

# COMPUTATION OF WAVE-MAKING RESISTANCE OF WIGLEY HULL FORM USING MICHELL'S INTEGRAL

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## ABSTRACT

*The aim of the present paper is to compute the wave-making resistance of the Wigley mathematical hull of different cross-sections (parabolic, rectangular and triangular). Michell gave the first approximate hydrodynamic solution to the problem of wave resistance experienced by a slender ship-shaped body moving on the surface of an inviscid fluid of infinite depth. Based on Michell's integral a computer program has been developed and executed in order to analyze the hydrodynamic characteristics of ships in an unbounded liquid and the validity of the computer scheme is verified by comparing it with the experimental results.*

**Keywords:** *Linearised Free Surface Condition, Michell's Integral, Thin Ship, Wave-Making Resistance*

## 1. INTRODUCTION

The determination of wave resistance of ship hulls is one of the most important and interesting subjects in ship theory. Model testing facilities were primarily built for the purpose of investigations of this part of ship resistance. Since Froude's time, an immense amount of experimental results have been published. It can, however, be safely stated that the purpose of representing the resistance of a ship in terms of its form has not been served by experimental methods in a general manner. The appreciable differences in resistance observed at times between ship models of seemingly insignificant variations of shape can not be satisfactorily explained on the basis of our experimental experience.

Michell [1] was the first to treat the wave resistance of ships moving at constant speed in smooth water of infinite depth. The work of Michell was unfortunately overlooked and forgotten for many years until it was rescued by the work of Havelock [2]. Since 1909, Havelock [2] published many papers on ship waves and wave resistance with the ship replaced by a moving pressure distribution. In 1923, Havelock [2] began to exploit Michell's [1] results.

In 1925 Havelock [2] gave an alternative derivation based on the Green function instead of the modified Fourier-integral theorem developed and used by Michell [1]. The alternative method developed by Havelock [2], in which the wave-making resistance is measured by the energy in the wave system, makes use of the idea of sources and sinks. Michell's [1] thin ship can be simulated by a distribution of sources on the centerline plane of the forebody and of sinks in the afterbody, the sum of their total strength being zero. Therefore, Michell's [1] results could be considered as a special case of the source and sink distribution method. The power of the source and sink distribution method is that it could be used to simulate the flow around different body shapes to find the wave pattern, pressure distribution, and resistance. Havelock [2] found the wave pattern far astern by this method, and from energy considerations, obtained the wave-making resistance.

Wehausen [3] gave a more comprehensive introduction to the subject of wave resistance of ships, which includes the measurement and the analytical theories of wave resistance. Dawson [4] came up

with a numerical method in calculating the linear wave resistance, which was a major development in numerical analysis of wave resistance problems. The general approach to solve the wave-resistance problem before Dawson [4] was based on the concept of systematic perturbation. The perturbation potential which perturbs the known basic-flow potential should be a small quantity compared to the basic flow potential. The basic flow used in thin-ship theory is the uniform stream flow, whereas in Dawson's [4] method, the basic flow is the double-body flow, in which the free surface is treated as a wall (no waves) and the solution is sought below the water level around the hull. According to the mirror image method in fluid mechanics, it could be imagined as if there were a symmetric hull above the water plane which forms a double body together with the hull below the water plane. The solution could be found by solving the problem of the flow around the double body with no free surface in infinite water region. For the wave-resistance problem of ships with low forward speed, Dawson's [4] method is very reasonable in theory, therefore, it is called the low-speed theory. The use of double-body flow as the basic flow is one of the characteristics of Dawson's [4] method.

With the rapid development in computer resources, Raven [5] improved linear Dawson's [4] method in gridding and numerical processing. Bertram [6] used Dawson's [4] method to predict ship motions in waves. Peng [7] developed a computational method for the resistance in calm water and for sea keeping performance in waves of multi-hull ship. The wave resistance of multi-hull ships was expressed as an extension of the traditional wave resistance of Michell Integral [1] for thin nonsymmetrical ship hull forms by including, in addition to the centre plane sources, a camber surface dipole distribution where the strength of which is related to the asymmetry of the hull forms.

The aim of the present paper is to calculate the wave-making resistance of ships in an unbounded fluid using Michell's [1] integral. Based on Michell's [1] integral a computer program has been developed and executed in order to analyse the hydrodynamic characteristics of ships in an unbounded liquid and the validity of the computer scheme is verified by comparing it with the experimental results.

