Application of Boundary Element Method for the Analysis of Potential Flow Field and Wave Resistance in Finite Depth of Water

Md. Shahjada Tarafder and Gazi Md. Khalil
Department of Naval Architecture and Marine Engineering,
Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh, India

ABSTRACT

A boundary element method is presented for solving a nonlinear free surface flow problem for a ship moving with a uniform speed in shallow water. The free surface boundary condition is linearized by the systematic method of perturbation in terms of a small parameter. The surfaces are discretized into flat quadrilateral elements and the influence coefficients are calculated by Morino's analytical formula. Dawson's upstream finite difference operator is used in order to satisfy the radiation condition. A verification of the numerical modelling is made using the Wigley hull. The peak of the wave resistance curve and the wave pattern at the critical speed demonstrate the validity of the computer scheme.

***Keywords :** Boundary element method, free surface, perturbation method, shallow water effect and wave making resistance*

INTRODUCTION

The resistance of a ship is quite sensitive to the effects of shallow water. In the first place, there is an appreciable change in potential flow around the hull. If the ship is considered as being at rest in a flowing stream of restricted depth, but unrestricted width, the water passing below it must speed up more than in deep water, with a consequent greater reduction in pressure and increased sinkage, trim and resistance. If the water is restricted laterally as in a river or canal, these effects are further exaggerated. The sinkage and trim in very shallow water may set an upper limit to the speed at which ships can operate without touching the bottom.

A second effect is the changes in the wave pattern which occur in passing from deep to shallow water. These changes have been studied by Havelock [4] for a pressure point impulse travelling over a free water surface. He has also proposed different approaches to obtain an appropriate formula for the ship resistance in shallow water in 1922 [5].

Kinoshita and Inui [9] extend Havelock's theory to satisfy the bottom boundary condition more exactly and Inui [7] further develops the theory to solve the channel problem. Kirsch [10] uses linearized wave theory to calculate the wave making resistance for simplified hull form in various water depths and channel widths. Muller [16] has carried out extensive experiments and theoretical calculations based on linearized wave theory to investigate the effect of shallow water on wave resistance. The wave resistance is determined by Guilloton's method and Havelock's integral.

More recently the Rankine source panel method based on linear wave theory has been applied to compute the wave resistance of ships in shallow water. Yasukawa [23] has developed a first order panel method based on Dawson's approach for the linear free surface condition. The shallow water effect is taken into consideration by replacing the bottom surface with Rankine sources. Lee [11] improves the solution efficiency over Yasukawa's work by choosing the Green function to be the sum of the Rankine source and its image with respect to bottom surface.

Kim et al. [8] analyze the free surface potential flow around ships in shallow water with linear and nonlinear free surface boundary conditions and simulated the shallow water effect by applying a symmetry condition on the bottom surface. The linear method is developed on the basis of double body approximation and thus the free surface condition is linearized with respect to the flow with an undisturbed free surface. In the nonlinear method the exact free surface conditions are approached in an iterative process, where in each iteration a condition linearized about the previous solution, is satisfied on the previously calculated wavy surface. The process starts with the free surface as a rigid lid and stops when the change in wave height between two iterations is below a given value.

Xinmin and Xiuheng [22] have applied Rankine source method to the computation of hydrodynamic forces and wave patterns of ship hulls moving in restricted water. Parabolic curved panels are adopted to model the hull surface and a body fitted grid is chosen to divide the local free surface. The other researchers who have made important contributions in the hydrodynamic characteristics of ships in shallow water are Monacella [15], Tuck and Taylor [21], Maruo and Tachibana [13] and Pettersen [17].

The aim of the present work is to investigate the influence of finite depth on the wave making characteristics of ships using a potential based panel method. The free surface conditions are linearized about the mean water surface by means of Taylor series expansion. A computer program PAFS (Panel Method Applied to Free Surface) is originally developed in the department of Naval Architecture and Ocean Engineering in Yokohama National University (Japan) for calculating the wave making resistance of ship in deep water. This program has been further extended by the author to take into account the effect of finite depth on wave making resistance. The interested reader may find the details for the computer program in Tarafder [20]. The validity of the computer program is examined with the Wigley hull.

