BED LOAD TRANSPORT MODEL FOR FIELD CONDITIONS – USING CHARACTERISTIC DIAMETER APPROACH

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ABSTRACT

A deterministic bed load transport model based on mixed-size (non-uniform) bed material (MBB) has been developed using characteristic particle diameter and published laboratory and field data from the North American and European continents (slope ranges from 0.0023 to 0.047 m/m and bed material size, D_{50} from 0.033 to 0.106 m) and excess discharge theory. Before developing the model performance of the existing bed load transport models was tested and was generally found to be poor, though discharge based models performed relatively better. In this model fitted values of the relationship exponent (β) were used instead of considering a fixed value, as in the existing models. Performance of the model was found satisfactory when tested with the field data and compared with another existing model. Subsequently, the model has been extended through the involvement of an active bed width function for non-uniform cross-sectional flow depth (single channel) rivers and for the braided rivers.

Keywords : *Characteristic Diameter, Excess Discharge Theory, Deterministic Model, MBB Model*

INTRODUCTION

The transport of bed load in mountain rivers with gravel, cobble and boulder bed material formations is difficult/complex to deal with, not only because of the sites' remoteness, large slopes, and lack of gauging stations but primarily due to the non-uniform and unpredictable flow conditions, non-uniformity of bed material formations and the interdependence of various parameters. The complexity of bed load transport phenomenon, in coarse bed material rivers compared with sand bed rivers, is due to their typical features like wider size distribution of bed material and bed load, movement of bed load (generally) for a short period of time (i.e. during high flows), less susceptibility to aggradation and degradation, slow response to modest changes in discharge and discharge duration, and presence of riffle/pool or step/pool type bed formations.

In order to relate different parameters of the bed load transport process, relationships between sediment, fluid, flow, and channel parameters are developed and are usually named sediment transport models/functions/equations. For bed load initiation and transport rate computation in fluvial streams such models have been developed, generally, using four basic theories of 1) discharge [22], 2) shear stress [10], 3) stream power [6], and 4) velocity [9], in conjunction with different calculation approaches (i.e. empirical, semi-empirical, probabilistic, deterministic and dimensional analysis). However, the existing sediment transport models for the mountain rivers are not accurate/efficient and reliable enough as investigators originally thought them to be.

As a large variety of characteristic size based bed load transport models exist a question arises why is it necessary to develop another model using the same approach. It has been developed because: a) computed results from different existing models often differ drastically from each other and from measurements; b) some of the models' results are contradictory partly because the ranges of measured data upon which these models are based were limited; c) they have been developed mostly using data collected under laboratory conditions (commonly with uni-size bed materials) while they are being applied to the field conditions (with mixed-size bed materials); d) models have been developed (mostly) using shear stress and stream power theories, which are less practical compared with the discharge theory (as proved by the recent studies); e) existing discharge theory based models, which are few in number, have been developed assuming a value of the relationship exponent, β taken as 1 in the Schoklitsch [22] model, and 2 in the Milhous [18] model, rather than using the fitted exponent values; and f) some of the models have been developed without (directly) involving the bed material relative size effects. Owing to these deficiencies/flaws with the existing bed load transport models it was required to develop a model for the field conditions.

To meet this requirement uni-size bed material data from the flume studies (i.e. CSU, Colorado State University; EPFL, Ecole Polytechnique Fédérale de Lausanne; and ETHZ, Eidgenossische Technische Hochschule Zürich) have been used to develop a philosophy and basis for the model development for the field conditions (i.e. for mixed size bed materials). For this purpose, first of all various individual empirical models (defining the relationship between the unit bed load discharge, qs and water discharge parameter, q-qc) have been developed. These use different slopes (S) and grain sizes (D_{50}), and assume four different cases for the relationship exponent β (β = fitted values; $\beta = 1$; $\beta = 1.5$; and $\beta = 2$). These are then transformed into four models (i.e. one for each β value) by using an appropriate statistical technique. The performance of these four models has been tested with the data from the River Severn, Wales (UK) and North Fork of South Platte River at Buffalo, Colorado (USA) and with the computed results of the Milhous [18] model. On the basis of their performance a model (with β = fitted values) has been selected as a base for model development for the mixed-size bed material (field conditions). Using this base and mixed-size (non uniform) bed material data from different rivers/streams studies (i.e. Elbow, Oak, Aare, Little South Fork-stations C-F, Gaula, Roaring - two sites, and Pitzbach) a deterministic model has been developed for the field conditions. Subsequently the performance of this model has been tested with the observed data of the River Severn and North Fork of South Platte River and with the Milhous [18] model results and has been found to be satisfactory. Later on, the developed model has been extended for the non-uniform cross-sectional flow rivers (single channel) through the incorporation of the active bed width function (for determining active bed width parameter, which plays important role in the transport of bed load when channel cross-section is non-uniform). The developed model has also been extended for the braided rivers with multiple channels.

However, before developing the model the performance of four existing (characteristic size based) models, belonging to different theories/approaches, has been tested.

PERFORMANCE TEST OF EXISTING (CHARACTERISTIC SIZE BASED) BED LOAD TRANSPORT MODELS

For the purpose of performance testing, the following models were selected, representative of different approaches.

1- Milhous [18] model, excess discharge based;
2- Meyer-Peter and Mueller [17] model, excess shear stress based;
3- Bagnold [6] model, excess stream power based; and
4- Parker et al. [21] model, based on concept of equal mobility and similarity approach.