## 2. MATHEMATICAL MODELING OF THE WAVE RESISTANCE PROBLEM

Consider a ship moving with a constant speed  $U$  in the direction of the negative  $x$ -axis as shown in Figure 1. The  $z$ -axis is upward and the  $y$ -axis extends to starboard. The origin of the coordinates is located in the undisturbed free surface at amidship so that the undisturbed incident flow appears to be a streaming flow in the positive- $x$  direction. It is assumed that the fluid is incompressible and inviscid and the flow irrotational. Consequently, we can define a velocity potential  $\Phi$  which satisfies the Laplace equation throughout the fluid domain

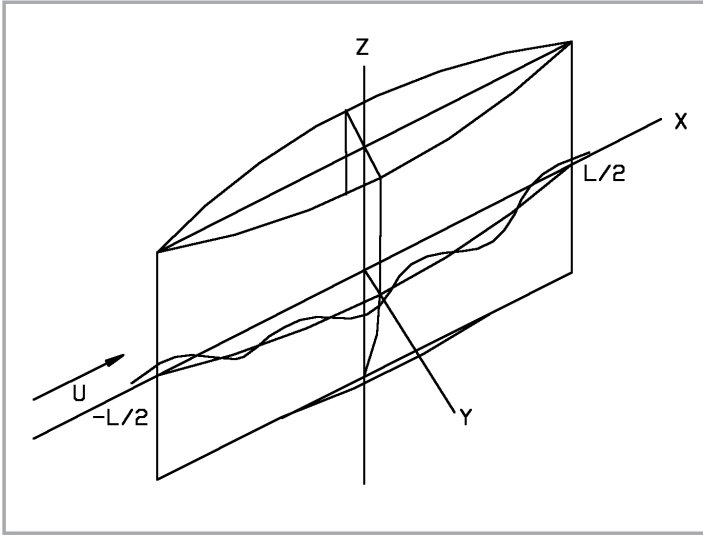


Figure 1: Definition sketch of the coordinate system

$$\nabla^2 \Phi = 0 \quad (1)$$

and is related to the velocity potential  $\Phi$  by

$$\Phi = Ux + \phi \quad (2)$$

where  $\phi$  is the perturbation velocity potential due to the existence of the hull. Now the problem can be constructed by specifying the boundary conditions as follows:

### A. HULL BOUNDARY CONDITION :

The kinematical boundary condition to be satisfied by an inviscid fluid is that the velocity of a particle on a bounding surface must be tangential to it. If the ship hull surface is represented by

$$y = F(x, z) \quad (3)$$

Now the boundary condition (3) can be expressed by Equation (2) as

$$v = (U + u) \frac{\partial F}{\partial x} + w \frac{\partial F}{\partial z} \quad (4)$$

where  $u$ ,  $v$  and  $w$  are the components of  $\nabla \phi$ . If we introduce the restriction that the tangent plane of the ship surface makes a small angle with the  $x$ - $z$  plane, that is,

$$\frac{\partial F}{\partial x} \ll 1 \quad ; \quad \frac{\partial F}{\partial z} \ll 1$$

then the boundary condition on the surface of the ship simplifies to

$$\frac{\partial \phi}{\partial y} = U \frac{\partial F}{\partial x} = Uf(x, z)$$

$$\frac{\partial \phi}{\partial y} = Uf(x, z) \quad \text{at } y = 0 \quad (5)$$

### B. FREE SURFACE CONDITION:

The kinematic and dynamic boundary conditions on the free surface can be respectively written as,

$$\Phi_x \xi_x + \Phi_y \xi_y - \Phi_z = 0 \quad \text{on } z = \xi(x, y) \quad (6)$$

$$g\xi + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi = \frac{1}{2} U^2 \quad \text{on } z = \xi(x, y) \quad (7)$$

Combining Equations (6) and (7) we get

$$\frac{1}{2} (\nabla \Phi \cdot \nabla \Phi)_x \nabla \Phi_x + \frac{1}{2} (\nabla \Phi \cdot \nabla \Phi)_y \nabla \Phi_y + g\Phi_z = 0 \quad \text{on } z = 0 \quad (8)$$

Substituting Equation (2) into Equation (8) and expanding the potential  $\phi$  in a Taylor series about the mean free surface  $z = 0$ , the linearised Kelvin free-surface boundary condition can be obtained as (see Appendix A):

$$\phi_{xx} + K_0 \phi_z = 0 \quad \text{on } z = 0 \quad (9)$$

where  $K_0 (= g/U^2)$  is the wave number.