MATHEMATICAL MODELLING OF THE PROBLEM

Consider a ship moving in a finite depth of water h with a constant speed U in the direction of the positive x-axis, as shown in Figure 1. The z-axis is vertically upwards and the y-axis extends to starboards. The origin of the co-ordinate system is located in an undisturbed free surface at a midship, so that the undisturbed incident flow appears to be a streaming flow in the positive-x direction. It is assumed that the fluid is incompressible and inviscid and the flow irrotational. The total velocity potential function Φ can be expressed as

$$\Phi = Ux + \phi = Ux + \sum_{n=1}^{\infty} \varepsilon^n \phi_n \quad (1)$$

where, ε is a perturbation parameter and ϕ is the perturbation velocity potential due to the existence of the body. The disturbance velocity potential satisfies the Lapace equation $\nabla^2\phi = 0$ in the fluid domain V (2)

The fluid domain V is bounded by the hull syrface S_H, free surface S_F, sea bottom S_B and the surface at infinity S_∞. Now the problem can be constructed by specifying the following boundary conditions as follows:

(a) *Hull boundary condition:* The hull boundary condition simply expresses the fact that the flow must be tangential to the hull surface i.e. the normal component of the velocity must be zero.

$$\varepsilon : \nabla\phi_1.n = -Un_x$$
$$\varepsilon^2 : \nabla\phi_2.n = 0 \quad (3)$$

in which denotes the unit normal vector on the surface and is positive into the fluid.

(b) *Free surface condition:* The free surface condition is nonlinear in nature and should be satisfied on the true surface, which is unknown and can be linearized as a part of the solution using perturbation method. The free surface boundary condition for first and second order approximation can be written as (see Maruo, 1966) [12];

$$\varepsilon \; : \phi_{1xx} + K_0\phi_{1z} = 0 \qquad \text{at } z = 0$$
$$\varepsilon^2 \; : \phi_{2xx} + K_0\phi_{2z} = f(\phi_1) \quad (4)$$
$$f(\phi_1) = -\frac{1}{U}\frac{\partial}{\partial x}(\phi_{1x}^2 + \phi_{1y}^2 + \phi_{1z}^2) - \zeta_1 \frac{\partial}{\partial z}(\phi_{1xx} + K_0\phi_{1z})$$

(c) *Sea bottom condition:* The first and second order vertical velocity component on the sea bottom can be expressed as

$$\varepsilon \; : \nabla\phi_1.n = 0$$
$$\varepsilon^2 \; : \nabla\phi_2.n = 0 \qquad \text{at } z = -h \quad (5)$$

(d) *Radiation condition:* It is necessary to impose a condition to ensure that the free surface waves vanish upstream of the disturbance.

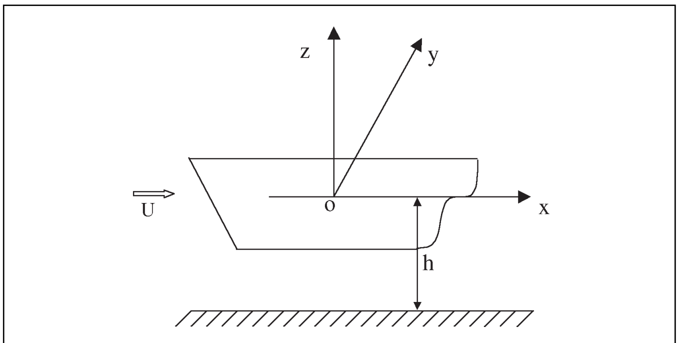


Figure 1: Definition sketch of the co-ordinate system

THE BOUNDARY ELEMENT METHOD

Applying Green's second identity Laplace's equation can be transformed into an integral equation as (see Curle & Davis, 1968) ;