Their performance was tested with the observed data of the Middle Fork of Boulder Creek at Netherland (Colorado) and Williams Fork near Leal (Colorado). These streams were selected for testing because their slopes (S) and bed material sizes (D_{50}) covered the range commonly available in coarse bed material streams (i.e. S = 0.5 - 1.7 % and D50 = 27 - 67 mm). Performance test results of these models (depicted in Figure 1) in comparison with the observed data of Middle Fork of Boulder Creek show that the Milhous [18] model performed better as the data points are scattered close to the line of perfect agreement (LPA). The Parker et al. [21] model performed worse since the data points generated by this model are located at the farthest distance from the LPA. Similarly, test of these models with the Williams Fork data, depicted in Figure 2, show that the performance of the Milhous model was relatively better, while the Meyer-Peter and Mueller model's performance was the worst. Its worst performance in this study confirmed the results of Yang [29] when he ranked the model at the bottom end. Likewise USGS [25] ranked this model at number three, during a performance test of five bed load transport models. This poor performance of the Meyer-Peter and Mueller model may be due to the reason that sediment size used in the model development was finer (ranged from 0.4 mm to 28.65 mm) than the streams' sediment sizes with which it was tested. For the Williams Fork, the performance of the Parker et al. [21] model was relatively better as data points were situated relatively closer to the LPA, which is different from its performance for the Middle Fork of the Boulder Creek. The Parker et al. model may have performed poorly partly because it is based on subsurface material and the concept of equal mobility and the value of τ^*_{ri} (= 0.0876) parameter in the model is site specific. This showed the inconsistency of performance of the existing models'. These inconsistent results are similar to those obtained by Gomez and Church [12] when they stated that "no formula performs consistently well". The poor performance of the Bagnold model confirmed the findings of Carson and Griffiths [8] study, according to which this model needs extensive calibration. From these two tests, with the Middle Fork and Williams Fork, a better performance of the Milhous [18] model is evident, although it is not good. These results, in general, are in agreement with the results of the recent studies carried out by Bathurst et al. [7], USGS [25], Milhous [18] and Inpasihardjo [15] who found that the

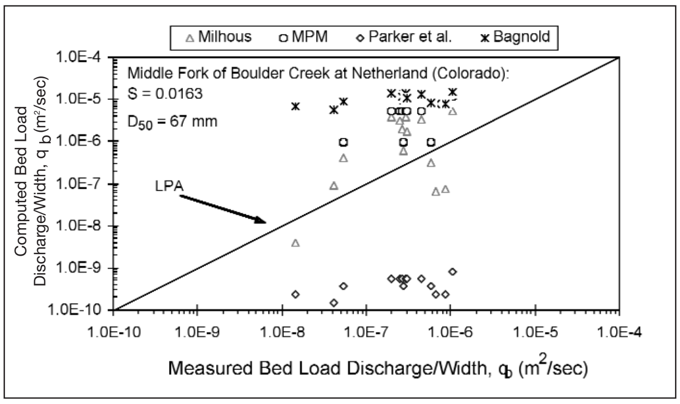


Figure 1: Comparison of measured bed load discharges of the Middle Fork of Boulder Creek at Netherland (Colorado) to the computed loads by four bed load transport models

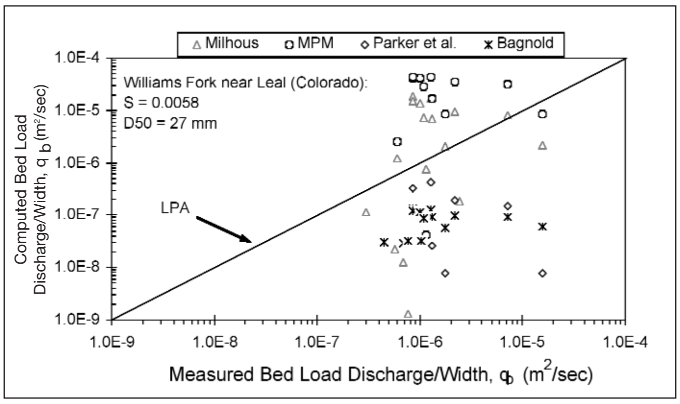


Figure 2: Comparison of measured bed load discharges of the Williams Fork near Leal (Colorado) to the computed loads by four bed load transport models

performance of excess discharge theory based models is better than the other theories (i.e. shear stress and stream power) based models. The Milhous model performed better partly because the data used in the development of it generally covers the sediment size range available in the coarse bed material rivers.

MODEL DEVELOPMENT FOR BED LOAD TRANSPORT (FOR FIELD CONDITIONS)

Basis for Model Development

From the results of different recent studies as mentioned above and the results of this study the inconsistent and poor performance of the existing characteristic size based bed load transport models, in general, is evident. The excess discharge theory based models performed (relatively) better but even so their performance was not satisfactory. It is, therefore necessary to develop this theory further using a sufficiently large data base, so that optimum results may be obtained. For this purpose the following deterministic model, based on excess discharge theory, is proposed.

$$q_s = \alpha\,(q - q_c)^{\beta} \qquad (1)$$

where

q_s = bed load discharge/width (m²/sec); α = coefficient; q = water discharge/width (m²/sec); q_c = critical water discharge/width (m²/sec).

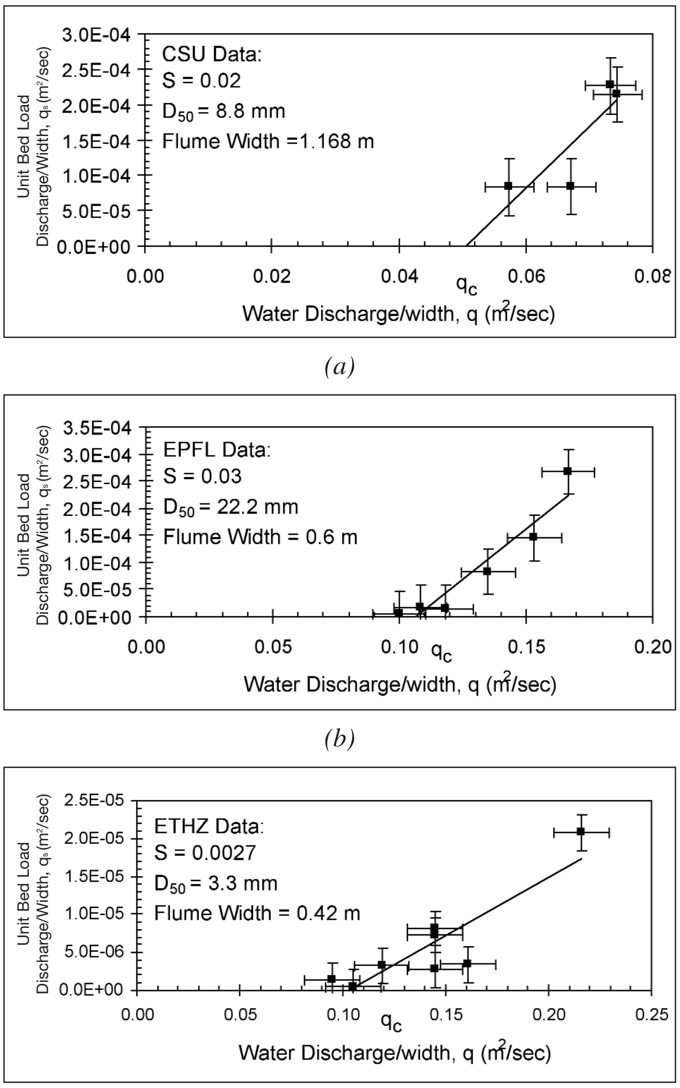
So far as methodology is concerned values of α and β will first be determined by using the uni-size bed material data for each case and then it will be checked how α and β vary. After examining their variations a model for field conditions has been developed, using mixed-size (non-uniform) bed material data.