The Australian mathematician, J. H. Michell [1], has given the first approximate solution to the problem of wave resistance experienced by a slender ship-shaped body moving on the surface of an inviscid fluid of infinite depth. The approximation consisted of replacing the exact free-surface boundary condition by the usual linearized Equation (9) and the boundary condition on the ship's surface by Equation (5) on the center plane section analogous to the thin-wing boundary condition. The solution is derived from a double Fourier expansion of the velocity potential. From the resultant velocity potential, the velocity and pressure distributions over the hull can be obtained. And by integrating the fore-and-aft components of the pressure, an expression for the wave-making resistance can be derived as follows:

$$R_w = \frac{4}{\pi} \rho U^2 v^2 \int_1^\infty \frac{\lambda^2}{\sqrt{\lambda^2 - 1}} |A(\lambda)|^2 d\lambda \quad (10)$$

which is the well-known Michell's [1] integral for the wave resistance of a ship and

$$A(\lambda) = -iv\lambda \iint_S e^{v\lambda^2 z + iv\lambda x} y(x, z) dz dx \quad (11)$$

where  $v$  is the wave number,  $U$  is the velocity of the ship,  $\rho$  is the density of water and  $g$  is the acceleration due to gravity,  $S$  is the submerged area of the hull surface and  $L$  is the half-width of the ship. Now the wave-making resistance coefficient can be obtained as

$$C_w = \frac{R_w}{\frac{1}{2} \rho S U^2} \quad (12)$$

### 3. NUMERICAL SCHEME

To allow numerical integration of Equation (10), the singularity of the integrand at  $\lambda = 1$  can be separated as:

$$\begin{aligned} & \int_1^2 \frac{\lambda^2}{\sqrt{\lambda^2-1}} |A(\lambda)|^2 d\lambda \\ &= \int_1^2 \frac{|A(1)|^2}{\sqrt{\lambda^2-1}} d\lambda + \int_1^2 \frac{\lambda^2 |A(\lambda)|^2 - |A(1)|^2}{\sqrt{\lambda^2-1}} d\lambda \\ &= |A(1)|^2 \int_1^2 \frac{\lambda^2}{\sqrt{\lambda^2-1}} d\lambda + \int_1^2 \frac{\lambda^2 |A(\lambda)|^2 - |A(1)|^2}{\sqrt{\lambda^2-1}} d\lambda \end{aligned} \quad (13)$$

The first integral on the right-hand side of Equation (13) is  $\ln(2 + \sqrt{3})$  and the second integral has a non-singular integrand. Thus for numerical evaluation of Equation (10) is transformed to

$$\begin{aligned} R_w &= \frac{4}{\pi} \rho U^2 v^2 \left[ |A(1)|^2 \ln(2 + \sqrt{3}) + \int_1^2 \frac{\lambda^2 |A(\lambda)|^2 - |A(1)|^2}{\sqrt{\lambda^2-1}} d\lambda + \right. \\ & \left. \sum_{n=1}^{\infty} \int_{2^n}^{2^{n+1}} \frac{\lambda^2 |A(\lambda)|^2}{\sqrt{\lambda^2-1}} d\lambda \right] \end{aligned} \quad (14)$$

The term  $A(\lambda)$  of Equation (14) denotes the integration over the wetted surface area of the hull surface and can be written from Equation (11) as

$$A(\lambda) = -iv\lambda \int e^{iv\lambda x} p(x) dx = -iv\lambda q \quad (15)$$

where

$$p = \int e^{v\lambda z} y(x, z) dz \quad (16)$$

$$q = \int e^{iv\lambda x} p(x) dx$$

The integration along the vertical direction can be obtained by using the following formula:

$$\begin{aligned} p &= \int_{z_1}^{z_1+2h} e^{kz} y(x, z) dz \\ &\approx \frac{e^{kz_1}}{k} \left[ e^{2hk} \left( -\frac{0.5y_1 - 2y_2 + 1.5y_3}{hk} + y_3 + \frac{\Delta^2 y}{(hk)^2} \right) - \right. \\ & \left. \frac{0.5y_1 - 2y_2 + 1.5y_3}{hk} - y_1 - \frac{\Delta^2 y}{(hk)^2} \right] \end{aligned} \quad (17)$$