$$4\pi E\phi(p) = \sum_{j=1}^{N_H} \int_{S_H} \phi(q)\frac{\partial G}{\partial n_q} dS - \sum_{j=1}^{N_H} \int_{S_H} \frac{\partial\phi(q)}{\partial n_q} G dS + \sum_{j=1}^{N_B} \int_{S_B} \phi(q)\frac{\partial G}{\partial n_q} dS - \sum_{j=1}^{N_F} \int_{S_F} \phi(q)\frac{\partial\phi(q)}{\partial n_q} G dS \quad (6)$$

$$\text{where, } E = \begin{cases} 1/2 \text{ on } S_H, S_B \\ 1 \text{ on } S_F \end{cases}$$

The Green's function G satisfies the Laplace equation and can be approximated (see Faltinsen, 1993) as [3];

$$G = \frac{1}{R(p;q)} + \frac{1}{R'(p;q)}$$

where R is the position vector between the field point p and the point of singularity q on the surface and R´ is its image. The surface S_∞ is a control surface at a large distance from the body and is chosen as the surface of a circular cylinder of large radius. So the integral over the surface S_∞ must be zero as the radius of cylinder increases infinitely.

The integral over the element in equation (6) is calculated by Morino's analytical expression (see Suciu and Marino, 1976) [19] based on the assumption of quadrilateral hyperboloid element. After satisfying the boundary conditions as stated in equations (3) to (5), the integral equation (6) can be written into a matrix form as:

$$[A]x = [B].$$

where [A] and [B] are the matrices built up by the Green's function and its derivatives, and x is the column matrix formed by the strength of the sources and dipoles respectively. The second derivatives of velocity potentials in the left side of free surface condition (4) are computed by Dawson's upstream finite difference operator (see Dawson, 1977) [2] in order to satisfy the radiation condition.

The derivatives of the velocity potentials (ϕx, ϕy) in right side of equation (4) are evaluated by fitting a second-degree polynomial function passing through the potentials at the centroid of three adjacent panels on the surfaces. The derivative of velocity potential along the z-direction is obtained after calculating the velocity potentials at three points on the normal vector. The matrix of linear system of equations is solved by LU decomposition method as described by Press et al. [18]. The advantage of using LU technique is that the authors only need to partition LU in one time for first order problem and subsequently, the authors shall apply it for higher order problems. Therefore, the CPU time can be saved.

CALCULATION OF WAVE PROFILE AND RESISTANCE

The linearized equation of wave profile for first and second order approximation can be obtained as

$$\zeta_1 = -\frac{U}{g}\phi_{1x} \tag{7}$$

$$\zeta_2 = -\frac{U}{g}\phi_{2x} - \frac{U}{g}\zeta_1\phi_{1xz} - \frac{1}{2g}(\phi_{1x}^2 + \phi_{1y}^2 + \phi_{1z}^2) \tag{8}$$

The wave resistance can be calculated by integration of pressure over the area of the hull up to the mean water level. After including the water line integral the wave making resistance can be obtained as below:

$$R_w = -\frac{1}{2}\int\left[\rho\,(U^2 - \nabla\Phi.\nabla\Phi) - \rho g z\right]dS - \frac{1}{2}\rho g\oint \zeta^2 n_x dl \tag{9}$$

RESULTS AND DISCUSSION

To investigate the shallow water effect on wave resistance and wave pattern around the ship-like body, the method has been tested for the Wigley hull. The equation of this type of hull surface (see Hofman and Kozarski, 2000) is

$$y(x,z) = \frac{B}{2}S(z)\left(1 - \frac{4x^2}{L^2}\right) \tag{10}$$

where B and T are the vessel width and draft respectively and $S(z)$ is a function defining the shape of the cross-sections. For rectangular (wall sided) cross-section $S(z) = 1$, for triangular cross-section $S(z) = 1+z/T$, for parabolic cross-sections $S(z) = 1-(z/T)^2$, etc. The symmetric parabolic ship section and parabolic waterlines are used in the present numerical treatment of the problem. The principal particulars of this model are shown in Table 1.