Developing Individual Empirical Models to Examine the Basic Relationships Using Uni-Size Bed Material Data and to Quantify Equation (1)

In the development of these models (from the flume data) critical discharges were required which were determined for each set of values (i.e. with one D_{50} value and different slopes, S) which are given in Table 1. The method of "back extrapolation" (in which a line is fitted to data and its coefficient with the x-axis is noted) followed for determining critical discharges (q_c) is illustrated in Figure 3; however, only three values of critical discharges are shown in this figure for demonstration purpose i.e. one case for each of the CSU, EPFL, and ETHZ studies having 4, 6, and 8 data points, respectively. Since in the figure there are not many data points, therefore in order to show the likely errors, standard error bars (i.e. both X and Y error bars, positive and negative) have been plotted. However, one thing should be remembered that the critical discharge values determined here by (regression) fitted lines can be and are usually determined by eye fitted lines (e.g. as done by Bathurst et al. [7] and Inpasihardjo [15]), therefore use of this regression is not important at all - that is why the

Table 1: Critical discharges for the uni-size bed material data (i.e. CSU, EPFL, and ETHZ studies data) and other relevant parameters

Study (1)	No of Data Points (2)	Bed Material Size (D_{50}) (m) (3)	Slope (S) (m/m) (4)	Critical Discharge (q_c) (m²/sec) (5)
CSU	4	0.0088	0.02	0.051
	6	0.0088	0.05	0.010
	6	0.0088	0.08	0.0095
	6	0.034	0.08	0.046
EPFL	3	0.0222	0.01	0.265
	6	0.0222	0.03	0.105
	7	0.0222	0.05	0.065
	8	0.0222	0.07	0.035
	6	0.0222	0.09	0.025
	4	0.0443	0.03	0.225
	6	0.0443	0.05	0.127
	4	0.0443	0.07	0.092
	6	0.0443	0.09	0.072
	6	0.0115	0.005	0.171
	4	0.0115	0.0075	0.133
	4	0.0115	0.01	0.114
ETHZ	4	0.02865	0.0107	0.520
	8	0.0033	0.0027	0.105

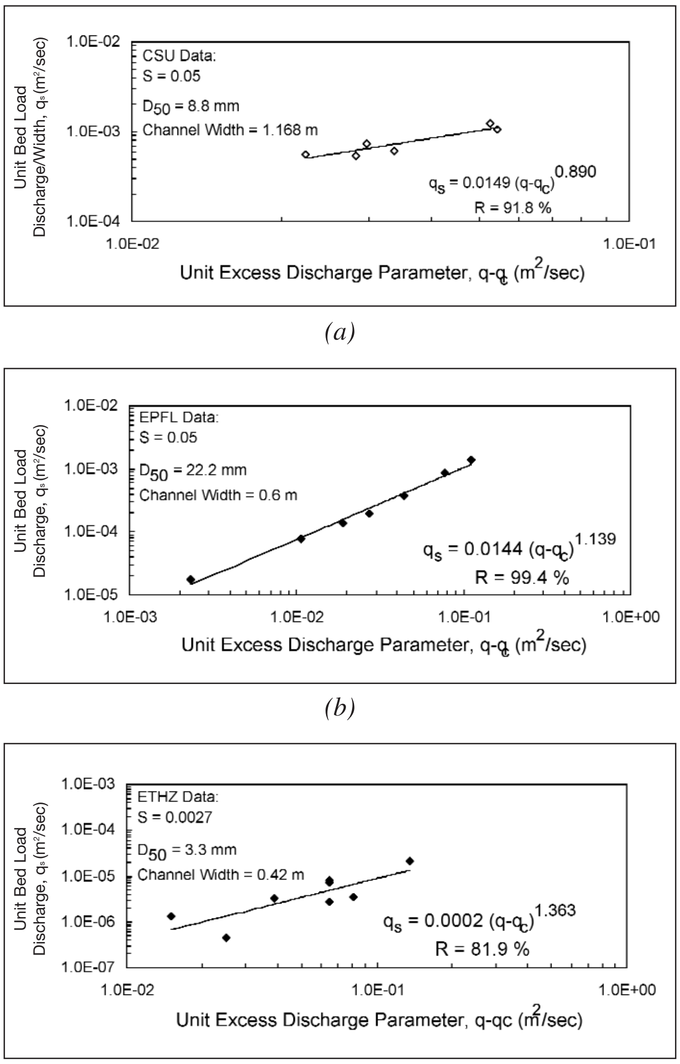


(a)

(b)

(c)

Figure 3: Critical discharge determination for uni-size bed materials; a) CSU data, b) EPFL data, and c) ETHZ data



(a)

(b)

(c)

Figure 4: Individual empirical models for bed load discharge for uni-size bed materials; a) CSU data, b) EPFL data, and c) ETHZ data

correlation coefficient, confidence limit and other regression techniques are not investigated here. By using these critical discharges (given in Table 1) empirical models quantifying Equation (1) for each D50 data set (and respective slope) were developed which are given in Table 2. In the development of these models three data sets of CSU, EFPL and ETHZ were used. The statistical technique used was the Power Model as it was found best among all the six techniques (i.e. Linear, Power, Polynomial, Logarithmic, Exponential, Through Origin) that were tested. These models consist of 4 based on CSU data, 12 based on the EPFL data; and 2 based on ETHZ data (for detail see Table 2). Figure 4 illustrates the development of these models, taking one case for each flume, along with their respective correlation coefficient (R) values. These individual models were developed considering four different cases (conditions), which are:

1. first case: models were developed without any condition i.e. with the fitted exponent (β) and coefficient (α) and are given in column 6 of Table 2;
2. second case: comprises development of models when β was fixed equal to 1 and α was adjusted to the best fit under this constraint. These models are presented in column 7 of Table 2;
3. third case: in the development of these models β was fixed equal to 1.5 and α was adjusted to the best fit under this constraint. The models developed in this case are given in column 8 of Table 2; and
4. fourth and final case: includes models in which β was fixed equal to 2 and α was adjusted to the best fit under this constraint. Models developed for this case are given in column 9 of Table 2.