in which  $k = v\lambda^2$ ;  $y_1 = y(z_1)$ ;  $y_2 = y(z_1 + h)$ ;  $y_3 = y(z_1 + 2h)$ ;  $\Delta^2 y = y_1 - 2y_2 + y_3$  and  $h$  is the spacing between two waterlines. Equation (17) results from approximating the function by a second-degree polynomial passing through  $y_1$ ,  $y_2$  and  $y_3$ . Similarly, the integration along the horizontal direction can be found by using the following formula:

$$\begin{aligned} q &= \int_{x_1}^{x_1+2d} e^{kx} p(x) dx \\ &\approx \frac{e^{kx_1}}{k} \left[ e^{2dk} \left( -\frac{0.5p_1 - 2p_2 + 1.5p_3}{dk} + y_3 + \frac{\Delta^2 p}{(dk)^2} \right) - \right. \\ & \left. \frac{0.5p_1 - 2p_2 + 1.5p_3}{dk} - y_1 - \frac{\Delta^2 p}{(dk)^2} \right] \end{aligned} \quad (18)$$

in which  $k = iv\lambda$ ;  $p_1 = p(x_1)$ ;  $p_2 = p(x_1 + d)$ ;  $p_3 = p(x_1 + 2d)$ ;  $\Delta^2 p = p_1 - 2p_2 + p_3$  and  $d$  is the spacing between two ordinates. The infinite sum in Equation (14) is truncated when a term is smaller than 5% of the sum of the previous terms. An algorithm and flow chart of the computer program based on Michell's [1] integral of Equation (14) have been presented in Appendix B and in Appendix C respectively.

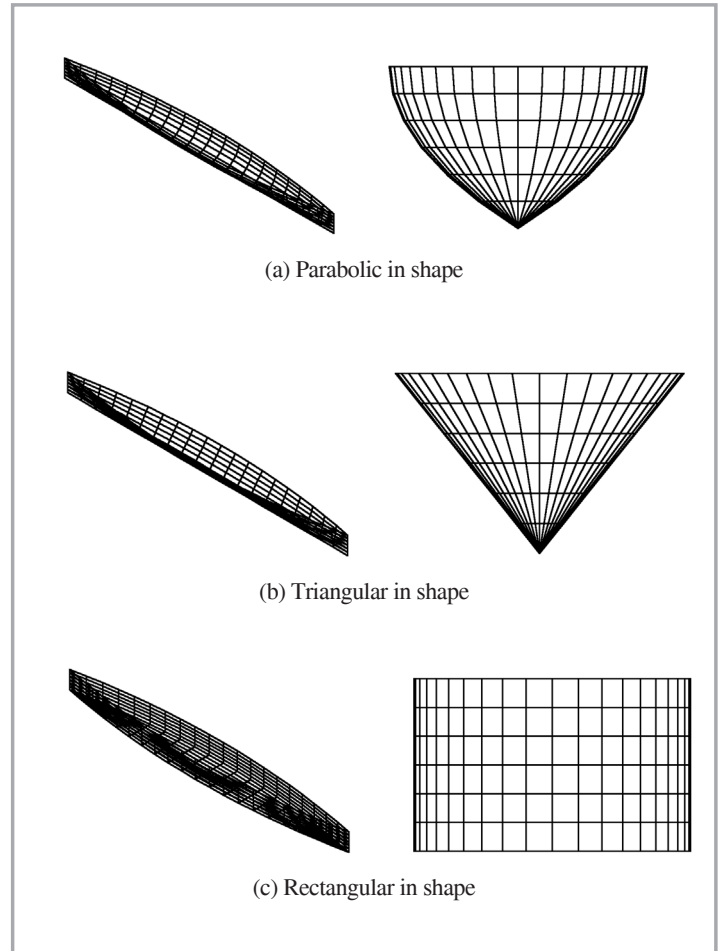


Figure 2: Wigley hulls of different cross-sections

### 4. RESULTS AND DISCUSSION

To compute the wave resistance around the ship-like body, the method has been applied to the Wigley hull. The equation of this type of hull surface is