Since the body is symmetric, one-half of the computational domain is used for numerical treatment. The panels from 1.0 ship length upstream to 2.5 ship length downstream cover the free surface domain. The transverse extension of the free surface is about 1.6 ship length. The extent of the sea bottom is $-3 \le x \le 5$, $2 \le y \le 0$. The number of panels on the hull, free surface and sea bottom are taken 40x7, 70x15 and 40x10 respectively as shown in Figure 2. A three-point upstream difference operator is used in both longitudinal and transverse directions to advent disturbances in the downstream direction. The method employs a clustering of panels on the free surface and sea bottom respectively.

Table 1: Principal particulars of the Wigley hull

Parameter	Magnitude
B / L	0.10
T / L	0.0625
C_B, Block coefficient	0.444
$C_{M,}$ Mid ship coefficient	0.667

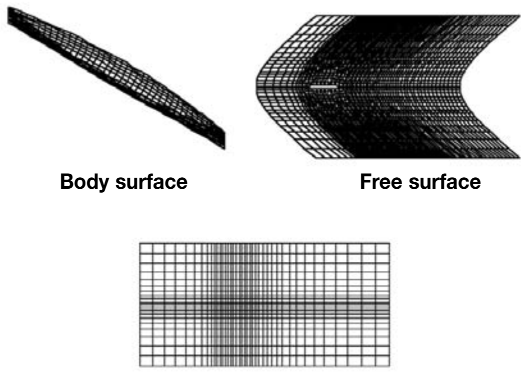


Figure 2: Panel arrangements for the Wigley model

Figure 3 presents a comparison of the wave profiles based on second order approximations with fixed sinkage and trim (abbreviated as 2nd, Fixed) in deep and shallow water at various ship speeds. The shallow water effect on the profiles becomes evident when the Froude number F_n, based on the length L of the model, reaches 0.332 and the differences between the wave profiles in deep and shallow water are mainly located at the middle and stern.

The calculated wave profiles at various water depth-to-draft ratios ($h/T = 2.4, 2.8$ & ∞) are also plotted in Figure 4 for a particular speed $F_n = 0.289$. The Froude number based on water depth is obtained from the relation, $F_h = F_n\sqrt{(L/h)}$. The two shallow water, $h/T = 2.4$ and 2.8 are subcritical depths ($Fh < 1.0$) with respect to the speed $F_n = 0.289$. The differences among the wave profiles in deep and shallow water become larger as the depth of the water decreases. The expected steeper bow waves are generated. This indicates that the second order nonlinear effect seems to be very significant for shallow water although the hull is quite slender.

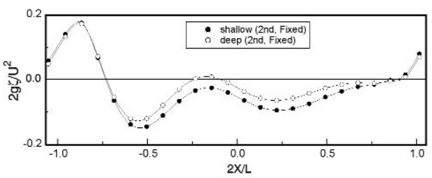
Figure 5 shows a comparison of wave making resistance based on first and second order approximation with fixed sinkage and trim in deep and shallow water. The general effect of shallow water is to cause an increase in resistance at the lower speeds compared with the deep-water value but a reduction in resistance at the higher speeds as might be expected from the previous work of Millward and Bevan [14]. Since the four shallow water curves correspond to different water depth/draft (h/T) ratios it can be also seen that the shallow water effect becomes more pronounced as the depth-to-draft ratio decreases, that is, as the water becomes shallower, although in the supercritical region the difference among the four depths of water is small.

The wave making resistance of the Wigley hull based on first and second order approximation at $h/T = 2.8$ are also plotted in Figure 6. The first order solution predicts the maximum wave making resistance not at the critical speed but at a slightly faster speed, $F_h = 1.004$ while the second order solution predicts the maximum resistance at $F_h = 0.98$.

