2-Cycle Moment Distribution For The Analysis of Continuous Beams And Multi-Storey Framed Structures

By: Ir. Chan Kam Leong, B.Sc(Eng.), MSc, P.Eng, C.Eng, MICE, MIStruct E, MIEM, MACEM
K.L. Chan & Associates, Consulting Engineers

INTRODUCTION

The usefulness of the 2-Cycle Moment Distribution cannot be over-emphasized. It is considered necessary to revise my article published 36 years ago to include beam and frame for easy reference.

As the name suggests, the 2-Cycle Moment Distribution only 'distributes' twice regardless of the number of spans in continuous beams and frames. And both D.L. and T.L. are distributed simultaneously to obtain critical moments at supports as well as at spans. It is simpler, faster and more flexible than the conventional Hardy Cross Method. This can be demonstrated by the following examples.

MAXIMUM MOMENTS AT SUPPORT AND SPAN

Figure 1 shows a continuous beam. The methods of calculating maximum moments at support and span are as follows:

Method for maximum moment at support

Step 1. Write down D.F., $M_{D.L}$ & $M_{T.L.}$

Step 2. Calculate and write down C/O. C/O = $\frac{1}{2}$ x D.F x ($M_{T.L.}$ - $M_{D.L}$)

Step 3. Obtain total unbalanced moment (ΣM) by adding $M_{T.L.}$ to C/O

Step 4. Balancing moments are of opposite sign to reduce larger moment.

Step 5. Add ΣM to balancing moment for maximum moment at support.

Method for maximum moment at span

Step 1. Write down mid-span moment due to total load as if beam is fully fixed at each end.

Step 2. Calculate & write down 'adjustment' due to left hand support, i.e. Adj. M_l = - $\frac{1}{2}$ (1 + D.F) x C/O

Step 3. Calculate & write down 'adjustment' due to right hand support, i.e. Adj. M_r = - $\frac{1}{2}$ (1 + D.F) x C/O

Step 4. Add both adjustments to span moment to obtain maximum moment at span.

The results of maximum moments at supports and spans are shown in Figure 1.

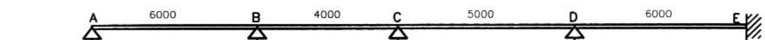
MINIMUM MOMENT AT SPAN

Span BC is shorter than the adjacent spans. It is possible that negative moment may extend across the shortest span. It is therefore necessary to calculate the minimum moment at mid span of span BC. It can be total loads on the adjacent spans obtained by allowing dead load on span BC. The procedure of calculating maximum and minimum moment at span is the same. The minimum moment at midspan is -7.06.

Legend
D.F – Distribution Factor
$M_{D.L}$ – Fixed-End Moment due to Dead Load
$M_{T.L}$ – Fixed-End Moment due to Total Load
C/O – Carry-Over Moment

Sign Convention
+VE for Sagging Moment
–VE for Hogging Moment

Beam–600x230, D.L=1.4G =15 KN/m , T.L=1.4G+1.6Q =22.5 KN/m



D.F	1	2/5	3/5	5/9	4/9	6/11	5/11	0
$M_{D.L}$	−45	−45	−20	−20	−31.25	−31.25	−45	−45
$M_{T.L}$	−67.5	−67.5	−30	−30	−46.88	−46.88	−67.5	−67.5
C/O	(a) −9.5	(b) −33.75	(c) +0.35	(d) +4.5	(e) −0.52	(f) −5.97	0	(g) −8.24
ΣM	−77.0	−101.25	−29.65	−25.5	−47.40	−52.85	−67.5	−75.74
Bal. M	(q) +77.0	(r) +28.64	(s) −42.96	(t) −12.17	(u) +9.73	(v) −7.99	(w) +6.66	(x) 0
Max Supp M	0	−72.61	−72.61	−37.67	−37.67	−60.84	−60.84	−75.74
Midspan $M_{T.L}$	+33.75		+15.0		+23.44		+33.75	
Adj. M_l	(h) +9.5		(j) −0.28		(l) +0.38		(n) 0	
Adj. M_r	(i) +23.65		(k) −3.50		(m) +4.61		(o) +4.12	
Max Span M	+66.90		+11.22		+28.43		+37.87	

c/o = ½ x D.F x ($M_{T.L}$ − $M_{D.L}$)

(a) ½ x 2/5 x (−67.5+20) = −9.5
(b) ½ x 1 x (−67.5−0) = −33.75
(c) ½ x 5/9 x (−30+31.25) = +0.35
(d) ½ x 3/5 x (−30+45) = +4.5
(e) ½ x 6/11 x (−46.88+45) = −0.52
(f) ½ x 4/9 x (−46.88+20) = −5.97
(g) ½ x 5/11 x (−67.5+31.25) = −8.24

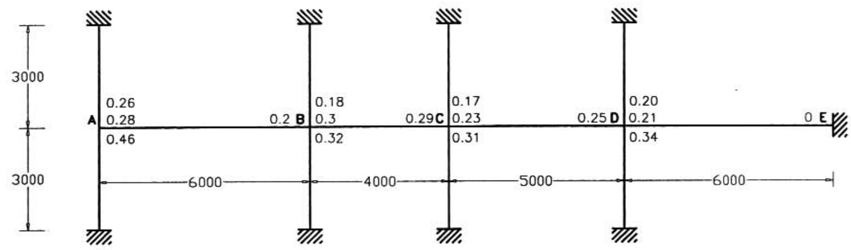
Adjustments due to supports:
−½(1+D.F) c/o