$$y(x, z) = \frac{B}{2} S(z) \left( 1 - \frac{4x^2}{L^2} \right) \quad (16)$$

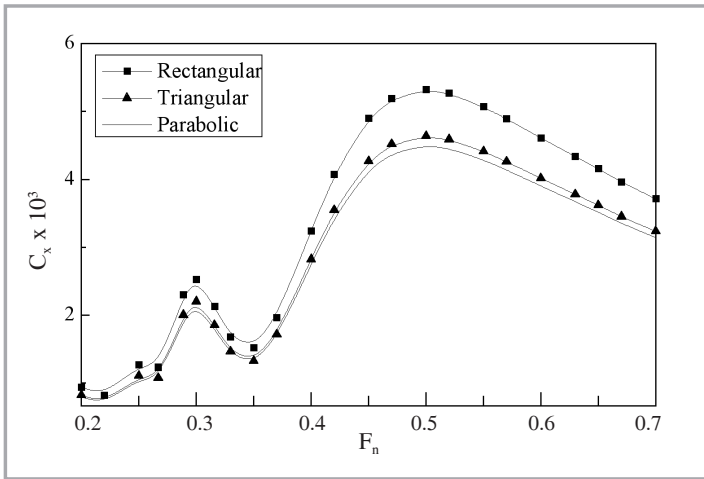


Figure 3: Comparison of wave-making resistance coefficients for Wigley hull of different shapes

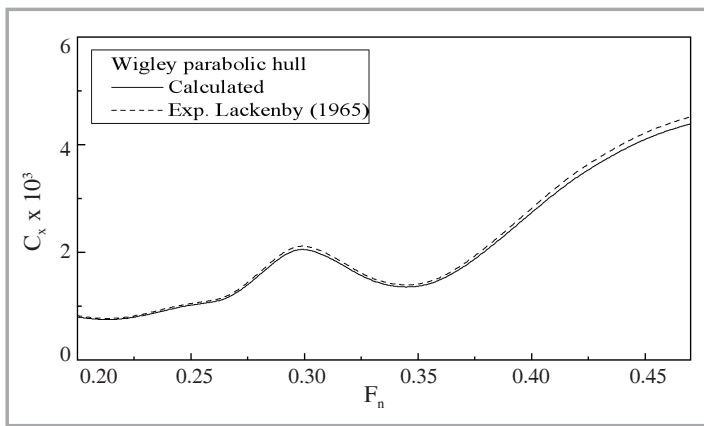


Figure 4: Comparison of calculated and experimental wave-making resistance coefficients of the Wigley parabolic hull

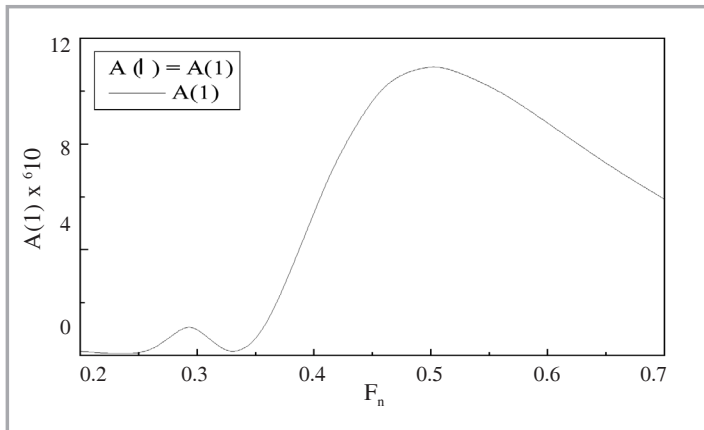


Figure 5: Variation of  $A(\lambda)$  with the Froude Number ( $F_n$ ) for the Wigley hull

The function  $S(z)$  defines the shape of the cross-section of the hull such as

$S(z) = 1$  for rectangular (wall-sided) cross-section,  $S(z) = 1 + z/T$  for triangular cross-section,  $S(z) = 1 - (z/T)^2$  for parabolic cross-sections.  $L$ ,  $B$  and  $T$  are the length, breadth and draft of the vessel respectively. The shapes of the Wigley hulls of different cross-sections are presented in Figure 2.

The wave making resistance coefficient  $C_w$  of the Wigley hulls of different cross-sections (parabolic, rectangular and triangular) is plotted to a base of Froude number  $F_n (= U/\sqrt{gL})$

in Figure 3. The value of  $C_w$  fluctuates with the increase of speed as the speed of the ship  $F_n$  approaches 0.35. With further increase in speed, the value of  $C_w$  begins to increase more and more rapidly approaching Froude number 0.50. However, an increase in  $C_w$  is accompanied by a number of humps and hollows in the resistance curve.

As the speed of the ship increases, the wave pattern changes, for the length of the waves will increase and the relative position of their crests and troughs will alter. In this process there will be a succession of speeds when the crests of the two systems reinforce one another, separated by other speeds at which crests and troughs tend to cancel one another. The former condition leads to higher wave heights, the latter to lower ones. The humps and hollows in  $C_w$  curve are due to these interference effects between the wave systems.