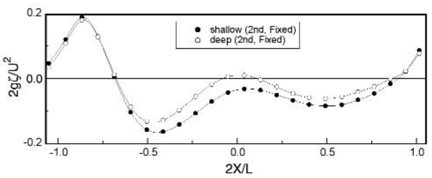
CONCLUSIONS

The present paper deals with the computation of free surface flow around a ship in shallow water using a potential based panel method. An attempt has been made to predict numerically the hydrodynamic characteristics of the Wigley hull in shallow water. The following conclusions can be drawn from the present numerical analysis:

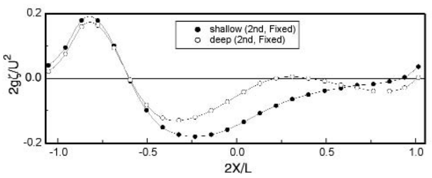
a) The wave making resistance in shallow water begins to increase appreciably as the critical speed ($F_h = 1.0$) is approached and then decreases.
b) The first order solution predicts the maximum resistance not at the critical speed but at a slightly higher speed ($F_h = 1.004$) while the second order solution predicts the maximum wave making resistance at a slightly lower speed ($F_h = 0.98$).
c) The second order solution significantly improves the wave profiles particularly at the bow and the first trough but after that the difference between the first and second order results seems to be insignificant.
d) In lower water depths the water surface in the midship region is remarkably displaced downward for all velocities.