The reason for β =1, β =1.5, β = 2 (in the latter three cases) is that this is typical of the range in other models (i.e. existing models).

Transformation of Individual Models to Four Deterministic Models to Develop Basis for Model Development for Field Conditions.

After developing the individual empirical models (given in Table 2) there were different ways to combine them in one deterministic model that could be representative for all. One well known approach was the similarity approach but it involves graphical work which may introduce more error, as too many fitted lines have to collapse in this approach. Another

Table 2: Uni-size bed-material based sediment transport models (empirical) along with bed-material sizes and slopes for different flume studies including CSU, EPFL, and ETHZ studies

Data Set	No Of Data Points	Bed Material Size D_{50} (m)	Channel Width (W) (m)	Slope (S) (m/m)	Bed Load Discharge Empirical Models			
					with fitted slope and coefficient Case - 1	with constant slope of 1 and changing coefficient Case - 2	with constant slope of 1.5 and changing coefficient Case - 3	with constant slope of 2 and changing coefficient Case - 4
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
CSU								
	4	0.0088	1.168	0.02	$qs = 0.086 * (q-qc)^{0.284}$	$qs = 0.011 * (q-qc)^{1}$	$qs = 0.004 * (q-qc)^{1.5}$	$qs = 0.001 * (q-qc)^{2}$
	6	0.0088	1.168	0.05	$qs = 0.0149 * (q-qc)^{0.890}$	$qs = 0.013 * (q-qc)^{1}$	$qs = 0.003 * (q-qc)^{1.5}$	$qs = 0.0005 * (q-qc)^{2}$
	6	0.0088	1.168	0.08	$qs = 0.057 * (q-qc)^{1.001}$	$qs = 0.061 * (q-qc)^{1}$	$qs = 0.011 * (q-qc)^{1.5}$	$qs = 0.002 * (q-qc)^{2}$
	6	0.034	1.168	0.08	$qs = 0.095 * (q-qc)^{0.842}$	$qs = 0.070 * (q-qc)^{1}$	$qs = 0.026 * (q-qc)^{1.5}$	$qs = 0.095 * (q-qc)^{2}$
EPFL								
1	6	0.0222	0.6	0.01	$qs = 1.632E-05 * (q-qc)^{1.797}$	$qs = 0.0005 * (q-qc)^{1}$	$qs = 0.00004 * (q-qc)^{1.5}$	$qs = 0.000003 * (q-qc)^{2}$
	3	0.0222	0.6	0.03	$qs = 8.532E-05 * (q-qc)^{1.808}$	$qs = 0.001 * (q-qc)^{1}$	$qs = 0.0002 * (q-qc)^{1.5}$	$qs = 0.00002 * (q-qc)^{2}$
	7	0.0222	0.6	0.05	$qs = 0.0144 * (q-qc)^{1.139}$	$qs = 0.008 * (q-qc)^{1}$	$qs = 0.001 * (q-qc)^{1.5}$	$qs = 0.0002 * (q-qc)^{2}$
	8	0.0222	0.6	0.07	$qs = 0.007 * (q-qc)^{1.297}$	$qs = 0.016 * (q-qc)^{1}$	$qs = 0.0021 * (q-qc)^{1.5}$	$qs = 0.0003 * (q-qc)^{2}$
	6	0.0222	0.6	0.09	$qs = 0.045 * (q-qc)^{0.932}$	$qs = 0.028 * (q-qc)^{1}$	$qs = 0.004 * (q-qc)^{1.5}$	$qs = 0.0006 * (q-qc)^{2}$
2	4	0.0443	0.6	0.03	$qs = 4.859E-07 * (q-qc)^{2.625}$	$qs = 0.0004 * (q-qc)^{1}$	$qs = 0.00004 * (q-qc)^{1.5}$	$qs = 0.000004 * (q-qc)^{2}$
	6	0.0443	0.6	0.05	$qs = 5.512E-08 * (q-qc)^{3.343}$	$qs = 0.002 * (q-qc)^{1}$	$qs = 0.0001 * (q-qc)^{1.5}$	$qs = 0.00001 * (q-qc)^{2}$
	4	0.0443	0.6	0.07	$qs = 0.0004 * (q-qc)^{1.796}$	$qs = 0.007 * (q-qc)^{1}$	$qs = 0.0007 * (q-qc)^{1.5}$	$qs = 0.00008 * (q-qc)^{2}$
	6	0.0443	0.6	0.09	$qs = 0.003 * (q-qc)^{1.517}$	$qs = 0.016 * (q-qc)^{1}$	$qs = 0.0018 * (q-qc)^{1.5}$	$qs = 0.0002 * (q-qc)^{2}$
3	6	0.0115	0.6	0.005	$qs = 4.801E-05 * (q-qc)^{1.429}$	$qs = 0.0005 * (q-qc)^{1}$	$qs = 0.00008 * (q-qc)^{1.5}$	$qs = 0.000015 * (q-qc)^{2}$
	4	0.0115	0.6	0.0075	$qs = 1.181E-04 * (q-qc)^{1.273}$	$qs = 0.0006 * (q-qc)^{1}$	$qs = 0.0001 * (q-qc)^{1.5}$	$qs = 0.00002 * (q-qc)^{2}$
	4	0.0115	0.6	0.01	$qs = 1.577E-05 * (q-qc)^{1.801}$	$qs = 0.0009 * (q-qc)^{1}$	$qs = 0.00014 * (q-qc)^{1.5}$	$qs = 0.00002 * (q-qc)^{2}$
ETHZ								
1	4	0.02865	2	0.0107	$qs = 0.0005 * (q-qc)^{1.462}$	$qs = 8.2*10-8 * (q-qc)^{1}$	$qs = 1.7*10-12 * (q-qc)^{1.5}$	$qs = 5*10-17 * (q-qc)^{2}$
2	8	0.0033	0.42	0.0027	$qs = 0.0002 * (q-qc)^{1.363}$	$qs = 8.5*10-8 * (q-qc)^{1}$	$qs = 3*10-11 * (q-qc)^{1.5}$	$qs = 4*10-14 * (q-qc)^{2}$

NB: With a few exceptions the values of correlation coefficient (R) for all these models were greater than 85%.
q, qc and qs are unit water discharge, critical water discharge and bed load discharge (m3/sec/m), respectively.

approach was to develop a representative empirical model for each data set (i.e. one for each CSU, EPFL and ETHZ) and then plot α and β versus slope (S, average value) and particle size (D_{50} , average value) - two variables that play important role in the transport of bed load. This approach (partly statistical and graphical) was tried up to the very last stage but due to the insufficient data points for slopes and particle sizes it was rejected. Finally, a decision was taken to combine the individual models (in terms of α and β) through multi-variate analysis (a statistical technique in which a response variable is related to more than one predictor variables - using the statistical package of MINITAB).

