EFFECT OF MATERIAL PROPERTIES ON DUCTILITY OF REINFORCED CONCRETE BEAMS

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ABSTRACT

Reinforced concrete (RC) is one of the most important building materials and is widely used in civil engineering structures. The constituents of reinforced concrete such as concrete and steel are very complex due to their different mechanical properties. The stress-strain behavior of concrete under short term compression loading is considered parabolic and that of the steel is elastic plastic. Concrete and reinforcing steel are represented by separate material models that are combined together to describe the behavior of the RC beam sections. The end displacements of the steel element are assumed to be compatible with the boundary displacements of the concrete element which implied perfect bond between them. This paper presents the effects of materials properties such as concrete compressive strength, yield strength of steel and reinforcement content on the curvature ductility of singly reinforced concrete beams as well as doubly reinforced concrete beams. From the analyses, it is observed that the ductility of a singly RC beam decreases as the tension steel content is increased, and the presence of compression steel increases the ductility significantly. For the same reinforcement content the increase in compressive strength of concrete increases curvature ductility of reinforced concrete section and for a doubly reinforced section rate of increase of curvature is more than that of a singly reinforced section. However, the moment ratio increases at a relatively slow rate than that of singly reinforced beams. On the other hand the ductility of RC beams decreases as the strength of reinforcement is increased.

Keywords: *Concrete Compressive Strength, Curvature Ductility, Moment-curvature Relationship, Nonlinear behavior, RC Beams, Reinforcing Steel*

1.0 INTRODUCTION

Reinforced concrete (RC) sections exhibit highly nonlinear load-deformation response due to the nonlinear stress-strain relationship of its constituent materials. Reinforced concrete structures are commonly designed to satisfy criteria of serviceability and safety. In order to ensure the serviceability requirement it is necessary to predict the cracking and the deflections of RC structures under service loads. In order to assess the margin of safety of RC structures against failure an accurate estimation of the ultimate load (collapse load) is essential and the prediction of the load-deformation behavior of the structure throughout the range of elastic and inelastic response is desirable. The ductility of reinforced concrete beams depends mainly on the shape of the moment-curvature relationship of the sections, since ductility may be defined as the ability to undergo deformations without a substantial reduction in the flexural capacity of the member. The strength of under-reinforced flexural members is not much affected by variations of concrete strengths due to the fact that section behavior is controlled by the yield strengths of steel. However, to ensure ductile failure of the members it is required to observe the behavior of the RC beams due to variations of material properties of its constituents. In earthquake resistant design, a prime consideration is given to the structures that should be capable of deforming in a ductile manner when subjected to loads. So, for the design of structures, structural engineers are given increasing attention to the design of structures that undergo large deformation without total collapse. For the development of advanced design and analysis of structures the need for experimental research continues. Experiments provide a firm basis for the design equations, which are invaluable in the preliminary design stages. The development of reliable numerical models can, however, reduce the number of required test specimens for the solution of a given problem, recognising that tests are time-consuming and costly. For the analysis of RC members, the development of suitable numerical models by many researchers [1-7] continues.

The ductility of a RC section is normally expressed as the curvature ductility factor φ_u/φ_y, where φ_y is the curvature when the tension steel first reaches the yield strength and φ_u is the ultimate curvature normally defined for unconfined concrete as when the concrete compressive strain reaches a specified limiting value. Figure 1 presents a typical moment-curvature relationship of a singly reinforced concrete section. It is assumed that plane sections remain plane after bending and that the stress-strain relationship of concrete and steel are known.

For RC beams with unconfined concrete the flexural strength and ductility mainly depends on the tension reinforcement ratio, ρ. The compressive strength of the concrete f'c has a less significant effect. When the tension reinforcement reaches yield strength there is some redistribution of load carrying capacity of concrete and steel.