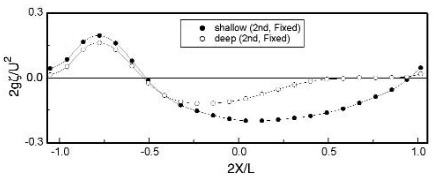
(a) Wave profile at $F_n = 0.26$



(b) Wave profile at $F_n = 0.289$

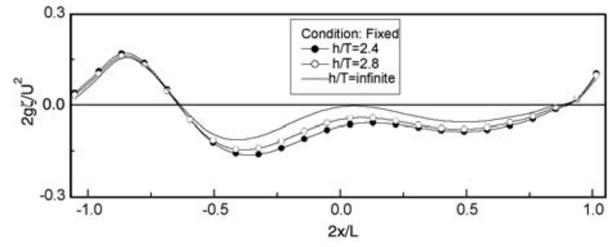


(c) Wave profile at $F_n = 0.332$

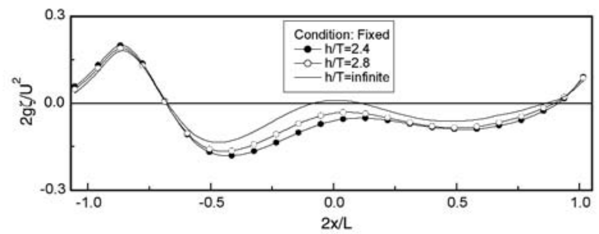


(d) Wave profile at $F_n = 0.369$

Figure 3: Wave profiles (2nd, Fixed) of the Wigley hull in deep and at h/T = 2.8

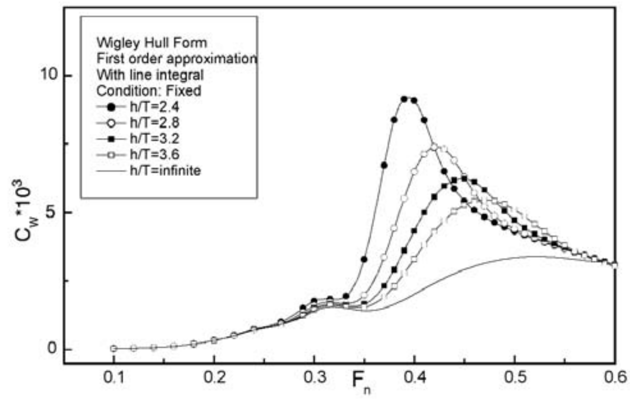


(a) First order approximation

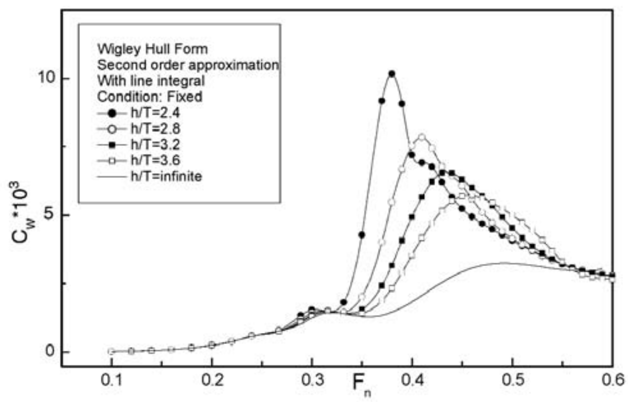


(b) Second order approximation

Figure 4: Comparison of calculated wave profiles in deep and subcritical depths for the Wigley hull at $F_n = 0.289$



(a) First order approximation



(b) Second order approximation

Figure 5: Comparison of computed wave making resistance of the Wigley hull in deep and shallow water

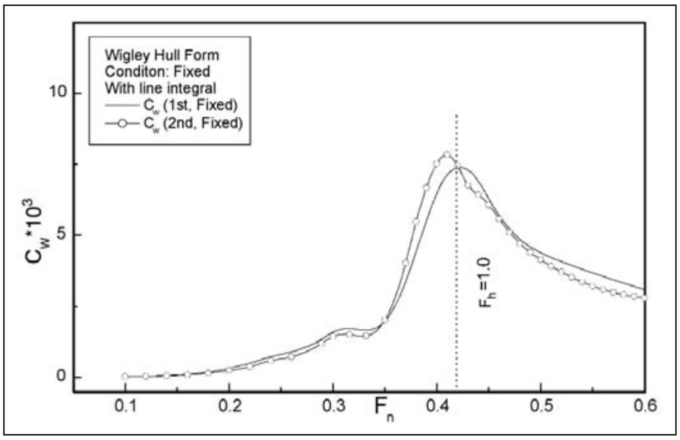


Figure 6: Wave making resistance of the Wigley hull at h/T = 2.8

REFERENCES

[1] Curle, N. and Davis, H. J. (1968), *Modern Fluid Dynamics*, Vol. 1, D. Van Nostrand Company Ltd., London, pp.108-111.

[2] Dawson, C. W. (1977), A practical computer method for solving ship-wave problems, *Proceedings of Second International Conference on Numerical Ship Hydrodynamics*, pp.30-38.

[3] Faltinsen, O. M. (1993), *Sea Loads on Ships and Offshore Structures*, Cambridge University Press, pp.110-122.

[4] Havelock, T. H. (1908), The propagation of groups of waves in dispersive media with application to waves on water produced by a traveling disturbance, *Proceedings of the Royal Society*, London, Vol. 81.

[5] Havelock, T. H. (1922), The effect of shallow water on wave resistance, *Proceedings of the Royal Society*, A, Vol. 100, pp.499-505.

[6] Hofman, M. and Kozarski, V. (2000), Shallow water resistance charts for preliminary vessel design, *International Shipbuilding Progress*, Vol. 47(449), pp.61-76.

[7] Inui, T. (1954), Wave making resistance in shallow sea and in restricted water with special reference to its discontinuities, *Journal of the Society of Naval Architects of Japan*, Vol. 76, pp.1-10.

[8] Kim, K, Choi, Y. Jansson, C. and Larsson, L. (1996), Linear and nonlinear calculations of the free surface potential flow around ships in shallow water, *20th Symposium on Naval Hydrodynamics*, pp.408-425.