A combined representative function (dimensionless) for the exponent, β (= f [S, D_{50}/W]) was obtained with a reasonable value of correlation coefficient, R (i.e. 77.3%). Nonetheless, the combined representative function for the coefficient, α (= f[S, D_{50}/W]) gave a poor value of correlation coefficient. Therefore for combining coefficients, α (= f[S]) a power model approach was used which provided a high value of the correlation coefficient. On the other hand the relationship between β values and slopes (S) was also explored and was found very poor. The representative functions for α and β for the above mentioned four cases are

Case - I

β = fitted values

Combining β (individual) values for one function (R = 77.3 %)

$$\beta = 1.02 - 7.16S + 22.5 \frac{D_{50}}{W} \tag{2}$$

Combining α (individual) values for one function (R = 50 %)

$$\alpha = 0.113 * S^{1.490} \tag{3}$$

Case - II

β = 1 (R = 84 %)

$$\alpha = 9.85 * S^{2.375} \tag{4}$$

Case - III

β = 1.5 (R = 77 %)

$$\alpha = 12.698 * S^{3.125} \tag{5}$$

Case - IV

β = 2 (R = 74 %)

$$\alpha = 10.34 * S^{3.711} \tag{6}$$

where

S = slope and ranged between 0.003 and 0.09 (m/m); D_{50} = characteristic size of the bed surface material (m) and ranged between 0.003 m and 0.045 m; and W = channel width (m) ranging between 0.6 and 2 m. In Equation (3) though there is not a strong relationship between α and S (slope), however, it should be remembered that the performance of Equation (1) (subsequently converted into Equations (7) to (10) depends not only upon the α parameter (function of S) but also upon the combined effect of the α and β parameters. This is best in the case of Equation (7), as proved by the performance test. On the other hand Equations (4) to (6) have good relationships also.

By substituting the values of α and β into Equation (1) the bed load discharge deterministic models for all the four cases can be obtained and are

when β = fitted values

$$q_s = 0.113 * S^{1.490} (q - q_c)^{1.02 - 7.16S + 22.5} \frac{D_{50}}{W} \tag{7}$$

when β = 1

$$q_s = 9.82 * \mathrm{S}^{2.375} (q - q_c) \tag{8}$$

when β = 1.5

$$q_s = 12.698 * S^{3.125} (q - q_c)^{15} \tag{9}$$

when β = 2

$$q_s = 10.34 * S^{3.711} (q - q_c)^2 \tag{10}$$

The above mentioned models (Equations (7) to (10) are based on the assumption that sediment is moving over the whole flow width of the channel cross-section (i.e. surface flow width = active bed width). The values of α and β vary significantly between data sets (and the data are anyway limited). The attempt to fit single α and β relationships is therefore in the context of a study, to investigate possible links between flow rate and sediment discharge.

Model Performance

To check the performance of the developed models (Equations (7) to (10)) they were applied to the River Severn and North Fork of South Platte River at Buffalo (Colorado) data to compute bed load discharges. These two streams were particularly selected for the test because their bed material sizes (D_{50}) were almost in the same range as that of the materials used in the flume studies, whose data were used in the development of the models. The computed bed load discharges by the developed models were plotted against the observed bed load discharges from the rivers and results are depicted in Figures 5 and 6, respectively. As evident from

Figure 5 (for River Severn) data points by Equation (7) (i.e. β = fitted values) are mostly located either on or in the close proximity of the line of perfect agreement (LPA), whereas data points generated by Equations (8) to (10) are located below the LPA which shows underestimation of computed loads by these models, nevertheless, among these models (Equations (8) to (10) results of Equation (8) (β=1) were better.

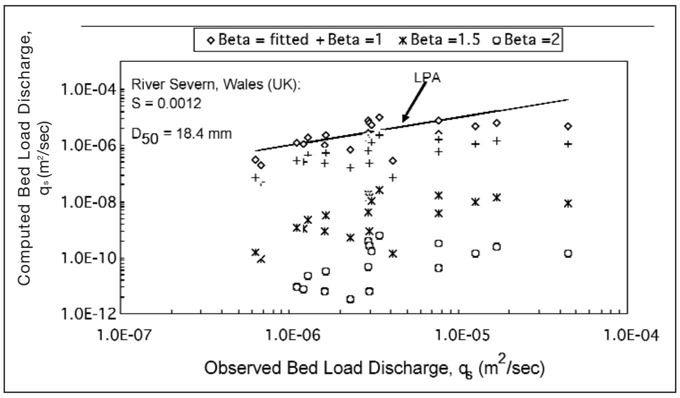


Figure 5: Comparison of bed load discharges computed by the four deterministic, uni-size based, models (Equations 7-10) with the observed loads of River Severn (UK)

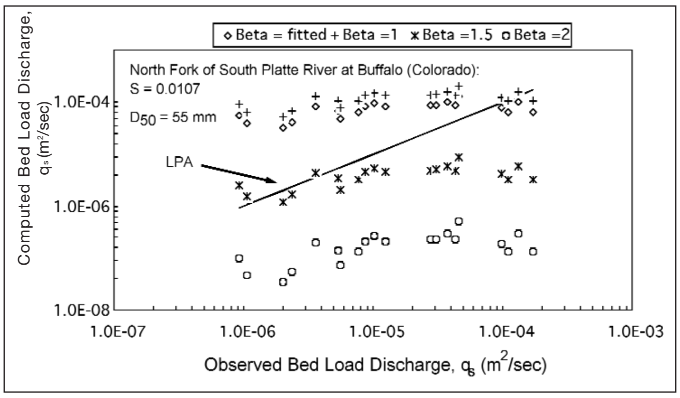


Figure 6: Comparison of bed load discharges computed by the four deterministic, uni-size based, models (Equations 7-10) with the observed loads of North Fork of South Platte River at Buffalo, Colorado (USA)

On the other hand, for the North Fork of South Platte River, Equation (7) performed better at the higher observed load discharges, compared with the lower discharges when it overestimated the computed loads. The performance of this equation was somewhat similar to Equation (8). However, Equation (8) overestimated the computed loads. Equation (9) (β = 1.5) performed better at the lower discharges but it underestimated the computed loads at the higher discharges. In other words it can be said that performance of Equation 7 improved with the increase in load discharges while performance of Equation 9 got worse with the increase in discharge, especially at the higher discharges. Thus, based upon these results, for River Severn and North Fork of South Platte River, it is not difficult to rank performance of the models in the following descending order:

Equation (7), with β = fitted values
Equation (8), with β =1
Equation (9), with β =1.5; and
Equation (10), with β = 2.