The objective of this paper is to observe the moment-curvature behavior of RC beams under loading for various steel contents by the model with comparison with analytical results. This also presents the effect of material properties such as concrete compressive strength and grade of steel on the

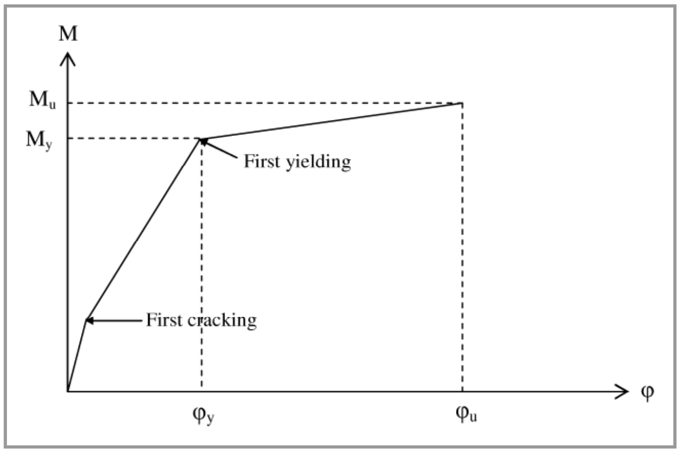

Figure 1: Typical moment-curvature curve for a singly reinforced concrete section falling in tension

moment ratio and curvature ductility ratio of the both singly and doubly RC beams.

2.0 PROPERTIES OF CONCRETE AND REINFORCING STEEL

The performance of a structure under loads depends to a large extent on the stress-strain relationship of the materials from which it is made. Concrete is used mostly in compression and its stress-strain curve is of primary interest. There are various mathematical models to idealise the stress-strain behavior of concrete. In the design of RC structural members, uniaxial compressive strength of concrete obtained from cylinder test is one of the most important design parameter and is widely used. There are various idealisations that are widely used for analysis of RC beams as presented in [8-9] for normal density and light weight concrete and for prestressed concrete in [10] and for structural brickwork by researchers [11]. The general trends for all the curves have somewhat similar character having an initial relatively straight elastic portion in which stress-strain are closely proportional, then begin to curve to horizontal and falling branch. The peak of the curve for high strength concrete is relatively sharp, but for low strength concrete the curve has a flat top. So the behavior of concrete may be represented by a second degree parabola for both the ascending and descending branch. The stress-strain curve may be represented by the following equations as suggested by Hognestad [12] with some modifications for descending branch as shown in Figure 2. In the present model stress-strain relationship of concrete is parabolic with an ultimate compressive strain of concrete is of 0.003.

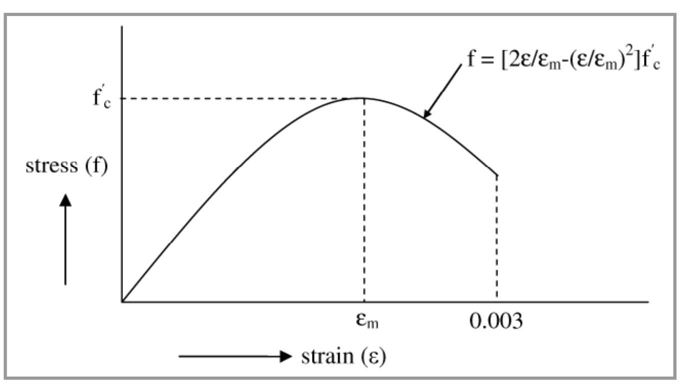

Figure 2: Stress-strain relationship of concrete used in the analysis and model

The stress-strain curves for reinforcing steel bars used in structure are obtained from the test of bars loaded in tension. For all practical purposes steel exhibits the same stress-strain curve in compression as in tension. The stress-strain relationship of steel exhibits an initial linear elastic portion, a yield plateau, a strain hardening range. The extent of the yield plateau is a function of the tensile strength of steel. The stress-strain relationship may be idealised elastic perfectly plastic as shown in Figure 3.

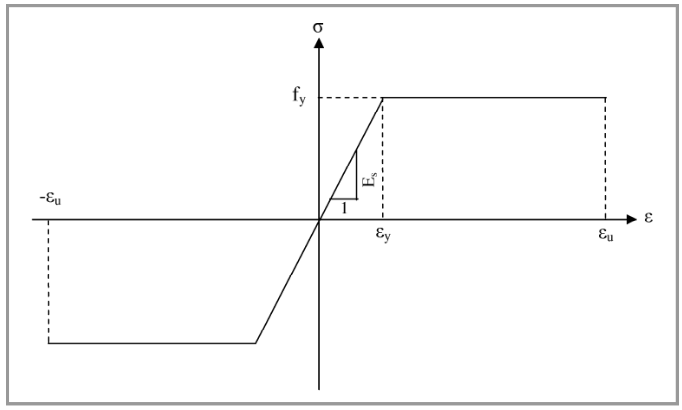

Figure 3: Stress-strain relationship of steel reinforecement used in the analysis and model