Figure 4 presents a comparison of the computed values of wave-making resistance co-efficient of the Wigley parabolic hull with the experimental results [8] and shows very close agreement with each other. The absolute values of  $A(\lambda)$  of Equation (9) are plotted in Figure 5. The trend of the curve of  $A(1)$ , where  $\lambda$  is equal to 1, is very similar to the  $C_w$  curve. The absolute values of  $A(\lambda)$  for different Froude numbers are also plotted to a base of  $\lambda$  in Figures 6 and 7. It is observed that the absolute value of  $A(\lambda)$  does not significantly change for large values of  $\lambda$ . So the infinite sum of  $A(\lambda)$  can be truncated after a certain value of  $\lambda$ .

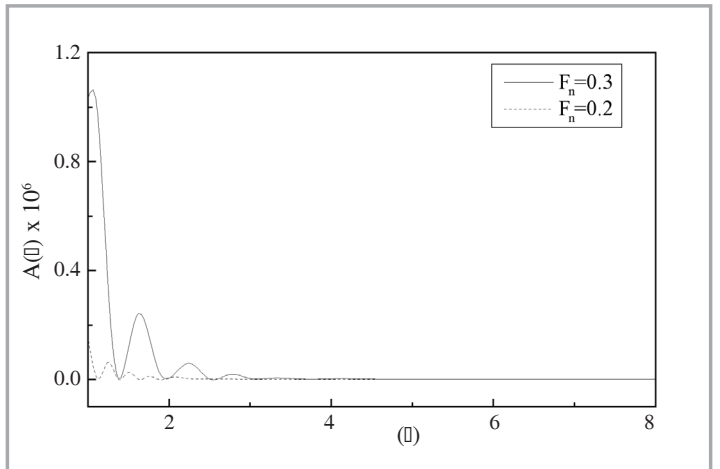


Figure 6: Variation of  $A(\lambda)$  with  $\lambda$  at different Froude numbers

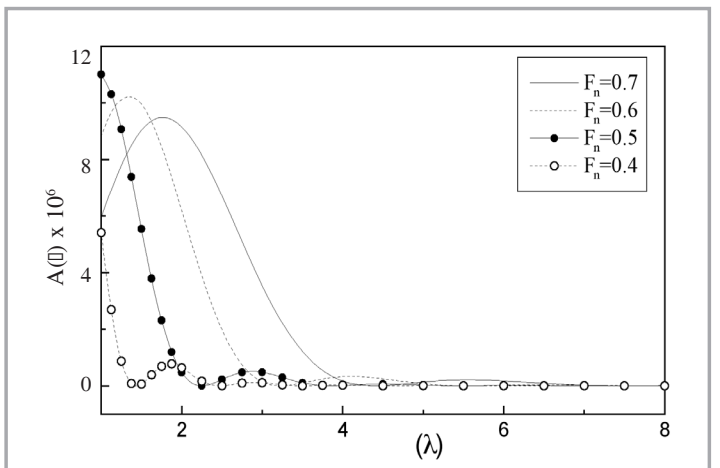


Figure 7: Variation of  $A(\lambda)$  with  $\lambda$  at different Froude numbers

## 5. CONCLUSIONS

The present paper has successfully put forward a numerical approach to predict the hydrodynamic resistance of Wigley hull in an unbounded liquid. The following conclusions can be drawn from the present numerical analysis:

- An increase in the co-efficient of wave-making resistance  $C_w$  is usually accompanied by a number of humps and hollows in the resistance curve.
- The absolute value of  $A(\lambda)$  does not appreciably change for large values of  $\lambda$ . So the infinite sum of  $A(\lambda)$  can be truncated after a certain value of  $\lambda$ .
- The present numerical approach leads to results which are in satisfactory agreement with the experimental ones.
- The present numerical scheme could be a useful tool at the preliminary design stage for the prediction of wave-making resistance of the Wigley type hull. ■