(h) −½ (1+1) x (−9.5) = +9.5
(i) −½ (1+2/5) x (−33.75) = +23.63
(j) −½ (1+0.6) x (+0.35) = −0.28
(k) −½ (1+5/9) x (+4.5) = −3.50
(l) −½ (1+4/9) x (−0.52) = +0.38
(m) −½ (1+6/11) x (−5.97) = +4.61
(n) −½ (1+5/11) x (0) = 0
(o) −½ (1+0) x (−8.24) = +4.12

Balancing Moments are of opposite sign to reduce larger moment

(q) = −(−77.0−0) x 1 = +77.0
(r) = +(101.25−29.65) x 2/5 = +28.64
(s) = −(101.25−29.65) x 3/5 = −42.96
(t) = +(25.5−47.40) x 5/9 = −12.17
(u) = −(25.5−47.40) x 4/9 = +9.73
(v) = +(52.85−67.5) x 6/11 = −7.99
(w) = −(52.85−67.5) x 5/11 = +6.66
(x) = +(75.74−0) x (0) = 0

Figure 1 : Max. M (Support & Span)
for a continuous beam over supports providing no restraint to rotation.



D.L=1.4G =15 KN/m , T.L=1.4G+1.6Q =22.5 KN/m

Beam−600x230, Col up−460x230, Col low−560x230

Load Pattern A		A	B	B	C	C	D	D	E
		T.L.		D.L.		T.L.		D.L.	
D.F	Upper col	0.26	0.18		0.17		0.2		
	Lower col	0.46	0.32		0.31		0.34		
	Beam	0.28	0.2	0.3	0.29	0.23	0.25	0.21	0
$M_{D.L}$				−20	−20			−45	−45
$M_{T.L}$		−67.5	−67.5			−46.88	−46.88		
C/O		−4.75	−9.45	+3.90	+7.13	−0.24	−3.09	0	−0.20
ΣM		−72.25	−76.95	−16.10	−12.87	−47.12	−49.97	−45	−45.20
M	Upper col	+18.79 (72.25x0.26)	+10.95 (76.95−16.10)x0.18			+5.82 (47.12−12.87)x0.17	+0.99 (49.97−45)x0.2		
	Lower col	+33.24 (72.25x0.46)	+19.47 (76.95−16.10)x0.32			+10.62 (47.12−12.87)x0.31	+1.69 (49.97−45)x0.34		

Load Pattern B		A	B	B	C	C	D	D	E
		D.L.		T.L.		D.L.		T.L.	
D.F	Upper col	0.26	0.18		0.17		0.2		
	Lower col	0.46	0.32		0.31		0.34		
	Beam	0.28	0.2	0.3	0.29	0.23	0.25	0.21	0
$M_{D.L}$		−45	−45			−31.25	−31.25		
$M_{T.L}$				−30	−30			−67.5	−67.5
C/O		−1.5	−6.3	+0.18	+2.25	+4.53	−0.14	0	−3.81
ΣM		−46.5	−51.3	−29.82	−27.75	−31	−31.39	−67.5	
M	Upper col	+12.09 (46.5x0.26)	+3.87 (51.3−29.82)x0.18		+0.55 (31−27.75)x0.17		+7.22 (67.5−31.39)x0.2		
	Lower col	+21.39 (46.5x0.46)	+6.87 (51.3−29.82)x0.32		+1.01 (31−27.75)x0.31		+12.28 (67.5−31.39)x0.34		

Figure 2 : For Max M in col

DETERMINATION OF COLUMN MOMENTS

For multi storey buildings, it is considered satisfactory to compute column moments under the same assumption used for beam moments, i.e. far lends of columns are fixed above and below the floor at which moments are to be determined. Column moments are computed for unbalanced floor loading, that is line load on one side only.

Figure 2 is a sub-frame of a multi-storey structure; live load is placed on the alternate spans as shown on load pattern A and load pattern B.

The method of calculating the maximum moments in columns is:

Step 1. Write down the Distribution Factors for the columns and for the beams and the $M_{D.L}$ and $M_{T.L.}$ according to the load pattern.

Step 2. Calculate and write down the carry-over moment (C/O) and C/O = ½ x D.F x ($M_{T.L.}$ - $M_{D.L.}$)

Step 3. Obtain Total Unbalanced Moment (ΣM) by adding $M_{T.L.}$ to C/O

Step 4. Maximum moments in column are obtained by multiplying the difference of the beam moments at the joint by the distribution factors of the columns.

The sign of column moments should be opposite to the beam unbalanced moments at the joint. The results of the column moments are shown in Figure 2.

It can be seen that the maximum moments in columns A, B and C are obtained from load pattern A, whereas load pattern B gives maximum moments in column D.

CONCLUSIONS

1. Moment coefficients may be used only if loads and spans meet the code requirement.
2. The Hardy Cross Method is too time consuming.
3. The 2-Cycle Method is simpler and faster. Not only support moment but also span moment and column moment can be obtained fairly quickly using this method.
4. A structure basically consists of beams and columns, the 2-Cycle Method helps us to understand structures better and thus gain confidence. Hence, we can feel and appreciate them better.
5. It can be used to counter check computer software. No structural engineer should ever use unfamiliar software without applying some verification.
6. It helps us to be a computer-aided design engineer and not just a computer operator!

REFERENCES

[1] Portland Cement Association, Continuity in Concrete Building Frames: Practical Analysis for Vertical Load and Wind Pressure, 1959.

[2] Chan Kam Leong, "Two Cycle Moment Distribution." Bulletin of the Institution of Engineers, Malaysia, August, 1978.

[3] Leong Tuck Wah, "Analysis of Multi-Storey Frame Structures Using The Two Cycle Moment Distribution Method." Bulletin of the Institution of Engineers, Malaysia, December 1978.