3.0 MOMENT-CURVATURE RESPONSE OF RC BEAMS

Theoretical moment-curvature (M-φ) curves for RC sections under flexure with and without axial load can be derived on the basis of assumptions used in the model. The curvatures associated with a range of bending moments and axial loads may be determined using these assumptions and from the requirements of strain compatibility and equilibrium of forces. The relationship between bending moment, M and curvature, φ is given by the classical elastic equation as:

$$EI = \frac{M}{\varphi} \qquad (1)$$

With increase in moment, cracking of the concrete reduces the flexural rigidity of the sections. The curvature varies along the length of member because of the fluctuation of the neutral axis depth. For a given curvature, the position of the neutral axis depth can be determined by trial and error; i.e. assuming neutral axis depth, calculating the strain and stress at various points of the sections and equating the compressive and tensile forces. Once the neutral axis depth is obtained, the moment, M can be calculated by summing the moments of all forces on the section. The behavior of the section after cracking is dependent mainly on the steel content.

A nonlinear numerical model, NLRCF [13] has been developed considering the material properties of reinforced concrete and adopted the stiffness method. The methodology may be summarised as: at first stiffness matrix of structural element of RC member is developed. Then for a particular load, resulting deflection and forces of an element at node are obtained by iterative method. Since behavior is nonlinear, the stiffness of the RC sections become a function of the state of strain and hence stress. Stiffness properties of cracked and uncracked sections derived by researchers [14-16] have been utilised. The steps involved are: the formation of the current stiffness matrix, the solution of the equilibrium equations for the displacement increments, the determination of force equilibrium of all elements in the model and the convergence check. Since the global stiffness matrix of the structure depends on the displacement increments, the solution of equilibrium equations is typically accomplished with an iterative method through convergence check. The nonlinear problem has been solved by the modified Newton-Raphson approach. The stiffness matrix has been updated once in each load increment and it was left constant for rest of the iteration. Convergence criteria in terms of incremental nodal forces are adopted in order to terminate the iterative cycle when the solution is considered to be sufficiently accurate. The criterion for measuring the convergence of the iterative solution is based on the accuracy of satisfying the global equilibrium equations and on the accuracy of determining the total displacements. The accuracy of satisfying the global equilibrium equations is controlled by the magnitude of unbalanced element forces. The accuracy of nodal displacements depends on the magnitude of the additional displacement increment after each iterative cycle. The failure load has been assumed to occur at a load level where a large number of iteration is required and the solution does not converge so as to allow application of next load step. The numerical model thus developed can be effectively used to capture the load-deflection and moment-curvature response of RC beams as presented in [17].

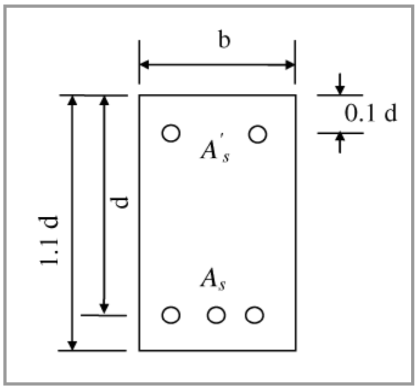

Figure 4: Cross section of RC beams [9]

An investigation is carried out by the model NLRCF to observe the effect of percentages of tensile and compressive steel on the moment-curvature response of singly and doubly RC beams. For this, a RC section as presented in Figure 4 with steel content as presented in Table 1 is considered. The material properties are concrete crushing strength, 27.6 MPa, concrete crushing strain 0.002, ultimate concrete strain 0.004, yield strength of steel 276 MPa and yield strain of steel 0.00133 [9]. From the theory, it is well known that the ductility of a singly RC section decreases as the tension steel content is increased and the presence of compression steel increases the ductility significantly. Figure 5 presents the moment-curvature relationship of the RC beams due to percentages of reinforcement as shown in Table 1. From the Figure, it is observed that there exists a good correlation between the numerical and analytical results.

