau_{cre}\tau_y} \leq \tau_y \tag{11}$$
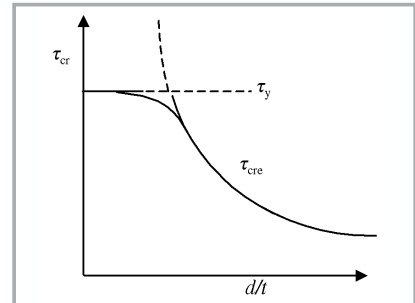

Figure 6: The shear buckling curve

3.0 BENDING CAPACITY

Due to the slenderness of the web, the bending resistance of the web is neglected, hence the bending moment of the section is resisted by the flange only, i.e:

$$M_{cx} = z_c\, p_y \tag{12}$$

Where Z_x is the elastic modulus of the section with the web neglected. From the experimental and finite element analysis, it was found that the deflection of beam with trapezoid web is less than flat web section. However, for conservativeness in design, this advantage of the trapezoid web section is neglected.

4.0 LATERAL TORSIONAL BUCKLING

The lateral torsional buckling resistance moment is given by:

$$M_b = p_b\, Z_x \tag{13}$$

in which p_b is the bending strength allowing for susceptibility to lateral-torsional buckling and Z_x is the plastic section modulus.

The bending strength, p_b is related to the equivalent slenderness, λ_{LT}, given by,

$$\lambda_{LT} = uv\lambda\,(\beta)^{1/2}, \tag{14}$$

where u is the buckling parameter,

$$u = \left(\frac{I_y S_x^{\;2} \gamma}{A^2 H} \right)^{1/4} \qquad (15)$$

$$\gamma = \left(1 - \frac{I_y}{I_x} \right) \qquad (16)$$

H is the warping constant, defined as,

$$H = \left(I_y d^2 \right) \Big/ 4 \qquad (17)$$

v is the slenderness factor obtained from the BS 5950. The value of v depends on the value of λ/x where x is the torsional index:

$$x = 1.132 \left(\frac{AH}{I_y J} \right)^{1/2} \qquad (18)$$

J is the torsional constant given by,

$$J = \frac{1}{3} \sum b.t^3 \qquad (19)$$

λ is the slenderness of the beam. For steel grade S275, $\beta = 1$. Based on these formulae, λ_{LT} is governed by the warping constant H and torsional constant J.

For sections with uniform web throughout the length, the above torsional properties are well defined and can be referred to BS 5950: 2000. For sections with non-uniform web profile, there is no established formula for the design check of the lateral torsional buckling.

4.1 Torsion and warping behaviour of TWP section

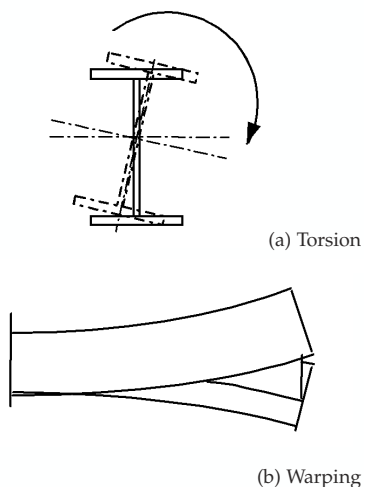When a section is subjected to torsional moment, it will rotate (Figure 7a). The angle of rotation depends on the torsional constant J. Experimental and analytical investigation were carried out to study the J value of the trapezoid web section compared to the normal web section.

The relationship between torsion and the angle of rotation was studied. The most significant result of the analysis is that the rotation of the trapezoid web is less than that of the flat web. Therefore, it can be predicted that the torsional constant J of the corrugated section is greater than the normal section which subsequently increases the lateral torsional buckling.

Warping is a phenomenon in a member of thin open section. The torsional response of a thin open section is very different from that of a solid circular shaft, where all cross sections normal to the beam axis remain plane under torsional loading. If the cross section is not circular, plane cross sections do not remain plane under torsion: they warp (Figure 7b).

Warping introduces longitudinal strains in the flanges as the section twists and significantly affects the torsional stiffness. From the analytical study, it was found that there is no significant difference in the strains developed in the flange of the trapezoid web section, compared to the flat web section.

4.2 Full-scale lateral buckling test

In the experimental test, a special test rig was designed, in which the beam was allowed to deflect horizontally while the loading point remained at the center of the section. Beam with trapezoid web and flat web with various spans of up to 5 meter length were tested by applying vertical load in the middle of the span.

The relationship between the bending moment resulted from the applied load and horizontal deflection were plotted, from which the critical buckling bending moment was defined. Results of the tests showed that the buckling bending moment of a trapezoid web is higher than the flat web sections.