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## NOMENCLATURE

B	=	width of the ship
$C_w$	=	wave-making resistance co-efficient
$F_n$ (FN)	=	Froude number
g	=	acceleration due to gravity
h	=	waterline spacing
L	=	length of the ship
NF	=	number of speeds (FN)
NP	=	number of offsets on a section
NSP	=	number of offset sections (ordinates)
$R_w$	=	wave-making resistance
T	=	draft of the ship
U	=	velocity of the ship
x	=	longitudinal co-ordinates of section x
y	=	half-breadth (first at waterline, last at keel)
$\Phi$	=	total velocity potential
$\phi$	=	perturbation velocity potential
$\xi$	=	wave elevation
$\nu$	=	wave number
$\rho$	=	density of the water

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## APPENDIX A

### Linearisation of free surface condition

In order to linearise the free surface Equation (8), the first step is to introduce appropriate smallness parameter. Hereby we define the parameter  $\varepsilon = B/L$ , where B and L are the beam and length of the ship. Then the surface of the ship can be written as

$$y = \varepsilon f(x, z)$$

As the perturbation velocity potential of the fluid motion generated by a thin ship is small in order of  $\varepsilon$ , one can expand the velocity potential in a power series such as

$$\phi = \sum_{n=1}^{\infty} \varepsilon^n \phi_n = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \dots \quad (A1)$$

and the corresponding wave elevation can be expressed as

$$\zeta = \sum_{n=1}^n \varepsilon^n \zeta_n = \varepsilon \zeta_1 + \varepsilon^2 \zeta_2 + \dots \quad (A2)$$

Substituting Equation (2) into Equation (8) we obtain

$$\frac{1}{2} (Ux + \phi)_x \{ (Ux + \phi)_x^2 + (Ux + \phi)_y^2 + (Ux + \phi)_z^2 \}_x + \quad (A3)$$

$$\frac{1}{2} (Ux + \phi)_y \{ (Ux + \phi)_x^2 + (Ux + \phi)_y^2 + (Ux + \phi)_z^2 \}_y +$$

$$g(Ux + \phi)_z = 0$$

Expanding Equation (A3) we obtain

$$\frac{1}{2} U \{ U^2 + 2U\phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2 \}_x + \quad (A4)$$

$$\frac{1}{2} \phi_y \{ U^2 + 2U\phi_x + \phi_x^2 + \phi_y^2 + \phi_z^2 \}_y + g\phi_z = 0$$

Substituting Equation (A1) into Equation (A4) we obtain

$$\frac{1}{2} U \{ U^2 + 2U(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_x + (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_x^2 +$$

$$(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_y^2 + (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_z^2 \}_x +$$

$$\frac{1}{2} (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_y \{ U^2 + 2U(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_x +$$

$$(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_x^2 + (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_y^2 +$$

$$(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_z^2 \}_y + \frac{1}{2} (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_z$$

$$\{ U^2 + 2U(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_x + (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_x^2 +$$

$$(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_y^2 + (\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_z^2 \}_y +$$

$$g(\varepsilon\phi_1 + \varepsilon^2\phi_2 + \dots)_z = 0$$

Equating the coefficient of  $\varepsilon$  from Equation (A5) we obtain

$$\frac{1}{2} U \{ 2U\phi_{1x} \}_x + g\phi_{1z} = 0 \quad \text{at } z = \zeta \quad (A6)$$

Simplifying the above equation we obtain

$$U^2 \{ \phi_{1x} \}_x + g\phi_{1z} = 0 \quad \text{at } z = \zeta \quad (A7)$$

The boundary condition is to be evaluated at the unknown surface  $z = \zeta$ . To avoid the difficulties we are to satisfy the free surface boundary condition at mean value of  $\zeta$  (say  $z = 0$ ) by using Taylor's theorem. The function  $\Phi(x, y, z)$  can be written as

$$\Phi(x, y, z) = \Phi(x, y, 0) + \zeta \Phi_{1z}(x, y, 0) + \frac{\zeta^2}{2} \Phi_{2z}(x, y, 0) + \frac{\zeta^3}{2} \Phi_{3z}(x, y, 0) + \dots \quad (A8)$$

Similarly,

$$\left[ \phi \right]_{z=\zeta} = \left[ \phi \right]_{z=0} + \zeta \left[ \phi_{1z} \right]_{z=0} + \frac{\zeta^2}{2} \left[ \phi_{2z} \right]_{z=0} + \dots \quad (A9)$$

Substituting the expression of  $\phi$  of Equation (A9) in Equation (A7)

$$U^2 \frac{\partial}{\partial x} [\varepsilon\phi_1 + \zeta\varepsilon\phi_{1x} + \dots]_{z=0} + g[\varepsilon\phi_1 + \zeta\varepsilon\phi_{1z} + \dots]_{z=0} = 0 \quad (A10)$$