Table 1: RC beams with percentages of steel (Park & Paulay, 1975)

Beam	ρ	ρ'
1	0.0375	0.0250
2	0.0375	0.0125
3	0.0375	0
4	0.0250	0.0125
5	0.0250	0
6	0.0125	0.0125
7	0.0125	0

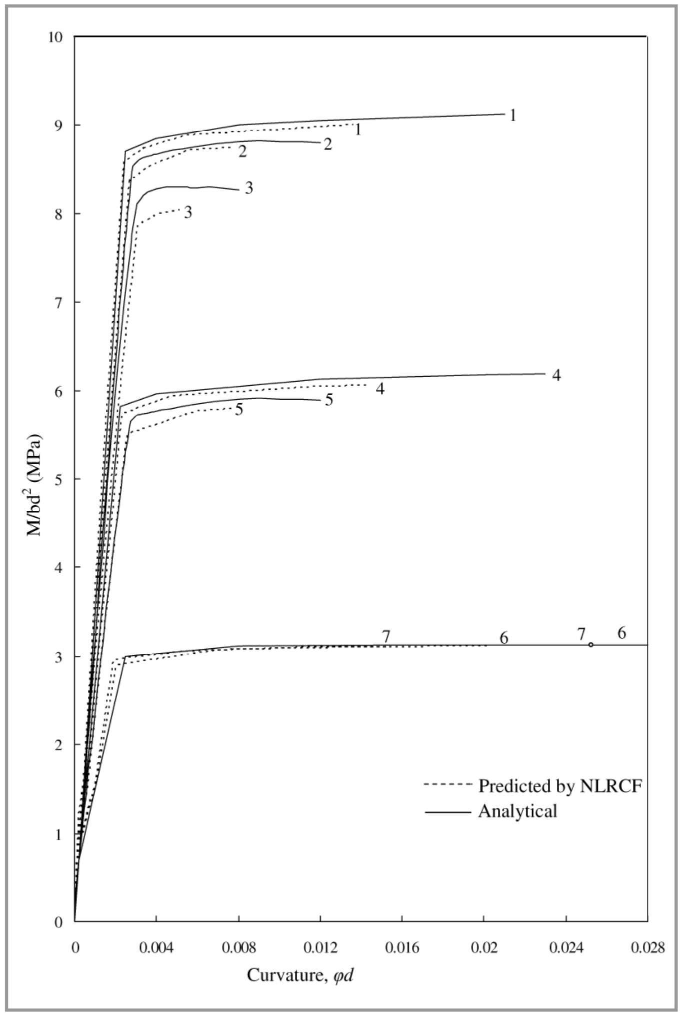

Figure 5: Moment-curvature relationships of the beam sections [9]

Figure 5 presents the effect of reinforcement content on the variation of curvature at various stages of loading for RC beams. It is shown that an increasing steel content increases the moment carrying capacity of the RC beam but decreases the curvature. However, the inclusion of compression steel increases the ductility quite effectively.

6.0 EFFECT OF CONCRETE COMPRESSIVE STRENGTH

To observe the effect of compressive strength of concrete on the curvature ductility and moment carrying capacity, doubly RC beams 1, 2 and singly RC beam 3 as presented in Table 1 are considered for analyses by the model. The strength of concrete is considered 20.7, 27.6 and 34.5 MPa for analysis. The ductility of reinforced concrete sections can be expressed by the curvature ductility ratio φ_u/φ_y where, φ_u = curvature at the end of the post elastic range and φ_y = curvature at first yield. The effect of concrete strength on the curvature ductility of the RC section is presented in Figure 6. On the other hand, the effect of concrete strength on the moment index Mu/My is presented in Figure 7 where, M_u = ultimate moment carrying capacity and M_y = moment at first yield.

From the Figure 6, it is observed that concrete compressive strength has a significant effect on the curvature ductility of RC beams. The curvature ductility of the RC beams both singly reinforced and doubly reinforced increase as the strength of the concrete is increased. For the same concrete strength, the curvature ductility is increased as the content of compression reinforcement increases.