From the analytical and experimental study, the lateral torsional buckling can be checked by assuming that the web is flat, i.e. neglecting the extra strength contributed by the trapezoid shape of the web profile.

5.0 SECONDARY BENDING MOMENT

When a web is subjected to a shear force, it will be resisted by the web through the development of a shear stress flow. For the corrugated web, the shear stress flow is as shown in Figure 8(a). Due to oblique orientation of alternate web sub-panels, component forces Q normal to the longitudinal axis of the section are created and they act in opposite directions [see Figure 8(b)].

These forces are transferred to the flanges, developing into couples and inducing a lateral bending moment in the flange, which is known as the secondary bending moment.

In a normal flat web plate, this phenomenon does not occur because the plane on which the shear flow is acting coincides with the longitudinal neutral axis and therefore, no reaction force component normal to the axis developed.
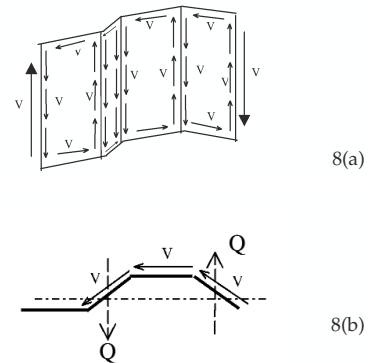
8(a)

8(b)

Figure 8: : Shear stress flow in the web and the secondary bending moment.

The secondary bending moment contributes towards a slight reduction in the bending strength of the web [refer to Eqn.(2)].

The secondary bending moment resulting from the couple is $M = V\,C_o$. The derivation of C_o is studied by carrying out a parametric study in finite element analysis. Results show that the secondary bending moment is higher for the section with lower depth. From the

(a) Torsion

(b) Warping

Figure 7: Rotation of section under torsion

parametric studies, the coefficient of secondary bending moment (unit in mm) is derived as,

$$C_o = 21725/d \qquad (20)$$

Where d is the depth of the section in mm. This equation is only applicable to a corrugation thickness of 80 mm.

6.0 AXIAL BUCKLING

There are few alternative formule for the derivation of axial buckling. The first option is by simply neglecting the contribution of the web, which is considered as the most conservative approach. The second option is by considering the contribution of the web, assuming that the web is flat, i.e. similar to normal section. However, it may also not be economic since there is indication that a section with trapezoid web has higher stiffness in the minor axis compared with a flat web section. The second moment of area *(I)* is a governing factor in the buckling formula.

6.1 Stiffness of column in minor axis

For two beams of the same material and length, the ratio of their moment of inertia, i.e. *I* is inversely proportional to deflection δ. For a flat web section, *I* can be directly calculated. For non-uniform sections such as the trapezoid web, there are a number of proposed formulae, none of which have been verified.

A load-deflection test was carried out for both beam specimens with trapezoid web and flat web of various lengths. The result showed lower deflection of trapezoid web section, which means that the *I* value of trapezoid section is higher than that of flat web. A numerical study was carried out using the finite element method, which showed a similar result with the experimental method. The deflection of a beam with trapezoid of lower *d/b* ratio is lower than the flat web, but is greater for low *d/b* ratios.

6.2 Critical buckling load

The eigenvalue buckling analysis in the finite element method was used to determine the elastic critical buckling load of a column.

It was proved that the buckling load increase linearly with the increase in the second moment of area of the sections, as expected in the Euler equation.

From a parametric study, a formula has been derived for the buckling capacity of a trapezoid web column, P_{cr}, by:

$$P_{cr} = P_e\left[1 + 10\left(\frac{h_r}{B}\right) - 4.5\right] \qquad (21)$$

where h_r is the corrugation thickness and B is the full width of the flange. P_e is the buckling capacity of the section assuming a flat web, obtained from the buckling curve in BS 5950:2000. In the calculation of P_e, it does not make any difference whether the web is included or not.
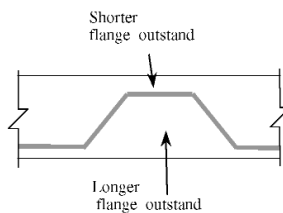

Figure 9: Different outstand length of flange in TWP section

7.0 LOCAL FLANGE BUCKLING

Due to the trapezoidal profile of the web, there are portions of flange with longer outstand, hence high slenderness, and other portions with shorter outstand and thus low slenderness (Figure 9). The portions with higher slenderness would tend to buckle more easily. In determining the section classification of flanges, the outstand is thus based on the longer outstand.

8.0 SUMMARY

The derivation of a design formula for bending and compression members of a trapezoid web section has been presented in this paper. In summary, the capacity of the section can be checked by assuming that the web is flat (similar to a normal section). However, for conservativeness, the web can be neglected. ■

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