[9] Kinoshita, M. & Inui, T. (1953), Wave making resistance of submerged spheroid ellipsoid and a ship in shallow sea, *Journal of the Society of Naval Architects of Japan*, Vol. 75, pp.119-135.

[10] Kirsch, M. (1966), Shallow water and channel effects on wave resistance, *Journal of Ship Research*, pp.164-181.

[11] Lee, S. J. (1992), Computation of wave resistance in the water of finite depth using a panel method, *The Journal SNAK92*, pp.130-135.

[12] Maruo, H. (1966), A note on the higher order theory of thin ships, *Bulletin of the Faculty of Engineering*, Yokohama National University, Vol. 15, pp.1-21.

[13] Maruo, H. and Tachibana, T. (1981), An investigation into the sinkage of a ship at the transcritical speed in shallow water, *Journal of the Society of Naval Architects of Japan, Vol. 150*, pp.56- 62.

[14] Millward, A. and Bevan, M. G. (1986), Effect of shallow water on a mathematical hull at high subcritical and supercritical speeds, *Journal of Ship Research, Vol. 30,* No.2, pp.85-93.

[15] Monacella, V. J. (1964), The disturbance due to a slender ship oscillating in waves in a fluid of finite depth, *Journal of Ship Research*, pp. 242-252.

[16] Muller, E. (1985), Analysis of the potential flow field and of ship resistance in water of finite depth, *International Shipbuilding Progress, Vol.32*, No.376, pp.266-277.

[17] Pettersen, B. (1982), Calculation of potential flow about three dimensional bodies in shallow water with particular application to ship maneuvering, *Journal of Ship Research, Vol.26,* No. 3, pp.149-165.

[18] Press, W. H., Teukolsky, S. A., Vetterling, W. T. and Flannery, B. P. (1999), *Numerical Recipes in Fortran 77: The Art of Scientific Computing, Vol. 1,* Cambridge University Press, pp.35-40.

[19] Suciu, E. O. and Morino L. (1976), A nonlinear finite element analysis for wings in steady incompressible flows with wake roll-up, AIAA Paper, No. 76-64, pp.1-10.

[20] Tarafder, M. S. (2002), Computation of Wave Making resistance of Ships in Shallow Water Using a Potential Based Panel Method, Doctoral Thesis, Department of Naval Architecture and Ocean Engineering, Yokohama National University, Japan.

[21] Tuck, E. O. and Taylor, P. J. (1970), Shallow wave problems in ship hydrodynamics, *8th Symposium on Naval Hydrodynamics*, pp. 627-658.

[22] Xinmin, X. and Xiuheng, W. (1996), A study on manoeuvring hydrodynamic forces acting on 3-d ship hulls with free surface effect in restricted water, *International Shipbuilding Progress, Vol.43,* No.433, pp.48-69.

[23] Yasukawa, H (1989), Calculation of free-surface flow around a ship in shallow water by rankine source method, *5th International Conference on Numerical Ship Hydrodynamics,* pp.643-653.

NOMENCLATURE

B	breadth of the ship model
C_w	wave making co-efficient
F_h	depth based Froude number
F_n	length based Froude number
g	acceleration due to gravity
G	Green's function
h	depth of water
K_0	wave number
L	length of the ship model
N_H	number of panels on the hull surface
N_F	number of panels on the free surface
N_B	number of panels on the bottom surface
S_H	hull surface
S_F	free surface
S_B	bottom surface
S_∞	surface at infinity
T	draft of the ship model
U	uniform velocity in the positive-x direction
Φ	total velocity potential
ϕ	perturbation velocity potential due to presence of the body
ϕ_x	velocity of the fluid in the x-direction
ϕ_y	velocity of the fluid in the y-direction
ϕ_z	velocity of the fluid in the z-direction
ϕ_1	first order perturbation velocity potential
ϕ_2	second order contributory part for perturbation velocity potential
ζ	wave elevation
ζ_1	first order wave elevation
ζ_2	second order contributory part for wave elevation