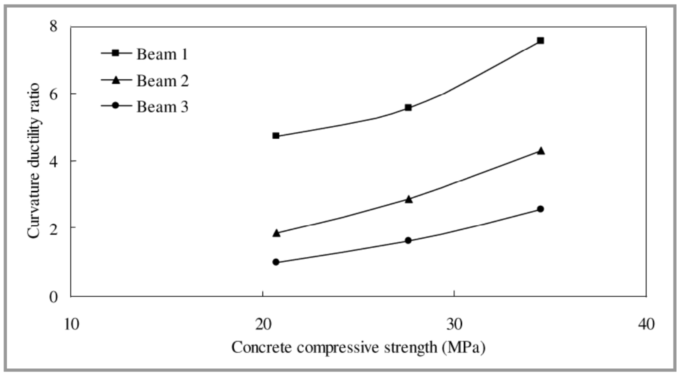

Figure 6: Effect of concrete compressive strength on curvature ductility ratio

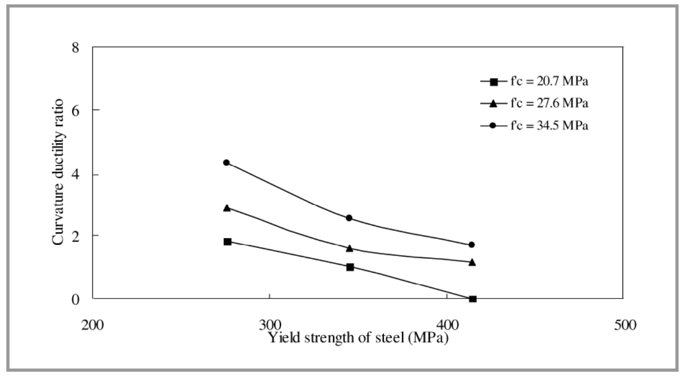

Figure 9: Effect of strength of steel on curvature ductility ratio of Beam 2 (ρ = 0.0375, ρ = 0.0125)

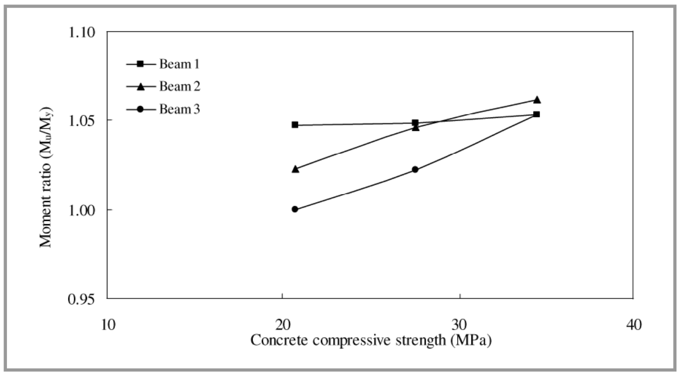

Figure 7: Effect of concrete compressive strength on moment ratio

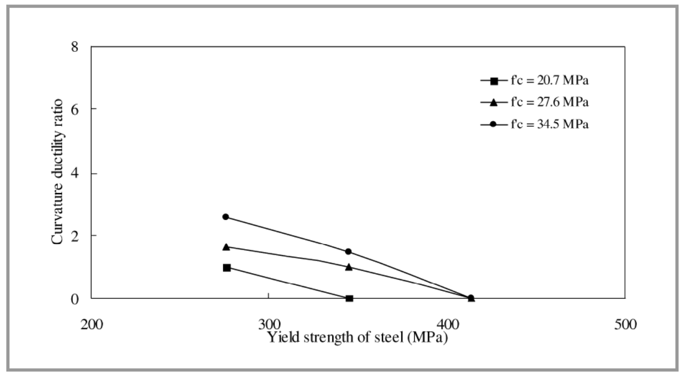

Figure 10: Effect of strength of steel on curvature ductility ratio of Beam 3 (ρ = 0.0375, ρ = 0.0)