Substituting Equation (A2) in the above equation

$$U^2 \frac{\partial}{\partial x} [\varepsilon\phi_1 + \zeta\varepsilon^2\phi_{1x} + \dots]_{z=0} + g[\varepsilon\phi_1 + \zeta\varepsilon^2\phi_{1z} + \dots]_{z=0} = 0$$

Now taking the coefficient of  $\varepsilon$  we obtain

$$U^2 \frac{\partial^2 \phi_1}{\partial x^2} + g\phi_{1z} = 0 \quad \text{at } z = 0$$

Dropping the subscript the above equation can now be expressed as

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{g}{U^2} \phi_z = 0 \quad \text{at } z = 0 \quad (A11)$$

Similarly, expanding Equation (7) the first order linear equation of the wave profile can be obtained as

$$\zeta = - \frac{U}{g} \phi_z = 0 \quad (A12)$$

## APPENDIX B

### Algorithm of Michell's Integral

Input values for  $\rho$ ,  $g$  and  $L$

DO FOR  $i = 1$  to  $NF$  STEP 1:

Input values for  $FN$

SET  $U = FN \sqrt{g \times L}$

SET  $v = g/U^2$

Input values for  $NSP$

SET  $\lambda = 1$

DO FOR  $j = 1$  to  $NSP$  STEP 1:

Input for  $NP$ ,  $x$ ,  $h$ ,  $y$

Calculate surface area  $S$

DO FOR  $k = 3$  to  $NP$  STEP 2:

SET  $y^1 = y(z_1)$ ,  $y_2 = y(z_1 + h)$ ,  $y_3 = y(z_1 + 2h)$ , and

$\Delta^2 y = y_1 - 2y_2 + y_3$

ENDDO (FOR  $k$ )

DO FOR  $m = 1$  to  $NP/2$

SET  $p = \int e^{v\lambda^2 z} y(x, z) dz$

ENDDO (FOR  $m$ )

DO FOR  $k = 2$  to  $NSP$  STEP 2:

SET  $p_1 = p(x_1)$ ,  $p_2 = p(x_1 + d)$ ,  $p_3 = p(x_1 + 2d)$ , and

$\Delta^2 p = p_1 - 2p_2 + p_3$

ENDDO (FOR  $k$ )

SET  $q = \int e^{iv\lambda x} p(x) dx$

ENDDO (FOR  $j$ )

SET  $A(\lambda) = -iv\lambda q$

SET  $SUM = |A(1)|^2 \ln(2 + \sqrt{3})$

SVOR = SUM

SET  $n = 0$

Set the limit of integration with respect to  $\lambda$

$a = 2^n$

20

$b = 2^{n+1}$

$\lambda = (a + b)/2$

DO FOR  $j = 1$  to  $NSP$  STEP 1:

DO FOR  $m = 1$  to  $NP/2$

SET  $p = \int e^{v\lambda^2 z} y(x, z) dz$

ENDDO (FOR  $m$ )

DO FOR  $k = 2$  to  $NSP$  STEP 2:

SET  $p_1 = p(x_1)$ ,  $p_2 = p(x_1 + d)$ ,  $p_3 = p(x_1 + 2d)$

and  $\Delta^2 p = p_1 - 2p_2 + p_3$

ENDDO (FOR  $k$ )

SET  $q = \int e^{iv\lambda x} p(x) dx$

ENDDO (FOR  $j$ )

SET  $A(\lambda) = -iv\lambda q$

IF  $\lambda > 1$  THEN

$F(\lambda) = 0$

ENDIF

IF  $\lambda > 1$  THEN

$$F(\lambda) = \frac{\lambda^2 |A(\lambda)|^2}{\sqrt{\lambda^2 - 1}}$$

ENDIF

IF  $n = 0$  THEN

$$F(\lambda) = F(\lambda) - \frac{|A(1)|^2}{\sqrt{\lambda^2 - 1}}$$

ENDIF

SET  $T = \int F(\lambda) d\lambda$

SUM = SUM + T

IF (ABSOLUTE (T) < 0.05 \* SUM) RETURN

SET  $n = n + 1$

GO TO 20

SET  $R_w = \frac{4}{\pi} \rho U^2 v^2 \times SUM$

SET  $C_w = \frac{R_w}{(1/2)\rho S U^2}$

ENDDO (FOR  $i$ )

### APPENDIX C Flow Chart of Michell's Integral