Since, the model with the fitted values of β (Equation (7)) performed better than the other models, it will be used as the basis for developing a (forthcoming) mixed-size bed-material model for the field conditions.

DEVELOPMENT OF MIXED-SIZE (NON UNIFORM) BED-MATERIAL BASED MODEL (MBB MODEL) FOR FIELD CONDITIONS

Development of Individual Models

Based on the uni-size bed material model's philosophy for development of these models only the fitted value case was followed. For developing the individual empirical bed load discharge models for different rivers the values of critical discharge were required for these rivers which were determined by back extrapolation of q_s/q and are given in Table 3. Then by using the same approach as used in the case of the uni-size bed-material based empirical models various empirical models were developed for the Elbow River, Oak Creek, Aare River, Little South Fork (Station C-F), River Gaula, Roaring River (two sites) and Pitzbach site. In the case of the Elbow and Gaula rivers two models for each river were developed, one for the surface flow width and other for the active bed width data. For demonstration purpose a surface flow width based model for the Elbow River is presented in Figure 7. Similarly, two models were developed for the Little South Fork Station-C since it has two values of critical discharge, qc as the data were into two separate groups (i.e. non linear variation). So far as all the other rivers are concerned only one model was developed for each river. The developed models for each river site are presented in column 6 of Table 4.

Generating a Single Deterministic Model

The individual empirical models given in Table 4 were combined (except active bed width based models) in terms of single α and β functions by using the multi-variate analysis approach (a statistical approach with which a response variable can be related to more than one predictor variables). The reasons why to use this approach have been explained under an earlier article "Transformation of Individual Models to.….". The statistical analysis package used for the multi-variate analysis was MINITAB. This approach was opted for both α and β as it provided higher correlation coefficient (R) values of 86% and 85% respectively, unlike the uni-size bed-material when it worked well only for the β function. The developed functions for α and β are

$$\alpha = Exp\left[-11.8 - 36.3S + 131\frac{D_{50}}{W}\right] \qquad (11)$$

Table 3: Critical discharges for the field studies data (i.e. for rivers) along with other relevant parameters

Study (1)	No. of Data Points (2)	Bed Material Size (D_{50}) (m) (3)	Slope (S) (m/m) (4)	Critical Discharge (q_c) (m^2/sec) (5)
Elbow River	23	0.076	0.00745	0.95
Oak Creek	12	0.054	0.01	0.29
Aare River	34	0.070	0.0023	3.20
Little South Fork Station - C	23	0.033	0.02	0.04
				0.24
Station - D	20	0.042	0.014	0.46
Station - E	22	0.042	0.0105	0.34
Station - F	12	0.038	0.015	0.295
River Gaula	52	0.080	0.0024	4.4
Roaring River a) Alluvial Fan Road Bridge Site (1985).	30	0.077	0.047	0.142
b) Ypsilon Lake Trail Bridge Site (1985).	22	0.106	0.037	0.163
Pitzbach (1991)	33	0.098	0.0395	0.138

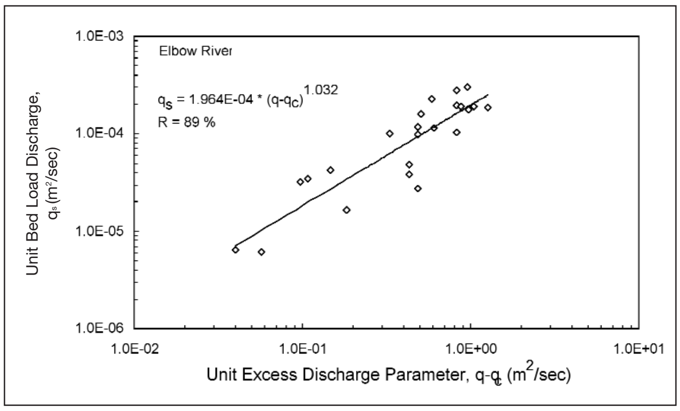


Figure 7: Individual empirical model development (for bed load discharge) for the Elbow River (Canada), using surface flow width of the channel

$$\beta = 1.01 - 14.5S + 51.2\,\frac{D_{50}}{W} \qquad (12)$$

where

S = slope, ranged between 0.2% and 5%; D_{50} = median size of the bed surface material (m) and ranged between 0.033 m and 0.106 m; and W = channel width, ranging between 6 and 120 m.

By substituting the values of α and β functions from Equations (11) and (12) into Equation (1) the generated deterministic model (i.e. MBB model) is

$$q_S = Exp\left[-11.8 - 36.3S + 131\,\frac{D_{50}}{W}\right] * \left\{q - q_C\right\}^{1.01-14.5S+51.2\,\frac{D_{50}}{W}} \qquad (13)$$

Performance Test of MBB Model

The relative performance of the developed model (Equation (13)) was investigated. First, unit bed load discharges were computed by using the model and data from the River Severn and North Fork of South Platte River. Then these computed loads were compared with the observed loads of the River Severn and North Fork and with the loads computed by the Milhous [18] model, a model that performed best in the performance test. The loads computed by the MBB and Milhous models were plotted against the observed loads recorded from both of the streams which are depicted in Figures 8 and 9. Figure 8 shows that sediment loads computed by the MBB model are situated close to the line of perfect agreement (LPA) for River Severn compared with the Milhous [18] model. Nevertheless, a trend of overestimation is dominant that is in-contrast to the Milhous model results that underestimate the loads, especially at lower flows. Therefore, it can be said that the overall performance of the model (i.e. MBB model) was better than that of the Milhous model. For the North Fork, data points generated by the model (MBB model) are almost horizontal and the model is therefore not very sensitive to whatever the change is which causes the measured data to vary. Nonetheless, these data points are mostly scattered close to the LPA, though at higher flows some data points are located lower than the LPA, indicating underestimation of loads at the higher flow rates. On the other hand the Milhous model overestimates the sediment loads at lower flows, as the data points lie above the LPA, and underestimated the