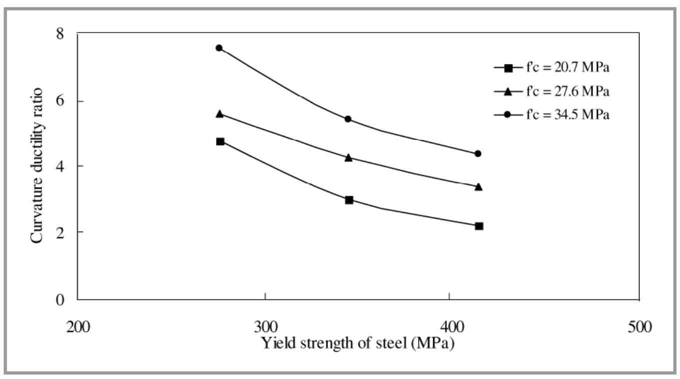

Figure 8: Effect of strength of steel on curvature ductility ratio of the Beam 1 (ρ = 0.0375, ρ = 0.0250,)

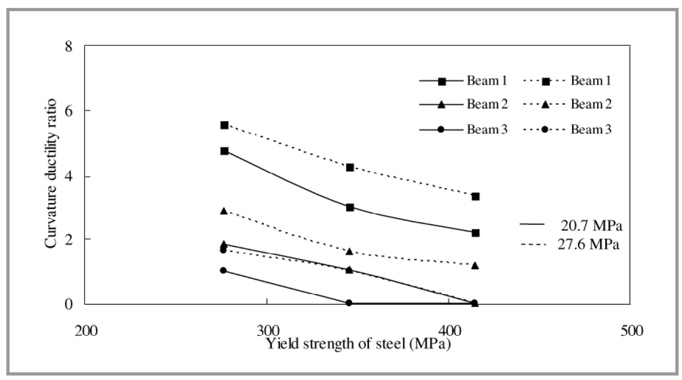

Figure 11: Effect of material properties on curvature ductility ratio of RC beams

From Figure 7, it is observed that the concrete compressive strength has less significant effect on the moment ratio but significant to curvature ductility ratio. As the concrete compressive strength increases, it increases the index at a relatively slow rate compared to that of curvature ductility. For a singly reinforced beam, the effect of concrete strength on the moment index is greater than that of doubly reinforced beams. As the content of compression reinforcement increases, it increases the moment ratio at a relatively lesser rate than that of the section with less compression steel content.

7.0 EFFECT OF STRENGTH OF STEEL REINFORCEMENT

In civil engineering construction, strength of steel as well as concrete may vary. The effect of strength of steel on curvature ductility ratio of the RC beams 1, 2 and 3 are observed by the model. The strengths of steel are assumed 276, 345 and 414 MPa for the analysis. The effect of two strength of concrete with three grades of steel on the curvature ductility is presented in Figures 8 to 10. From these figures, it is observed that the curvature ductility decreases as the yield strength of the reinforcement increased and this pattern is valid for the range of concrete strength considered. The curvature ductility ratio is higher for the RC beam with higher compression reinforcement content.

From the Figure 9, it is also observed that the curvature ductility ratio decreases to zero for Beam 3 with concrete strength 20.7 MPa and yield strength of steel of 414 MPa which means that for the beam, the concrete crushing is observed before the yielding of the steel reinforcement. This case is also observed for RC beam 3 as shown in Figure 10.

The effect of yield strength of steel on curvature ductility ratio for concrete strength 20.7 and 27.6 MPa is presented in Figure 11. From this Figure, it is observed that the curvature ductility ratio is greater for higher strength of concrete for the same yield strength of steel and this pattern is applicable to the range of strength of steel is considered for analysis.

CONCLUSIONS

The effect of material properties on the curvature ductility of reinforced concrete beams is analysed by the model. The numerical model used in the analyses has been developed considering nonlinear stress-strain relationships of the constituents of reinforced concrete. The model also considered combined stiffnesses of the RC members and cracking of concrete. From the analyses, it is observed that there exists a good correlation between the predicted results by the model and analytical results. From the analyses, it is also observed that for the same concrete strength, the curvature ductility of the RC member increases as the content of compression steel is increased. The concrete strength has a significant effect on the curvature ductility of RC beams. However, the rate of increase of ductility for a doubly reinforced section is more than that of a singly reinforced section and this pattern increases as the concrete strength is increased. As the strength of steel increases, it decreases the curvature ductility ratio and this pattern is valid for both doubly reinforced and singly reinforced beams but for the same strength of steel, the ductility is higher for higher concrete strength. ■

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