Table 4: Mixed-size bed-material based bed load transport models along with bed-material sizes and slopes for different rivers

Site (1)	No. of Data Points (2)	Bed Material D_{50} (m) (3)	Channel Width (m) (4)	Slope S (m/m) (5)	Bed Load Discharge Models (6)
Elbow River	23	0.076	43	0.00745	$q_s = 1.964\text{E-}04 * (q-q_c)^{1.032}$ (surface width based) $q_s = 2.367\text{E-}05 * (q-q_c)^{1.609}$ (active width based)
Oak Creek	12	0.054	3.66	0.01	$q_s = 4.868\text{E-}05 * (q-q_c)^{1.709}$
Aare River	34	0.070	14.8	0.0023	$q_s = 4.6271\text{E-}05 * (q-q_c)^{1.5}$
Little South Fork Station -C	23	0.033	6.54	0.02	$q_s = 1.607\text{E-}06 * (q-q_c)^{0.84}$ (using lower limit of q_c) $q_s = 5.915\text{E-}06 * (q-q_c)^{0.94}$ (using upper limit of q_c)
Station - D	20	0.042	12.64	0.014	$q_s = 1.831\text{E-}06 *(q-q_c)^{0.744}$
Station - E	22	0.042	11.34	0.0105	$q_s = 2.92\text{E-}06 * (q-q_c)^{1.39}$
Station - F	12	0.038	15.8	0.015	$q_s = 7.127\text{E-}06* (q-q_c)^{0.946}$
River Gaula	52	0.080	119.93	0.0024	$q_s = 1.491\text{E-}06* (q-q_c)^{0.663}$ (surface width based) $q_s = 8.580\text{E-}09 * (q-q_c)^{3.79}$ (active width based)
Roaring River					
a) Alluvial Bridge Site	30	0.077	6.1	0.047	$q_s = 5.165\text{E-}05* (q-q_c)^{1.403}$
b)- Ypsilon Lake Trial Bridge Site	22	0.106	6.25	0.037	$q_s = 6.553\text{E-}06* (q-q_c)^{1.006}$
Pitzbach	33	0.098	8	0.0395	$q_s = 4.23\text{E-}06* (q-q_c)^{0.883}$

NB: With a few exceptions the values of the correlation coefficient (R) for all these models were greater than 85%.
q, q_c and q_s are unit water discharge, critical water discharge and bed load discharge (m^3/sec/m), respectively.

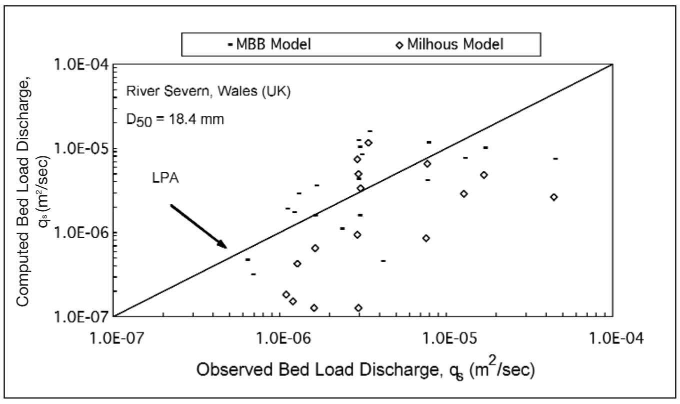


Figure 8: Comparison of observed and computed bed load sediment discharges by MBB and Milhous Models for the River Severn, Wales (UK)

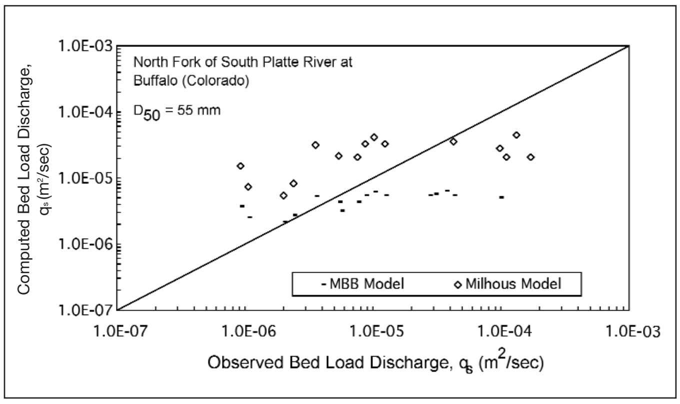


Figure 9: Comparison of observed and computed bed load sediment discharges by MBB and Milhous Models for North Fork of South Platte River, Colorado (USA)

loads at the higher flow rates. Generally, a trend of overestimation was found for this model, which is opposite to the MBB model results. Thus from both of the analyses (i.e. for River Severn and North Fork) a better performance of the developed model (i.e. MBB) is evident. However, while applying this model the critical discharge (qc) value should be determined carefully as a minor error in it could affect the computed loads considerably. The Milhous model's unsatisfactory performance for the North Fork stream is contrary to its performance for the Middle Fork and Williams Fork, which could be due to variation among the channel characteristics (i.e. D50 and S etc.). This varying performance of the Milhous model for different streams is in accordance with the study results obtained by Ashiq et al. [2] and Gomez and Church [12] who found that no formula (model) perform consistently well under varying field conditions. During these performance tests it has been found that the bed material size and slope were the two significant parameters that affected the models' performance.

MBB MODEL EXTENSION FOR NON UNIFORM CROSS-SECTIONAL CHANNELS

If a river channel cross-section is non uniform then transport of bed load does not take place uniformly across the width i.e. only some part (or parts) of the flow width (i.e. active bed width) acts as a sediment transport carrier while remaining part (or parts) acts just as a flow passage. For such rivers bed load transport computation by using surface flow width will lead to overestimation of the loads. To compute true amount of bed load discharge passing in a river it is therefore necessary to know how much width of the channel is active or transporting bed load (i.e. active bed width), which can be computed by Equation (1) developed by Ashiq [3].

$$Log\left[\frac{W_A}{W}\right] = -0.173\ (1 + 0.54\ q) \tag{14}$$

where

q = unit water discharge (m^3/sec/m); W_A = Active bed width (m); and W = surface flow width (m).

The computed active bed width parameter, W_A than be used to replace the surface width parameter, W in the MBB model (Equation (13)) which then may be used to compute bed load discharge for rivers with non uniform cross-sectional flow depth channels (i.e. surface flow width ≠ active bed width). While using Equation (14) one should bear in mind that there is spurious correlation, as W is involved in both q (i.e. Q/W) and Wa/W parameters. The effect of the spurious correlation has not been investigated here. Details regarding spurious correlation may be seen in statistics books written by Hald [14] and Aitchison [1].

MBB MODEL EXTENSION FOR BRAIDED RIVERS

The model developed earlier for the field conditions for the uniform cross-sectional rivers (Equations (13)) and its application to non-uniform cross-sectional flow depth rivers in conjunction with Equation (14) is useful only when rivers have single channel. If the river is braided (i.e. water flows in multiple channels) then it is necessary to apply Equations (13) and (14) to each individual channel, according to the prevailing condition and then sum up all the loads. Thus the model for the braided rivers would be

$$Q_b = \sum_{i=1}^{n} W_i q_{si} + \sum_{j=1}^{n} W_{AJ} q_{s(A)j} \tag{15}$$

where

Q_b = total bed load discharge in a braided river (m^3/sec); W_i = water surface flow width for channel i (m); q_{si} = unit bed load discharge (m^3/sec/m) for channel i for which whole channel flow width acts as sediment carrier (surface flow width = active bed width) and can be computed by Equation (13); W_{Aj} = active bed width of channel j; and $q_{s(A)j}$ = unit bed load discharge (m^3/sec/m) for channel j (surface flow width ≠ active bed width) and can be computed by Equation 13 in conjunction with Equation 14.

RESULTS AND DISCUSSION

The performances of four existing bed load transport models (characteristic diameter based), belonging to different theories/approaches, were tested with the observed data of the Middle Fork of Boulder Creek at Netherland (Colorado) and Williams Fork near Leal (Colorado). It was found that the performance of the excess discharge theory based model is better. These results are in accordance with the results of the studies carried out by USGS [25], Milhous [18] and Inpasihardjo [15]. The poor performance of the Bagnold's model confirmed the viewpoint of Carson and Griffiths [8] who suggested a need of extensive calibration of the model. The performance of the Meyer-Peter and Mueller model was found poor perhaps because the data used in the model development was of finer size than that of the streams used for the testing purpose. The Parker et al. [21] model performed unsatisfactorily, probably because it is based on the subsurface material, concept of equal mobility, and the τ^*_{ri} (= 0.0876) parameter used in the model is site specific. These are reasons that could have pursued Parker [20] to develop his surface material based model. During the investigation, models generally performed inconsistently with different data sets in agreement with the study results of Schulits and Hill [23], White et al [26&27], Yang [29], Nakota [19], Gomez and Church [12], USGS [25], Woo et al. [28], Ashiq [2, 4 and 5], Sun and Donahue [24], Kleinhans and Rijn [16], Habersack and Laronne [13], Espinosa et al. [11] etc. The reasons why these models performed poorly have been mentioned earlier. Likewise, Carson and Griffith [8] have stated two main reasons for the models' poor performance: a) shortage of data for coarse bed material rivers; and b) assumptions and boundary conditions used in the development of models. Bed load transport studies tend to be empirical and therefore are constrained by lack of data.

A deterministic model (Equation (13), MBB model) using the characteristic diameter approach and the excess discharge theory, based on the mixed size bed material (non uniform) and on the philosophy of the fitted exponents - developed from the uni-size bed-materials (using flume data), has been developed for field conditions. This model performed satisfactorily when tested with the observed data of the River Severn (Wales, UK) and North Fork of South Platte River (Colorado) and with the results of the Milhous [18] model- a model that performed best in the earlier test of models. However, when using this model one should be very careful about the computation of critical discharge value (q_c) as a small variation in it could considerably influence the computed bed load discharge. This model was tested with data that fell within the size range of bed material that was used in the model development; how it would behave outside this size range is still to be investigated. Therefore, before generalising the model for common use it should be tested with different data sets having a wide size range.

The MBB model has been extended for determining the bed load transport for channels with non-uniform cross-sections flow depths (i.e. surface flow width ≠ active bed width). Likewise, the model has also been further extended for the braided rivers. Owing to the lack of data, it was not possible to test the performance of the extended models' therefore they should be tested before further use. No other researcher has investigated this topic and the study here is therefore only an initial investigation of a possible relationship between bed load transport and water discharge for non-uniform cross-sectional flow depth channels.

SUMMARY

Performance of the existing bed load transport models, based upon different theories/approaches, was investigated. It was found that results from the excess discharge theory based models were relatively better though not satisfactory. To further improve this theory the MBB model (Equation (13)) for field conditions based on the mixed-size bed material data (published) has been developed by using the fitted exponent values (a new approach) instead of following the customary assumed exponent values approach. This model is based on the assumption that sediment is moving over the whole flow width of the channel cross-section (i.e. surface flow width = active bed width). Model's performance was tested with the observed data from the River Severn (Wales, UK) and North Fork of South Platte River at Buffalo (Colorado) and found satisfactory. Likewise, this model performed better when results were compared with the Milhous model. This model, later on, was extended for the non-uniform cross-sectional channels and braided channels.

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NOTATIONS

D_{50}	median size of bed surface material (m).
LPA	line of perfect agreement.
MBB	mixed-size bed material based model.
q	unit water discharge (m^3/sec/m).
q_c	critical unit water discharge (m^3/sec/m).
q_s	unit sediment discharge (m^3/sec/m).
$q_{s\,(A)j}$	unit bed load discharge (m^3/sec/m) for channel j.
q_{si}	unit bed load discharge (m^3/sec/m) for channel i.
Q_b	total bed load discharge (m^3/sec).
R	correlation coefficient.
S	channel slope (m/m).
W	mean channel (surface) width (m).
W_A	mean active bed width (m).
W_i	mean water surface flow width for channel i (m).
W_{Aj}	active bed width of channel j (m).
α	coefficient.
β	relationship exponent.
τ^*_{ri}	reference shear stress parameter used in Parker et al. (1982).

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